MATH1061 Semester 2, 2002 Solution Ass 8.

Exercise Set 10.1

Q5 a) Yes as $3 \mid (10-1)$. Yes as $3 \mid |(1-10)$. Yes as $3|(2-2)$. No as $3 \nmid (8-1)$.

b) Five possible examples are $3T0,\ 6T0,\ -6T0,\ 0T0,\ -99T0$.

c) Five possible examples are $1T1,\ -5T1,\ 4T1,\ 100T1,\ 10T1$.

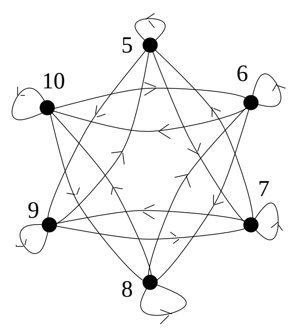
d) Five possible examples are $2T2,\ 5T2,\ -10T2,\ -100T2,\ -4T2$.

Q10 a) No as $\{a\} \cap \{c\} = \emptyset$.

b) Yes as $\{a,b\} \cap \{b,c\} = \{b\} \neq \emptyset$.

c) Yes as $\{a,b\} \cap \{a,b,c\} = \{a,b\} \neq \emptyset$.

Q26



Exercise Set 10.2

Q17 $\forall\, m,n \in \mathbb{Z} \quad m0n \Leftrightarrow m-n$ is odd.

Not reflexive since $m-m=0$ which is even $\forall\, m \in \mathbb{Z}$.

Yes, symmetric. To prove this, assume $m0n$ that is $m-n=2p+1$ for some p. It follows that $n-m=-2p-1=2(-p-1)+1$, which is odd. So $n0m$.

Not transitive as $22-1$ is odd and $1-22$ is odd, but $22-22$ is even.

Exercise Set 10.3

Q21 b) $\forall\, m,n \in \mathbb{Z},\ mDn \Leftrightarrow m^2 \equiv n^2 (mod\ 3)$.

We shall prove this is an equivalence relation.

First note $\forall\, m,\ m^2-m^2=0=0\times 3$. Hence $3|(m^2-m^2)$. Thus $m^2 \equiv m^2 (mod\, 3)$ and the relation is reflexive.

Next we shall prove $\forall\, m,n \in \mathbb{Z}$ if $m^2 \equiv n^2 (mod\, 3)$ then $n^2 \equiv m^2 (mod\, 3)$. If $m^2 \equiv n^2 (mod\, 3)$ then $3|(m^2-n^2)$ or $m^2-n^2=3k$ where $k \in \mathbb{Z}$. But this implies that $n^2 - m^2 = 3l$ where $l = -k \in \mathbb{Z}$. Thus $3|(n^2 - m^2)$ and so $n^2 \equiv m^2 (mod\, 3)$. So the relation is symmetric.

Next we must prove the relation is transitive. That is, we must prove $\forall\, m, n, p \in \mathbb{Z}$ if $m^2 \equiv n^2 (mod\, 3)$ and $n^2 \equiv p^2 (mod\, 3)$ then $m^2 \equiv p^2 (mod\, 3)$. So assume that $m^2 \equiv n^2 (mod\, 3)$ and $n^2 \equiv p^2 (mod\, 3)$. Implying that $m^2 - n^2 = k3$ and $n^2 - p^2 \equiv l3$ where $k, l \in \mathbb{Z}$. By adding these equations we get $m^2 - n^2 + n^2 - p^2 = k3 + l3 = (k + l)3$, where $k + l \in \mathbb{Z}$. Hence $m^2 \equiv p^2 (mod\, 3)$ and the relation is transitive.

Since the relation is reflexive, symmetric and transitive, it is an equivalence relation.

Q16 Equivalence classes are:

$[0] = \{4p | p \in \mathbb{Z}\}$;

$[1] = \{4p + 1 | p \in \mathbb{Z}\}$;

$[2] = \{4p + 2 | p \in \mathbb{Z}\}$; and

$[3] = \{4p + 3 | p \in \mathbb{Z}\}$.

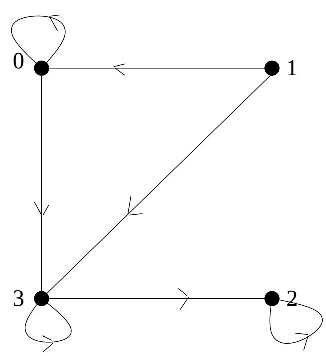
Q21 Equivalence classes are:

$[0] = \{q | q = 3p, \text{ for some } p \in \mathbb{Z}\}$

$[1] = \{q | q = 3p + 1 \text{ or } q = 3p + 2, \text{ for some } p \in \mathbb{Z}\}$.

Exercise Set 10.5

Q1 c) From the diagram below we can see that this relation is antisymmetric.



d) This relation is not antisymmetric, as $(1, 2), (2, 1) \in R_4$ but $1 \neq 2$.

Q9 The relation R is definied by $x, y \in \mathbb{R}, \quad xRy \Leftrightarrow x^2 \leq y^2$.

Since $x^2 \leq x^2$, xRx and R is reflexive.

Assume xRy and yRz, then $x^2 \leq y^2$ and $y^2 \leq z^2$, so $x^2 \leq z^2$ and R is transitive.

Assume $x, y \in \mathbb{R}$ and xRy and yRx. Then $x^2 \leq y^2$ and $y^2 \leq x^2$. Thus $x^2 = y^2$. But x is not necessarily equal to y. For example $x = -2$ and $y = 2$. Consequently R is not antisymmetric.

Thus R is not a partial order.