MATH1061 Semester 2, 2002 Solution Ass. 10

Exercise Set 6.3

Q13 $(26+26.37+26\cdot 37^2+\ldots+26.37^{29})-82=26(1+37+37^2+\ldots+37^{29})-82=$ $26\left(\sum_{k=0}^{29}37^k\right)-82=26\left(\frac{37^{30}-1}{37-1}\right)-82.$

Q16 b) Let S denote the sample space and $A,B\subseteq S$. Then

$$\begin{aligned} LHS &= P(A\cup B)=\frac{n(A\cup B)}{n(S)} \\ RHS &= P(A)+P(B)-P(A\cap B)=\frac{n(A)}{n(S)}+\frac{n(B)}{n(S)}-\frac{n(A\cap B)}{n(S)}. \end{aligned}$$

And since $n(A\cap B)=n(A)+n(B)-n(A\cup B)$

$$\begin{aligned} RHS &= \frac{n(A)}{n(S)}+\frac{n(B)}{n(S)}-\frac{n(A)+n(B)-n(A\cup B)}{n(S)} \\ &= \frac{n(A\cup B)}{n(S)}. \end{aligned}$$

Therefore $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ as required.

Q20 a) Total number of n bit strings is 4^n. Number of strings with all consecutive bits different is $4.3.3\ldots 3=4.3^{n-1}$. So the number of strings with at least two consecutive bits the same is $4^n-4.3^{n-1}$.

b) When $n=10$ we obtain $\frac{4^{10}-4.3^9}{4^{10}}=1-(\frac{3}{4})^9\approx 92.5\%$.

Exercise Set 6.4

Q7 a) $\binom{14}{7}=\frac{14\times 13\times 12\times 10\times 9\times 8}{7\times 6\times 5\times 4\times 3\times 2\times 1}=3432.$

b) i) $\binom{8}{4}\binom{6}{3}=\frac{8\times 7\times 6\times 5}{4\times 3\times 2\times 1}\times\frac{6\times 5\times 4}{3\times 2\times 1}=1400.$

ii) No man is $\binom{8}{7}$ and so at least one man is $\binom{14}{7}-\binom{8}{7}=3432-8=3424.$

iii) Exactly one woman is $\binom{8}{1}\binom{6}{6}$, exactly two women is $\binom{8}{2}\binom{6}{5}$ and exactly three women is $\binom{8}{3}\binom{6}{4}$. So at most three women is

$$\binom{8}{1}\binom{6}{6}+\binom{8}{2}\binom{6}{5}+\binom{8}{3}\binom{6}{4}=1016.$$

c) $2\binom{12}{6}+\binom{12}{7}=2640.$

d) $\binom{12}{5}+\binom{12}{7}=1584.$

Q20 a) $\binom{12}{3}\binom{9}{2}\binom{7}{2}\binom{5}{2}\binom{3}{1}\binom{2}{1}\binom{1}{1} = 9979200$

b) $\binom{10}{3}\binom{7}{2}\binom{5}{2}\binom{3}{2}\binom{1}{1} = 75,600$

c) $\binom{9}{3}\binom{6}{2}\binom{4}{1}\binom{3}{1}\binom{2}{1}\binom{1}{1} = 30,240$

Exercise Set 6.5

Q6 Calculate the total number of possible 5 tuples with repetition allowed but discounting order. Then reorganise the tuples so that the co-ordinates are in descending order. This gives all possible 5 tuples with the required format. There are $\binom{5+n-1}{5}$ such tuples.

Q12 $\binom{4+32-1}{32} = 6545$.

Exercise Set 7.1

Q2 a) The domain is $\{1, 3, 5\}$ and the co-domain is $\{s, t, u, v\}$.

b) $g(1) = t$, $g(3) = t$, $g(5) = t$.

c) The range is $\{t\}$.

d) The inverse image of t is $\{1, 3, 5\}$.
The inverse image of u is $\emptyset$.

e) $\{(1, t), (3, t), (5, t)\}$

Q10 a) $i_z(e) = e$, b) $i_z(b_i^{jk}) = b_i^{jk}$, c) $i_z(K(t)) = K(t)$.

Q12 b) $F(\phi) = 0$, d) $F(\{2, 3, 4, 5\}) = 0$.