MATH1051 Semester 1 - 2001

Solutions to the Assignment One Questions

5. If $C(t)$ is the amount of Carbon C_{14} left after time t, we have approximately $C(t) = Ce^{-kt}$ where C is the original amount of C_{14} present and k is the decay constant. The half-life, T, is the time for the material to decay to one half of its initial amount. If $k > 0.0001$, what inequality must T satisfy?

Solution: When $C(t) = C/2$ we have $t = T$. So $C/2 = Ce^{-kT}$ or $1/2 = e^{-kT}$. So $\ln(1/2) = \ln(e^{-kT})$ or $-\ln 2 = -kT$. Therefore, $k = \ln 2/T$. Now since $k > 0.0001$ we have $\ln 2/T > 0.0001$. That is $T < 10^4(\ln 2) = 6931$ years.

6. If $y = sin(x)$, $0 \leq x < \pi/2$ find $tan(x)$ as a function of y.

Solution: First note that $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{y}{\cos x}$. Now $\sin^2 x + \cos^2 x = 1$ implies $\cos x = \pm\sqrt{1 - \sin^2}$. Since $0 \leq x < \pi/2$, we have $\cos x > 0$. So $\cos x = \sqrt{1 - \sin^2 x} = \sqrt{1 - y^2}$. Therefore, $\tan x = \dfrac{y}{\sqrt{1 - y^2}}$. Note that since $x \neq \pi/2$ we have $y \neq 1$.