1.3 Graphs with circular symmetry eg. cone, potential well

Surfaces that can be formed by rotating a curve in the $(z,\, x)$ plane about the z axis are easy to visualise. For instance an (upside down) cone can be formed by rotating $z = x$ about the z axis.

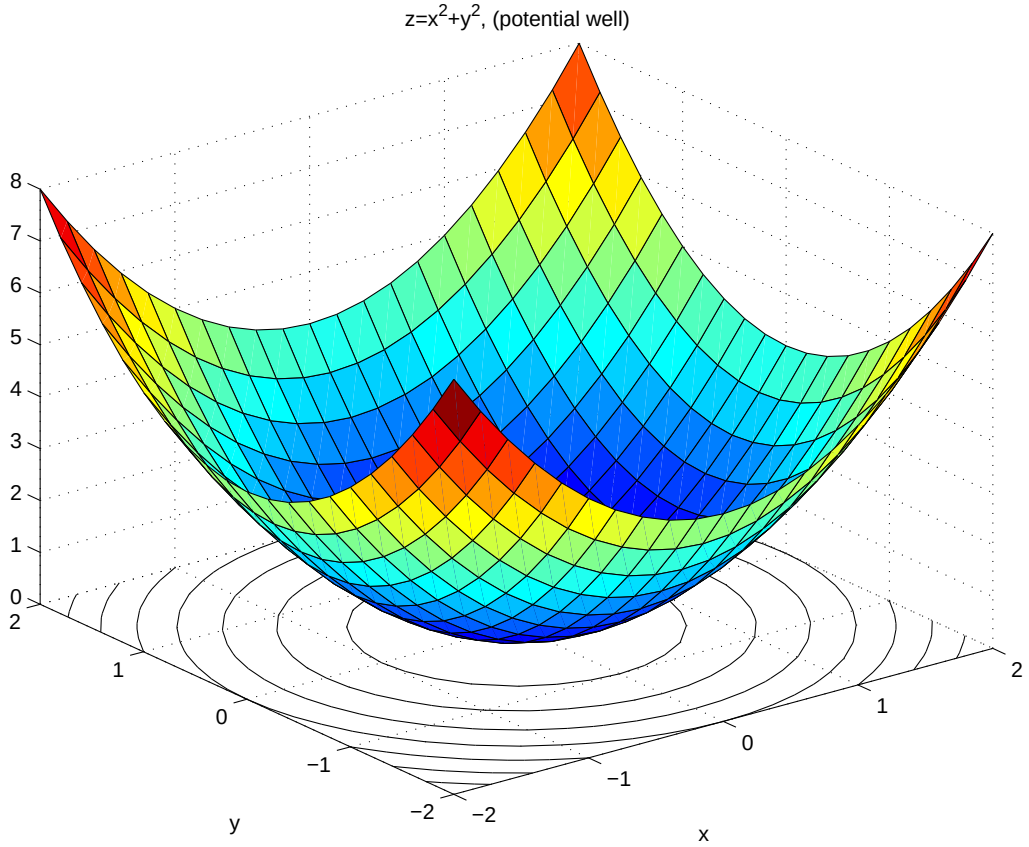
After rotating $z = r$, where r is the distance in the $(x,\, y)$ plane from the origin. Now $r = \sqrt{x^2 + y^2}$, so the (upside down) cone becomes

$$z = f(x,\, y) = \sqrt{x^2 + y^2}.$$

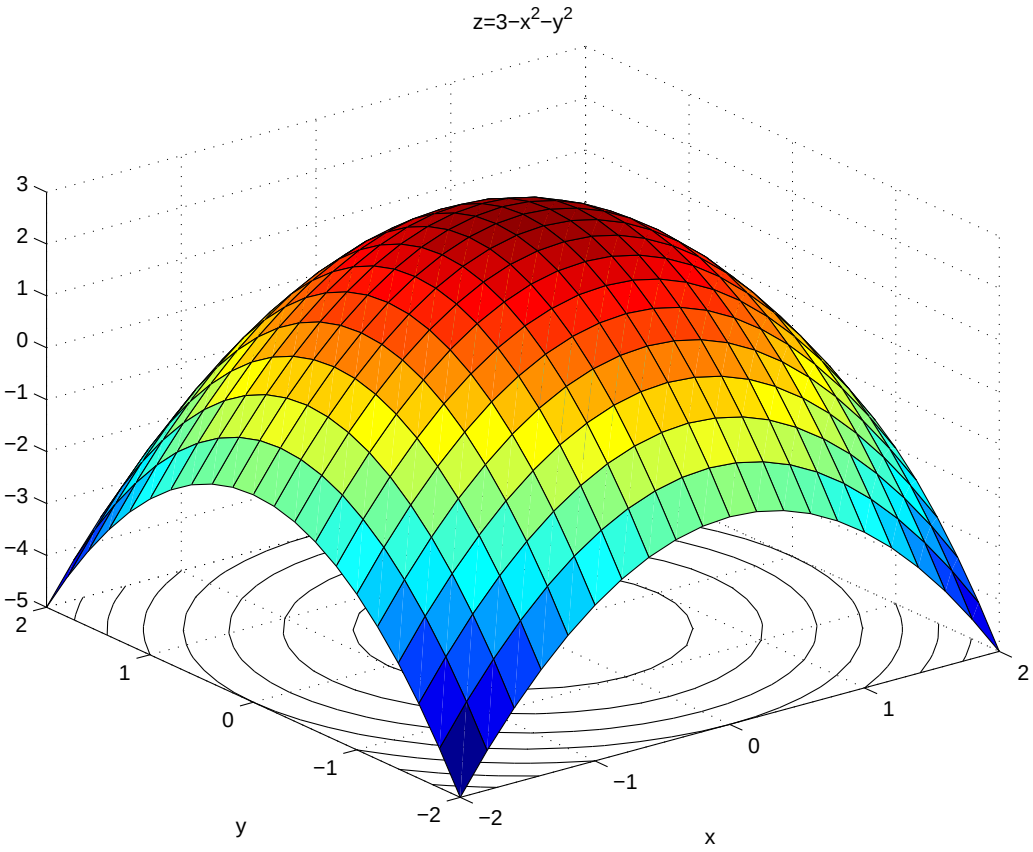
A surface $z = f(x,\, y) = F(x^2 + y^2)$ in which x and y only appear as $(x^2 + y^2)$ has **circular symmetry**.

Example: Sketch $z = x^2 + y^2$ potential well.

The intersection of this surface with a vertical plane, say $y = 1$, is always a parabola.



Example: Sketch $z = 3 - x^2 - y^2$.



Example: Sketch $z = x^2 - 4x + y^2 + 2y + 4$.

First complete the square

$$z = (x-2)^2 + (y+1)^2 - 1.$$

Now you can see that this is a potential well where the axis of symmetry is the vertical line $x = 2$, $y = -1$. It has lowest point at $z = -1$, when $x = 2$ and $y = -1$.

Example: Sketch $z = e^{-(x^2+y^2)}$.

Hint: First sketch $z = e^{-x^2}$ then rotate about z axis.

$z = e^{-(x^2+y^2)}$