MT152

1. Functions of Several Variables.

1.4 Cross sections of a surface

Many surfaces are easy to visualise in terms of their intersection with a series of vertical planes, say $y =$ constant.

For instance a vibrating guitar string

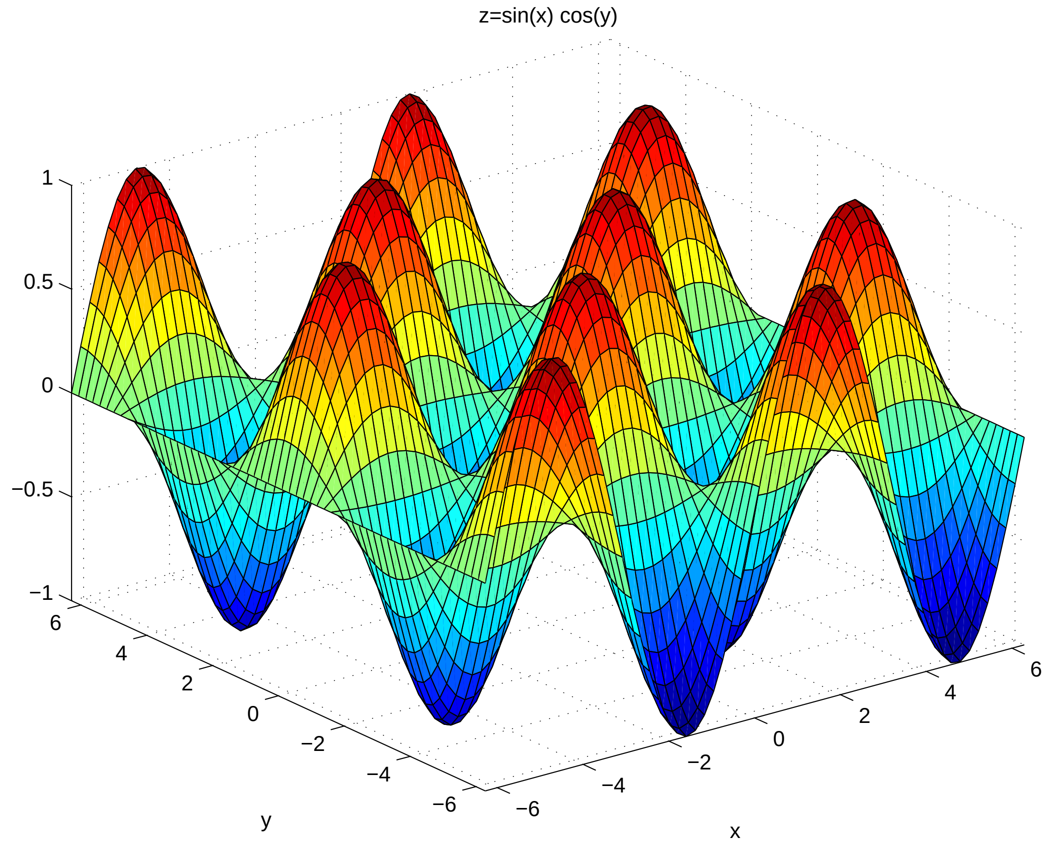
$$z = A \sin x \cos t \quad , \quad x \in [0, \pi]$$

is just a half a sine wave for $t =$ const.

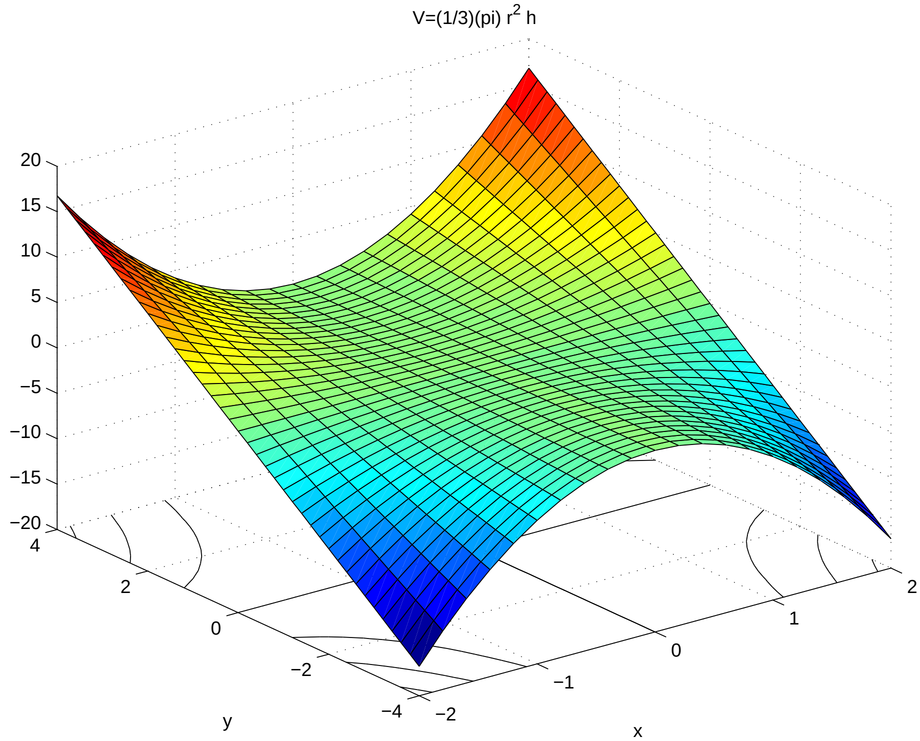
If

$$\left.\begin{array}{ll} t = 0 & z = A \sin x \\ t = \frac{\pi}{4} & z = \frac{A}{\sqrt{2}} \sin x \\ t = \frac{\pi}{2} & z = 0 \\ t = \frac{3\pi}{4} & z = \frac{-A}{\sqrt{2}} \sin x \end{array}\right\} \quad \begin{array}{l} \text{all sine curves, but} \\ \text{with different amplitudes} \end{array}$$

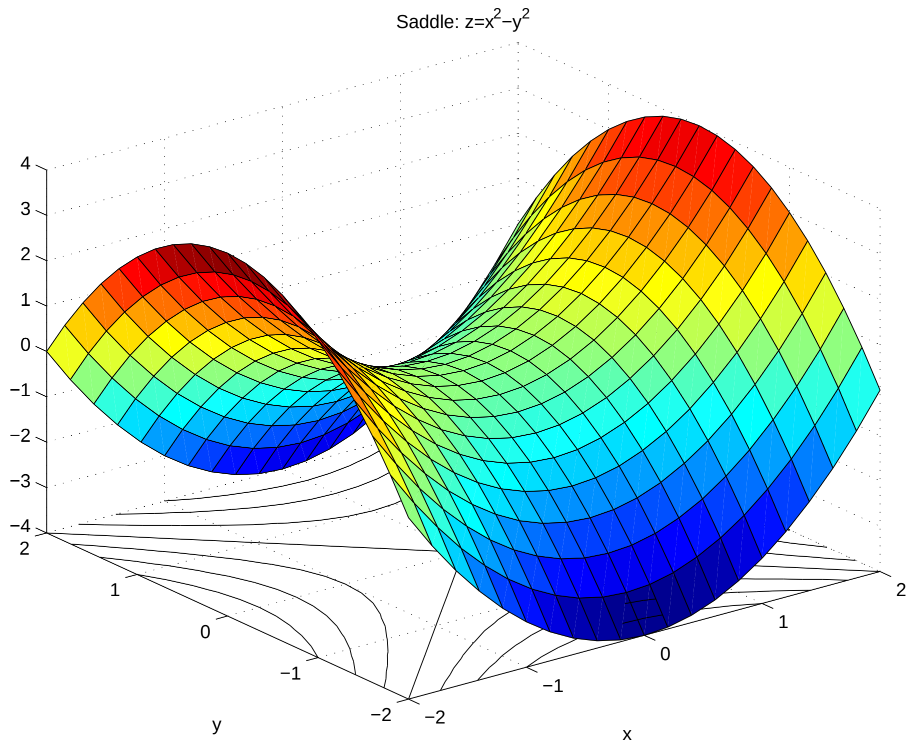
To really get the picture you can also consider the cross sections in x. For instance $x = \frac{\pi}{2}$ (at the top of the sine wave). Then $z = A \cos t$ which equals the amplitude of the sine wave.


z=sin(x) cos(y)
1
0.5
0
−0.5
−1
6
4
2
0
−2
−4
−6
y
−6
−4
−2
0
2
4
6
x

Example: Use Cross Sections to sketch the surface $V = \frac{1}{3}\pi r^2 h$ (volume of a cone). In the following picture $x = r$ and $y = h$.



The cross sections of a saddle $z = x^2 - y^2$ are parabolas. But for $y = y_0$ they point up $z = x^2 + (y_0)^2$ and for $x =$ constant they point down $z = -y^2 + x_0^2$. The surface is difficult to draw. Think of a saddle for horse riding.



Example: Use cross sections to sketch

$$z = f(x) = x^2$$