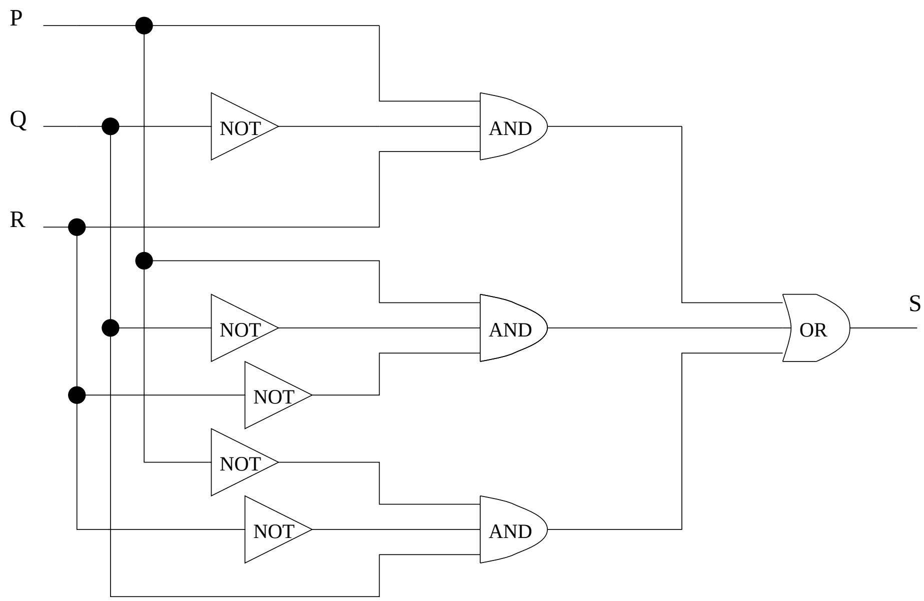
MASTH1061 Semester 2, 2002 Solution Ass. 2

Exercise Set 1.4

Q19 $(P \wedge \sim Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R) \vee (\sim P \wedge Q \wedge \sim R)$ (Note that this can be simplified to $(P \wedge \sim Q) \vee (\sim P \wedge Q \wedge \sim R)$.)



Exercise Set 2.1

Q15d $\forall x, y \in \mathbb{Z}$ if x and y are odd then $x.y$ is odd.

Q16b $\forall x$, if x is a computer science student, then x needs to take assembly language programming. $\forall$ computer science students x, x needs to take assembly language programming.

Q28d True. If it were false we would need to exhibit a number belonging to D which is of the form $y2$, where y is replaced by a digit other than 3 or 4. But the only element of D which is of the form $y2$ is 32. So we cannot find a counter example. Thus the statement is true.

Q32 Let p represent the statement n *is prime*, o represent the statement n *is odd*, t represent the statement $n = 2$. Then the statement form is $p \to (o \vee t)$

The negation of this statement form is

$$\sim (p \to (o \lor t)) \equiv \sim (\sim p \lor (o \lor t)) \equiv p \land \sim (o \lor t)$$
$$\equiv p \land \sim o \land \sim t$$

Thus the negation is

$\exists n \in \mathbb{Z}$, such that n is prime and n is not odd and $n \neq 2$.

Exercise Set 2.2

Q4 There exists a book, which all people have read.

Negation: $\forall$ books b, $\exists$ a person p such that p has not read b.

Equivalently: Given any book, there exists a person who has not read that book.

Q8 There exists a real number x such that for all real numbers $y, x + y = 0$.

Negation: $\forall x \in \mathbb{R}$, $\exists y \in \mathbb{R}$ such that $x + y \neq 0$

Equivalently: Given any real number x there exists a real number y such that $x + y \neq 0$

Q44 b) Let $P(x)$ be the statement that x is positive and $Q(x)$ be the statement that x is negative.

Then $\exists x \in \mathbb{R}$, $(P(x) \land Q(x))$ means there exists a real number which is both positive and negative. This is a false statement (note 0 is neither positive or negative). But $(\exists x \in \mathbb{R},\ P(x)) \land (\exists x \in \mathbb{R},\ Q(x))$ means there exists a real number which is positive and there exists a real number which is negative. This is a true statement, for example, 2 is positive and -100 is negative. Hence these two statements do not have the same truth values.