MATH1061 Semester 2, 2002 Solution Ass. 4

Exercise Set 3.5

Q28 Let $n = 2m + 1$ where $m \in \mathbb{Z}$. Then

$$\frac{n^2}{4} = \frac{(2m+1)^2}{4} = \frac{4m^2+4m+1}{4} = m^2 + m + \frac{1}{4}.$$

Hence $\lfloor \frac{n^2}{4} \rfloor = m^2 + m = m(m+1)$. And

$$\frac{(n-1)}{2}\frac{(n+1)}{2} = \left(\frac{2m+1-1}{2}\right)\left(\frac{2m+1+1}{2}\right) = \left(\frac{2m}{2}\right)\left(\frac{2m+2}{2}\right) = m(m+1).$$

Thus $\lfloor \frac{n^2}{4} \rfloor = \left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)$ whenever n is odd.

Exercise Set 3.6

Q14 First we will prove this result by contradiction. Assume that $a \mid b$ and $a \nmid c$, but $a|(b+c)$. Then we have $b = ak$ and $b + c = am$, where $k, m \in \mathbb{Z}$. Now substituting $b = ak$, into $b + c = am$, gives $ak + c = am$ or $c = am - ak = a(m-k)$. Since $(m-k) \in \mathbb{Z}$, $a \mid c$. But this is a contradiction as we assumed that $a \nmid c$. So if $a \mid b$ and $a \nmid c$ then $a \nmid (b+c)$.

Using the hint given on the question paper we see that to prove the contrapositive we need to prove that

If not $a \nmid (b+c)$ and $a \mid b$ then not $a \nmid c$, or in other words

If $a \mid (b+c)$ and $a \mid b$ then $a \mid c$.

Which is precisely what we showed above.

Exercise Set 3.8

Q14

3510	$= 5 \times 672 + 150$	150	$= 3510 - 5 \times 672$	$-(1)$
672	$= 4 \times 150 + 72$	72	$= 672 - 4 \times 150$	$-(2)$
150	$= 2 \times 72 + 6$	6	$= 150 - 2 \times 72$	$-(3)$
72	$= 12 \times 6.$			

So $gcd(3510, 672) = 6$

Starting with equation (3) we have

$6 = 150 - 2 \times 72,$	and substituting values from eqn (2) gives
$6 = 150 - 2(672 - 4 \times 150).$	So
$6 = 9 \times 150 - 2 \times 672.$	Now substituting values from enq (1) gives
$6 = 9(3510 - 5 \times 672) - 2 \times 672$	or
$6 = 9 \times 3510 - 47 \times 672.$	So $m = 9$ and $n = -47$.

Exercise Set 4.1

Q4 $d_m = 1 - (\frac{1}{10})^m, \quad m \geq 1$, so $d_1 = 1 - (\frac{1}{10})^1 = \dfrac{9}{10}$, $d_2 = 1 - (\frac{1}{10})^2 = \dfrac{99}{100}$, $d_3 = 1 - (\frac{1}{10})^3 = \dfrac{999}{1000}$ and $d_4 = 1 - (\frac{1}{100})^4 = \dfrac{9999}{10000}$

Q7 $a_k = 2k + 1$, $b = (k-1)^3 + k + 2$

$$\begin{array}{lll} k = 0: & a_0 = 2 \times 0 + 1 = 1 & b_0 = (0-1)^3 + 0 + 2 = -1 + 2 = 1 \\ k = 1: & a_1 = 2 \times 1 + 1 = 3 & b_1 = (1-1)^3 + 1 + 2 = 3 \\ k = 2: & a_2 = 2 \times 2 + 1 = 5 & b_2 = (2-1)^3 + 2 + 2 = 5 \\ k = 3: & a_3 = 2 \times 3 + 1 = 7 & b_3 = (3-1)^3 + 3 + 2 = 15 \end{array}$$

Hence $a_0 = b_0$, $a_1 = b_1$, and $a_2 = b_2$ and $a_3 \neq b_3$.

Q36 $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \ldots + \frac{n}{(n+1)!} = \sum_{i=1}^{n} \frac{i}{(i+1)!}$

Q38 $n + \frac{n-1}{2!} + \frac{n-2}{3!} + \frac{n-3}{4!} + \ldots + \frac{1}{n!} = \sum_{i=1}^{n} \frac{n+1-i}{i!}$

Q49 $\frac{5!}{7!} = \dfrac{5 \times 4 \times 3 \times 2 \times 1}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{1}{7 \times 6} = \frac{1}{42}$

Q50 $\frac{6!}{0!} = \frac{6!}{1} = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Q56

$$\begin{aligned} \frac{n!}{(n-k-1)!} &= \frac{n(n-1)(n-2)\ldots(n-k)(n-k-1)!}{(n-k-1)!} \\ &= n(n-1)(n-2)\ldots(n-k) \end{aligned}$$