

# CURRICULUM VITAE

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## FIELDS OF INTEREST:

Differential Equations  
Spectral Theory  
Complex Analysis  
Mathematical Physics

## ACADEMIC POSITIONS:

2006– Assistant Professor, University of West Georgia, Carrollton, Georgia.  
2002–2006 Post Doctoral Fellow, University of Missouri, Columbia, Missouri.  
1998–2001 Teaching/Research Assistant, University of Illinois.  
1997–1998 Teaching Assistant, Wayne State University.

## EDUCATION:

1998–2002 Ph.D., Mathematics, University of Illinois, Urbana-Champaign.  
Advisor: Professor Richard S. Laugesen.  
Thesis: On some Schrödinger eigenvalue problems from mathematical physics.  
1996–1998 Graduate program, Mathematics, Wayne State University, Detroit, Michigan.  
1995–1996 Language Training, Language Center International, Southfield, Michigan.  
1993–1995 M.S., Mathematics, Chonnam National University, Kwang-Ju, South Korea.  
Advisor: Professor Dong-Soo Kim.  
Thesis: Submanifolds with constant mean curvature vector fields.  
1985–1992 B.S., Mathematics, Chonnam National University, Kwang-Ju, South Korea.  
Interrupted by 32 month mandatory military service, Nov. 1987 to Jun. 1990.

## AWARDS and HONORS:

- May 2006 Travel Award for attending "Workshop on Low eigenvalues of Laplace and Schrodinger operators", American Institute of Mathematics, Palo Alto, CA.
- Mar 2006 Travel Award for attending "Spectral Theory and Mathematical Physics" Conference in Honor of Barry Simon's 60th Birthday, Pasadena, California."
- July 2004 Travel Award for attending "Workshop on Spectral Theory of Schrödinger Operators," Montréal, Canada.
- 2002 University Fellowship, University of Illinois.
- 2001 Hohn-Nash Award in Mathematics, University of Illinois, which is given "in recognition of outstanding scholarship in applied mathematics."
- 1999–2001 Summer Research Assistantship, University of Illinois.
- 1997 Paul Catlin Endowed Mathematics Scholarship, Wayne State University.

## PUBLICATIONS and PREPRINTS:

- [1] S. H. Ahn, D. S. Kim and K. C. Shin, *Submanifolds with constant mean curvature vector fields*, Honam Mathematical Journal, 17: 49–55, 1995.
- [2] K. C. Shin, *On the eigenproblems of  $\mathcal{PT}$ -symmetric oscillators*, Journal of Mathematical Physics, 42: 2513–2530, 2001.
- [3] K. C. Shin, *On the reality of the eigenvalues for a class of  $\mathcal{PT}$ -symmetric oscillators*, Communications in Mathematical Physics, 229: 543–564, 2002.
- [4] K. C. Shin, *New polynomials  $P$  for which  $f'' + P(z)f = 0$  has a solution with almost all real zeros*, Annales Academiæ Scientiarum Fennicæ Mathematica, 27: 491–498, 2002.
- [5] K. C. Shin, *On Half-Line Spectra for a Class of Non-Self-Adjoint Hill Operators*, Mathematische Nachrichten, 261-262: 171–175, 2003.
- [6] K. C. Shin, *Trace Formulas for Non-Self-Adjoint Periodic Schrödinger Operators and some Applications*, Journal of Mathematical Analysis and Applications, 299: 19–39, 2004.
- [7] K. C. Shin, *On the shape of spectra for non-self-adjoint periodic Schrödinger operators*, Journal of Physics A: Mathematical and General, 37: 8287–8291, 2004.
- [8] K. C. Shin, *Eigenvalues of  $\mathcal{PT}$ -symmetric oscillators with polynomial potentials*, Journal of Physics A: Mathematical and General, 38: 6147–6166, 2005.
- [9] K. C. Shin, *The potential  $(iz)^m$  generates real eigenvalues only, under symmetric rapid decay boundary conditions*, Journal of Mathematical Physics, 46(8): 082110, 17 pages, 2005.
- [10] K. C. Shin, *Schrödinger type eigenvalue problems with polynomial potentials: Asymptotics of eigenvalues*, Preprint, 32 pages, 2004. [math.SP/0411143](#).
- [11] K. C. Shin, *Half-line non-self-adjoint Schrödinger operators with polynomial potentials: Asymptotics of eigenvalues*, Preprint, 15 pages, 2005. [math.SP/0502522](#).

Papers can be downloaded from [www.math.missouri.edu/~kshin/](http://www.math.missouri.edu/~kshin/).

## WORK in PROGRESS:

- [12] K. C. Shin, *Anharmonic oscillators in the complex plane*, in preparation.

## INVITED CONFERENCE TALKS:

- Jan. 2006, Special Session on Value Distribution Theory in Classical and p-Adic Function Theory, AMS and MAA Joint Meeting, San Antonio, Texas, *Eigenvalues of non-self-adjoint Schrödinger operators with polynomial potentials*.
- Oct. 2004, Special Session on Spectral Problems of Differential Operators, AMS Sectional Meeting, Evanston, Illinois, *Asymptotic expansions of the eigenvalues of anharmonic oscillators*.
- Jan. 2004, Special Session on Value Distribution Theory in Classical and p-Adic Function Theory, AMS and MAA Joint Meeting, Phoenix, Arizona, *Trace formulas for non-self-adjoint periodic Schrödinger operators and some applications*.
- Jan. 2001, Special Session on Functional Equations, AMS and MAA Joint Meeting, New Orleans, Louisiana, *On the eigenproblems of  $\mathcal{PT}$ -symmetric oscillators*.

## SEMINAR TALKS:

- Feb. 2006, University of West Georgia, *Eigenvalues of Schrödinger operators with a polynomial potential: Asymptotics of eigenvalues*.
- Jan. 2006, PDE Seminar, University of Missouri, *Eigenvalues of Schrödinger operators with a polynomial potential: Asymptotics of eigenvalues*.
- Jun. 2005, Georgia State University, *The Schrödinger type eigenvalue problem in the complex plane with a polynomial potential: Asymptotics of eigenvalues*.
- Oct. 2004, PDE Seminar, University of Missouri, *The Schrödinger type eigenvalue problem in the complex plane with a polynomial potential: Asymptotics of eigenvalues*.
- Oct. 2004, Analysis Seminar, University of Illinois, *The Schrödinger equations in the complex plane with polynomial potentials: Asymptotics of eigenvalues*.
- Jun. 2004, Chonnam National University, South Korea, *Some Schrödinger eigenvalue problems from mathematical physics*.
- Jun. 2004, Yonsei University, South Korea, *Reality of eigenvalues of some “non-standard” Schrödinger operators*.
- Mar. 2004, Analysis Seminar, Washington University, St. Louis, Missouri, *Reality of eigenvalues for certain non-self-adjoint  $\mathcal{PT}$ -symmetric oscillators*.
- Sep. 2003, PDE Seminar, University of Missouri, *On half-line spectra for a class of non-self-adjoint Hill operators*.
- May 2003, Applied Mathematics Seminar (Math 488), University of Missouri, *Floquet and spectral theory for periodic Schrödinger operators*.
- Sep. 2002, Analysis Seminar, University of Missouri, *On the reality of the eigenvalues for a class of  $\mathcal{PT}$ -symmetric oscillators*.
- Nov. 2000, Analysis Seminar, University of Illinois, *On the eigenproblems of  $\mathcal{PT}$ -symmetric oscillators*.

## TEACHING EXPERIENCE:

- 2002–2006, Postdoctoral Fellow, Department of Mathematics, University of Missouri, Columbia, Missouri.
  - I have had full responsibility for teaching 2 courses per semester, which include Elementary Differential Equations, Calculus I, II, and III, Discrete Mathematics.

- 1998–2001, Teaching Assistant, Department of Mathematics, University of Illinois, Urbana-Champaign, Illinois.
  - I have had full responsibility for teaching Calculus for Social Scientists, Small Group Learning Calculus, Calculus II.
- 1997–1998, Teaching Assistant, Department of Mathematics, Wayne State University, Detroit, Michigan.
  - I have had full responsibility for teaching Beginning and Intermediate Algebra.

#### MEMBERSHIP:

American Mathematical Society, Mathematical Association of America

#### REFERENCES:

- Professor Mark Ashbaugh, University of Missouri, Columbia, [mark@math.missouri.edu](mailto:mark@math.missouri.edu)
- Professor Fritz Gesztesy, University of Missouri, Columbia, [fritz@math.missouri.edu](mailto:fritz@math.missouri.edu) (post doctoral advisor)
- Professor Richard Laugesen, University of Illinois, Urbana-Champaign, [laugesen@math.uiuc.edu](mailto:laugesen@math.uiuc.edu) (thesis advisor)
- Professor Dorina Mitrea, University of Missouri, Columbia, [dorina@math.missouri.edu](mailto:dorina@math.missouri.edu) (teaching)
- Professor John Rossi, Virginia Polytechnic Institute and State University, [rossi@math.vt.edu](mailto:rossi@math.vt.edu)

#### PUBLICATIONS and PREPRINTS with ABSTRACTS:

Papers can be downloaded from [www.math.missouri.edu/~kcshin/](http://www.math.missouri.edu/~kcshin/).

- [1] S. H. Ahn, D. S. Kim and K. C. Shin, *Submanifolds with constant mean curvature vector fields*, Honam Mathematical Journal, 17: 49–55, 1995.
- [2] K. C. Shin, *On the eigenproblems of  $\mathcal{PT}$ -symmetric oscillators*, Journal of Mathematical Physics, 42: 2513–2530, 2001.

**Abstract** We consider the non-Hermitian Hamiltonian  $H = -\frac{d^2}{dx^2} - (ix)^{2n+1} + P(x^2)$  on the real line, where  $P(x)$  is a polynomial of degree at most  $n \geq 1$  with all nonnegative real coefficients (possibly  $P \equiv 0$ ). We impose zero boundary conditions at  $\pm\infty$ . It is proved that the eigenvalues  $\lambda$  must be in the sector  $|\arg \lambda| \leq \frac{\pi}{2n+3}$ . (This result was later improved to  $\arg \lambda = 0$  in Paper [3].) Also for the case  $H = -\frac{d^2}{dx^2} - (ix)^3$ , we establish a zero-free region of the eigenfunction  $u$  and its derivative  $u'$  and we find some other interesting properties of eigenfunctions.

A Hamiltonian  $H = -\frac{d^2}{dz^2} + V(z)$  is called  $\mathcal{PT}$ -symmetric if  $\overline{V(-\bar{z})} = V(z)$ . These  $\mathcal{PT}$ -symmetric Hamiltonians have been considered by many physicists in recent years, in particular by Bender.

- [3] K. C. Shin, *On the reality of the eigenvalues for a class of  $\mathcal{PT}$ -symmetric oscillators*, Communications in Mathematical Physics, 229: 543–564, 2002.

**Abstract** We study the eigenvalue problem  $-u''(z) + V(z)u(z) = \lambda u(z)$  with the boundary conditions that  $u(z)$  decays to zero as  $z$  tends to infinity along the rays  $\arg z = -\frac{\pi}{2} \pm \frac{2\pi}{m+2}$ , where  $V(z) = -[(iz)^m + P(iz)]$  and  $P(z) =$

$a_1z^{m-1} + a_2z^{m-2} + \dots + a_{m-1}z$  is a real polynomial, with  $m \geq 2$ . We prove that the eigenvalues are all positive real if for some  $1 \leq j \leq \frac{m}{2}$ , we have  $(j - k)a_k \geq 0$  for all  $k$ . We then sharpen this to a slightly larger class of polynomial potentials.

In particular, this implies that the eigenvalues are all positive real for the potentials  $V(z) = \alpha iz^3 + \beta z^2 + \gamma iz$  when  $\alpha, \beta, \gamma \in \mathbb{R}$  with  $\alpha \neq 0$  and  $\alpha \cdot \gamma \geq 0$ , and with the boundary conditions that  $u(z)$  decays to zero as  $z$  tends to infinity along the positive and negative real axes. This verifies the original conjecture of Bessis and Zinn-Justin.

- [4] K. C. Shin, *New polynomials  $P$  for which  $f'' + P(z)f = 0$  has a solution with almost all real zeros*, *Annales Academiæ Scientiarum Fennicæ Mathematica*, 27: 491–498, 2002.

**Abstract** Let  $a, b, c \in \mathbb{R}$ ,  $a \neq 0$  with  $ac \leq 0$ . We prove that there exists a sequence of positive real numbers  $\mu_k \rightarrow \infty$  such that for each  $k$ , the equation  $f''(z) + (az^3 + bz^2 + cz - \mu_k)f(z) = 0$  admits a solution with infinitely many real zeros and at most finitely many non-real zeros. This gives a new class of cubic polynomials  $P$  for which  $f'' + P(z)f = 0$  has a solution with almost all real zeros. We also find new quartic examples: for each  $a > 0$  and  $b \in \mathbb{R}$ , there exists a sequence of real numbers  $\mu_k \rightarrow \infty$  such that for each  $k$ ,  $f''(z) + (az^4 + bz^2 - \mu_k)f(z) = 0$  has a solution with almost all real zeros. The case  $a > 0$  and  $b > 0$  was discovered earlier by Gundersen.

- [5] K. C. Shin, *On Half-Line Spectra for a Class of Non-Self-Adjoint Hill Operators*, *Mathematische Nachrichten*, 261-262: 171–175, 2003.

**Abstract** In 1980, Gasymov showed that non-self-adjoint Hill operators with complex-valued periodic potentials of the type  $V(x) = \sum_{k=1}^{\infty} a_k e^{ikx}$ , with  $\sum_{k=1}^{\infty} |a_k| < \infty$ , have spectra  $[0, \infty)$ . In this note, we provide an alternative and elementary proof of this result.

- [6] K. C. Shin, *Trace Formulas for Non-Self-Adjoint Periodic Schrödinger Operators and some Applications*, *Journal of Mathematical Analysis and Applications*, 299: 19–39, 2004.

**Abstract** Recently, a trace formula for non-self-adjoint periodic Schrödinger operators in  $L^2(\mathbb{R})$  associated with Dirichlet eigenvalues was proved by Gesztesy. Here we prove a corresponding trace formula associated with Neumann eigenvalues.

In addition we investigate Dirichlet and Neumann eigenvalues of such operators. In particular, using the Dirichlet and Neumann trace formulas we provide detailed information on location of the Dirichlet and Neumann eigenvalues for the model operator with the potential  $Ke^{2ix}$ ,  $K \in \mathbb{C}$ .

- [7] K. C. Shin, *On the shape of spectra for non-self-adjoint periodic Schrödinger operators*, *Journal of Physics A: Mathematical and General*, 37: 8287–8291, 2004.

**Abstract** The spectra of Schrödinger operators with periodic potentials are studied. When the potential is real and periodic, the spectrum consists of at most countably many line segments (energy bands) on the real line, while when the potential is complex and periodic, the spectrum consists of at most countably many analytic arcs in the complex plane.

In some recent papers, such operators with complex  $\mathcal{PT}$ -symmetric periodic potentials are studied. In particular, the authors argued that some energy bands would appear and disappear under perturbations. Here, we

show that appearance and disappearance of such energy bands imply existence of nonreal spectra. This is a consequence of a more general result, describing the local shape of the spectrum.

- [8] K. C. Shin, *Eigenvalues of  $\mathcal{PT}$ -symmetric oscillators with polynomial potentials*, Journal of Physics A: Mathematical and General, 38: 6147–6166, 2005.

**Abstract** We study the eigenvalue problem  $-u''(z) - [(iz)^m + P_{m-1}(iz)]u(z) = \lambda u(z)$  with the boundary condition that  $u(z) \rightarrow 0$  as  $z \rightarrow \infty$  along the rays  $\arg z = -\frac{\pi}{2} \pm \frac{2\pi}{m+2}$ , where  $P_{m-1}$  is a polynomial of degree  $\leq m-1$  and  $m \geq 3$ . We provide an asymptotic expansion of the eigenvalues  $\lambda_n$  as  $n \rightarrow +\infty$ , and prove that for each *real* polynomial  $P_{m-1}$ , all but finitely many eigenvalues are real and positive.

This paper treats the case  $\ell = 1$  of Paper [10], and provides the induction basis for the proof in that paper. The proof of the main theorem in Paper [10] is a result of investigating the asymptotics of entire functions (Stokes multipliers) whose zeros are the eigenvalues, where I used induction on  $\ell$ .

- [9] K. C. Shin, *The potential  $(iz)^m$  generates real eigenvalues only, under symmetric rapid decay boundary conditions*, Journal of Mathematical Physics, 46(8): 082110, 17 pages, 2005.

**Abstract** We consider the non-Hermitian eigenvalue problems  $-u''(z) \pm (iz)^m u(z) = \lambda u(z)$ ,  $m \geq 3$ , under every rapid decay boundary condition that is symmetric with respect to the imaginary axis in the complex  $z$ -plane. We prove that the eigenvalues  $\lambda$  are all real and positive, with no exceptions.

- [10] K. C. Shin, *Schrödinger type eigenvalue problems with polynomial potentials: Asymptotics of eigenvalues*, Preprint, 32 pages, 2004. [math.SP/0411143](#).

**Abstract** For integers  $m \geq 3$  and  $1 \leq \ell \leq m-1$ , we study the eigenvalue problem  $-u''(z) + [(-1)^\ell (iz)^m - P(iz)]u(z) = \lambda u(z)$  with the boundary conditions that  $u(z)$  decays to zero as  $z$  tends to infinity along the rays  $\arg z = -\frac{\pi}{2} \pm \frac{(\ell+1)\pi}{m+2}$  in the complex plane, where  $P$  is a polynomial of degree  $\leq m-1$ . We provide asymptotic expansions of the eigenvalue counting function and the eigenvalues  $\lambda_n$ . We apply these expansions to the inverse spectral problem, reconstructing some coefficients of  $P$  from asymptotic expansions of the eigenvalues. Also, we show when  $m \geq 3$  and  $P$  has real coefficients (the  $\mathcal{PT}$ -symmetric situation) that under all symmetric decaying boundary conditions the eigenvalues are all real and positive, with only finitely many exceptions.

- [11] K. C. Shin, *Half-line non-self-adjoint Schrödinger operators with polynomial potentials: Asymptotics of eigenvalues*, Preprint, 15 pages, 2005. [math.SP/0502522](#).

**Abstract** For integers  $m \geq 3$ , we study the non-self-adjoint eigenvalue problems  $-u''(x) + (x^m + P(x))u(x) = Eu(x)$ ,  $0 \leq x < +\infty$ , with the boundary conditions  $u(+\infty) = 0$  and  $\alpha u(0) + \beta u'(0) = 0$  for some  $\alpha, \beta \in \mathbb{C}$  with  $|\alpha| + |\beta| \neq 0$ , where  $P(x) = a_1 x^{m-1} + a_2 x^{m-2} + \dots + a_{m-1} x$  is a polynomial. We provide asymptotic expansions of the eigenvalue counting function and the eigenvalues  $E_n$ . Then we apply these to the inverse spectral problem, reconstructing some coefficients of polynomial potentials from asymptotic expansions of the eigenvalues.