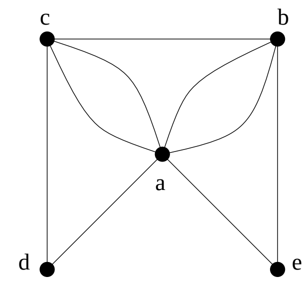
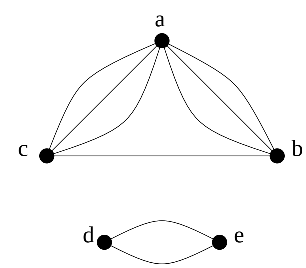
MATH1061 Semester 2, 2002 Solution Ass. 7

Exercise Set 11.2

Q9 b) One possible representation of the graph is given below and yes all possible representations have an Euler Circuit as the graph is connected and all vertices are of even degree.

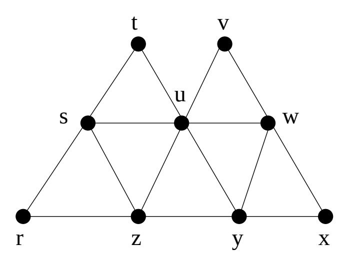


c) No not necessarily, as the graph may not be connected. For example

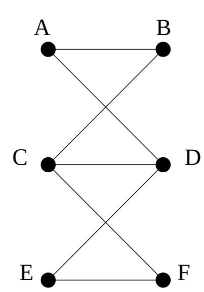


Q15 An Euler circuit exists as every vertex is of even degree. One such circuit is

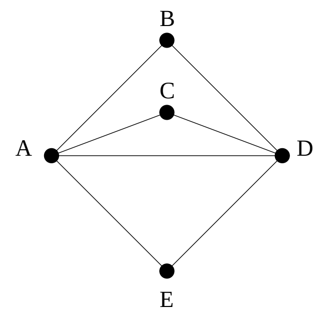
t u v w u y w x y z u s z r s t



Q17 No Euler circuit exists as vertices C and D are of odd degree.

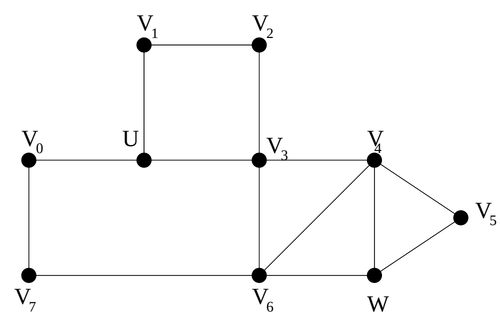


Q18 The corresponding graph is drawn below. An Euler Circuit exists as each vertex is of even degree. One such Euler circuit is ABDACDEA.



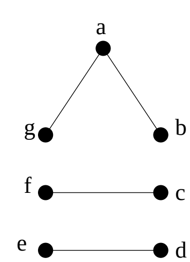
Q21 An Euler path exists as vertices U, and W are the only vertices of odd degree. One possible Euler path is

$$U\, V_1\, V_2\, V_3\, U\, V_0\, V_7\, V_6\, V_3\, V_4\, V_6\, W\, V_4\, V_5\, W$$



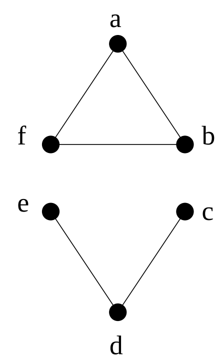
Exercise Set 11.5

Q15



Q16 No such tree exists as a tree on twelve vertices must have eleven edges.

Q17



Q18 No such tree exists, because a tree on five vertices must contain 4 edges. So the total degree must equal 2 times the number of edges, which is 8.

Q19 No such graph exists as any connected graph on 10 vertices and 9 edges must be a tree. Hence it has no non-trivial circuits.

Exercise Set 6.1

Q8 a) $3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$.

b) The probability that exactly two people become ill is $3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$. The probability that all three people become ill is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$. So the answer is $\frac{1}{8} + \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$.

c) $\frac{1}{8}$.

Q13 a) When n is even $\lfloor \frac{n}{2} \rfloor = \frac{n}{2}$. So the probability is

$$\frac{\frac{n}{2}}{n} = \frac{1}{2}.$$

b) When n is odd $\lfloor \frac{n}{2} \rfloor = \frac{n-1}{2}$. So the probability is

$$\frac{\frac{n-1}{2}}{n} = \frac{n-1}{2n}.$$

Q21 a) $\frac{365}{7} = 52\frac{1}{7}$. Hence 52 Sundays.

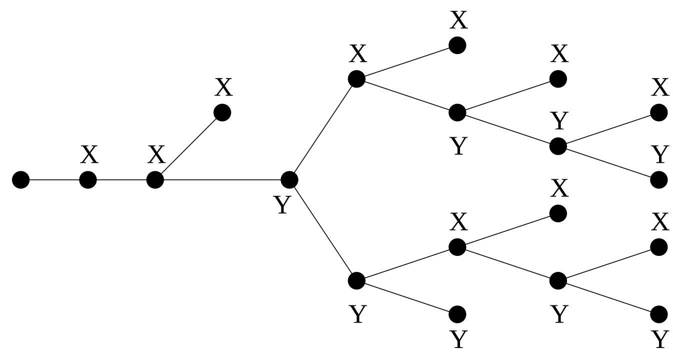
b) Since there are $52\frac{1}{7}$ weeks there will be 53 Mondays.

Q22 Let S consist entirely of integers from 1 to 100 such that if $a, b \in S$, then $a \nmid b$ and $b \nmid a$. Any element of S can be written in the form $2^k m$ where m is a positive odd integer. So if $a, b \in S$ where $a = 2^k m$ and $b = 2^q n$ then $m \neq n$. Otherwise $a \mid b$ or $b \mid a$. So the maximum number of distinct values for m (in $2^k m$) is 50. That is, the number of distinct odd integers between 1 and 100 is 50.

And since $S = \{51, 52, 53, \ldots, 100\}$ is a set of 50 numbers from 1 to 100 such that no one element divides another, the largest number of elements is 50.

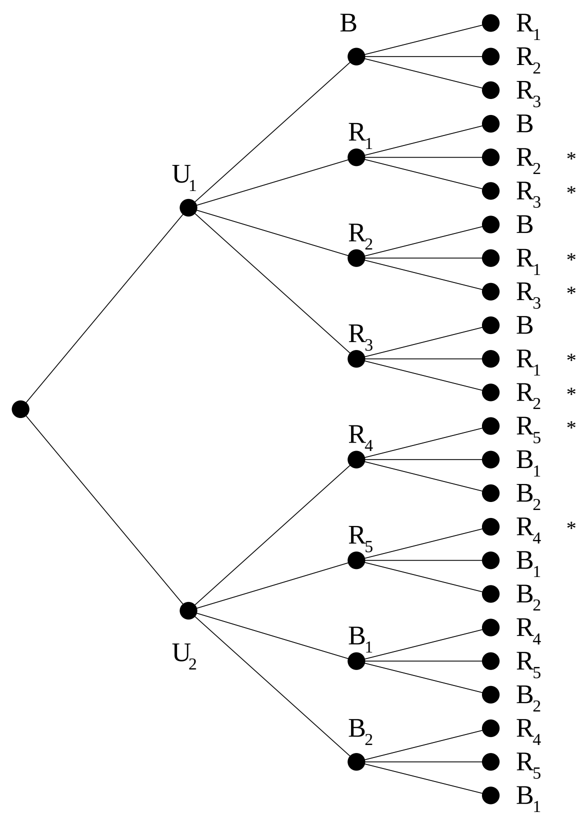
Exercise Set 6.2

Q5 9 ways, see tree below.



Q7

a)



b) $1 \times 4 \times 3 + 1 \times 4 \times 3 = 2 \times 4 \times 3 = 24$.

c) From above those branches marked with an *, give two red balls. Hence probability is $\frac{8}{24} = \frac{1}{3}$.

Q12 c) $8 \times 16 = 128$

Q23 $m \times n \times p = mnp$

Q25 $(d - c + 1)(b - a + 1)$

Q27 a) Let $n = s.r$, where $s = p_{j1}^{k_{j1}} \ p_{j2}^{k_{j2}} \ldots p_{jt}^{k_{jt}}$ for some t. That is s comprises of some of the factors $p_i^{k_i}$ and r comprises of the remaining factors. Thus choosing all possible representations of s amounts to choosing all possible subsets of the set $\{p_1, p_2, \ldots , p_m\}$.

There are 2^m possible subsets. So 2^m possible representations for s. Thus n can be written as the product of two positive integers, with no factors in common, in 2^m possible ways.

Q34 b) $6! = 720$

c) $6 \times 5 = 30$

d) 6