MATH1061 Semester 2, 2002 Solution Ass. 9

Exercise Set 7.3

Q3 b) G is not one-to-one as $G(a) = G(d) = y$ but $a \neq d$.
G is not onto as there exists no element m of X such that $G(m) = z$.

Q5 b) One possible example is $g = \{(1,1),(2,2),(3,1)\}$.

c) One possible example is $h = \{(1,1),(2,1),(3,1)\}$.

d) One possible example is $k = \{(1,2),(2,3),(3,1)\}$.

Q14 $f(x) = \dfrac{2x+1}{x}$, $x \neq 0$. Assume

$$\begin{aligned} f(x_1) &= f(x_2) \\ \Rightarrow \quad \frac{2x_1+1}{x_1} &= \frac{2x_2+1}{x_2} \\ \Rightarrow \quad (2x_1+1)x_2 &= (2x_2+1)x_1 \\ \Rightarrow \quad 2x_1x_2 + x_2 &= 2x_2x_1 + x_1 \\ \Rightarrow \quad x_2 &= x_1 \end{aligned}$$

Hence f is one-to-one.

Q44 Note that if $y = \dfrac{2x+1}{x}$ then $y = 2 + \dfrac{1}{x}$ or $x = \dfrac{1}{y-2}$ provided $y \neq 2$. So for all $y \in \mathbb{R} \setminus \{2\}$ there exists an $x \in \mathbb{R}$ such that $f(x) = y$, namely $x = \dfrac{1}{y-2}$. Hence $f(x)$ is onto.

Since $f(x)$ is one–to–one and onto, $f(x)$ is a one-to-one correspondence. The inverse function is

$$\begin{aligned} f^{-1} \quad &: \quad \mathbb{R} \setminus \{2\} \Rightarrow \mathbb{R} \setminus \{0\} \\ & \qquad f^{-1}(y) = \frac{1}{y-2}. \end{aligned}$$

Exercise Set 8.1

Q6 $t_k = t_{k-1} + 2t_{k-2}$, $t_0 = -1$, $t_1 = 1$. So $t_2 = 1 + 2(-1) = -1$ and $t_3 = -1 + 2(1) = 1$.

Q10 The sequence $b_0, b_1, b_2 \ldots$, is defined by $b_n = 5^n$, so $5b_{k-1} = 5.5^{k-1} = 5^k = b_k$. Hence $b_k = 5b_{k-1}$ for all $k \geq 1$.

Exercise Set 8.3

Q12 $e_k = 0e_{k-1} + 9e_{k-2}$ for all $k \geq 2$ with $e_0 = 0, e_1 = 2$. Therefore the characteristic equation is $t^2 - 9 = 0$. So $(t-3)(t+3) = 0$ and $t = -3$ or $t = 3$. Hence $e_n = C(3)^n + D(-3)^n$. Since $e_0 = 0$ and $e_0 = C(3)^0 + D(-3)^0$, we have

$$0 = C + D.$$

Since $e_1 = 2$ and $e_1 = C(3)^1 + D(-3)^1$, we have

$$2 = 3C - 3D.$$

Substituting $C = -D$ into $2 = 3C - 3D$ we get $2 = 3C + 3C$ or $C = \frac{1}{3}$. This in turn implyies that $D = -\frac{1}{3}$. Thus $e_n = \frac{1}{3}(3)^n - \frac{1}{3}(-3)^n = (3)^{n-1} + (-3)^{n-1} = 3^{n-1}(1 + (-1)^{n-1})$. So summarising

$$e_n = 3^{n-1}(1 + (-1)^{n-1}).$$