MT152 Semester 2 - 2000

Solutions to Problem Sheet 2

1. First find where the plane \( z = \frac{x}{2} - 3y + 3 \) intersects the axes.
The plane intersects \( x \)-axis \( (y = z = 0) \) at \( x = -6 \), \( y \)-axis \( (x = z = 0) \) at \( y = 1 \) and \( z \)-axis \( (x = y = 0) \) at \( z = 3 \). Then draw the triangle. The triangle defines the plane but it can still be hard to picture it!

2. Suppose the plane is \( ax + by + cz + d = 0 \).
If \( (x_0, y_0, z_0), (x_1, y_1, z_1) \) are any two points on the plane then
\[
a(x_1 - x_0) + b(y_1 - y_0) + c(z_1 - z_0) = 0.
\]
Using \( (1, 0, 0) \) and \( (3, 5, 1) \) gives \( 2a + 5b + c = 0 \). \( (1) \)
Using \( (1, 0, 0) \) and \( (4, 2, 0) \) gives \( 3a + 2b = 0 \). \( (2) \)
So from \( (2) \) \( b = -\frac{3}{2}a \) and from \( (1) \) \( c = -2a - 5\left(-\frac{3}{2}a\right) = \frac{11a}{2} \). Now substitute \( (1, 0, 0) \)
into the plane \( ax + by + cz + d = 0 \) to get \( d = -a \). So we have
\[
ax - \frac{3}{2}ay + \frac{11}{2}az - a = 0.
\]
Dividing through by \( a \) we obtain
\[
2x - 3y + 11z - 2 = 0.
\]
(Check that it works!)

3. (H & H Sec. 11.5 Q2) Find the equation of the linear function \( z = c + mx + ny \),
whose graph contains the points \( (0, 0, 0), (0, 2, -1), (-3, 0, -4) \).
Since \( (0, 0, 0) \) satisfies the linear function \( z = c + mx + ny \) we obtain \( c = 0 \). So \( z = mx + ny \). Since \( (0, 2, -1) \) satisfies the linear function \( z = mx + ny \) we obtain \( -1 = 2n \Rightarrow n = -\frac{1}{2} \).
So \( z = mx - \frac{1}{2}y \).
Since \((-3, 0, -4)\) satisfies the linear function \(z = mx - \frac{1}{2}y\) we obtain \(-4 = -3m\) \(\Rightarrow\) \(m = \frac{4}{3}\). Hence, \(z = \frac{4}{3}x - \frac{1}{2}y\).

4. (H & H Sec. 11.5 Q16) Sketch the graph of the linear function \(z = 4 + x - 2y\). 
\(z\) intercept \((x = y = 0)\) is 4, \(x\) intercept \((y = z = 0)\) is \(-4\) and \(y\) intercept \((x = z = 0)\) is 2.

5. Sketch the surface \(z = -x^2 - y^2 + 6y\).
First we complete the square in \(y\).

\[
z = -x^2 - \left((y - 3)^2 - 9\right) \\
  = -x^2 - (y - 3)^2 + 9
\]

Axis of symmetry: \(x = 0\) and \(y = 3\). If \(y = 3\) then \(z = -x^2 + 9\) which is a parabola pointing down with maximum at \(x = 0\) and \(z = 9\). Note that the surface goes through the origin and intersects the \(xy\)-plane in the circle \(x^2 + (y - 3)^2 = 9\).
6. (H & H Sec. 11.3 Q5) Since only one graph is a plane iv) is (c).
Both (i) and (v) have circular symmetry as do (a) and (b) as \( z = f(x^2 + y^2) \). Now (a) \( z = \frac{1}{x^2 + y^2} \rightarrow +\infty \) as \( x \) and \( y \) go to zero. But (b) \( z = \frac{1}{e^{x^2+y^2}} \rightarrow \frac{1}{e^0} \rightarrow -1 \) as \( x \) and \( y \) go to zero. So (i) is (a) and (v) is (b).
(ii) is (d) a parabola pointing down in \( yz \)-space.
(iii) must be (e) - you can just see the oscillations on \( y \) and the cubic curves in \( x \)!
7. (H & H Sec. 11.3 Q8) If \( b > 0 \) the parabola points up with min at origin.
If \( b < 0 \) the parabola points down with max at origin.
If \( b = 0 \) i.e. \( y = z = 0 \) which rules out (ii).
In fact the only surface that is always negative for \( y < 0 \) is (iv).
8. (H & H Sec. 11.3 Q12) Draw the graph of the traveling wave function

\[
 h(x, t) = 3 + \cos(x - 0.5t)
\]

If \( t = 0 \) \( h(x, t) = 3 + \cos x \)
If \( t = \frac{\pi}{2} \) \( h(x, t) = 3 + \cos(x - \frac{\pi}{4}) \)
If \( t = \pi \) \( h(x, t) = 3 + \cos(x - \frac{\pi}{2}) \)
If \( t = \frac{3\pi}{2} \) \( h(x, t) = 3 + \cos(x - \frac{3\pi}{4}) \)
If \( t = 2\pi \) \( h(x, t) = 3 + \cos(x - \pi) \)

Also if \( x = 0 \) then \( h(x, t) = 3 + \cos(\frac{t}{2}) \) and if \( x = 2\pi \) then \( h(x, t) = 3 + \cos(\frac{t}{2}) \).
9. Use cross-sections to sketch \( z = 4(x - 1)^2 - y^2 \).

If \( y = 0 \) then \( z = 4(x - 1)^2 \) is a parabola pointing up with min at \( x = 1 \) and \( z = 0 \). If \( y = \pm 1 \) we have the same parabola at \( x = 1 \) and \( z = 0 \) but shifted down.

If \( x = 1 \) then \( z = -y^2 \) is a parabola pointing down with max at \( y = 0 \) and \( z = 0 \).