Chapter 6
Risk, Return, and the Capital Asset Pricing Model

ANSWERS TO END-OF-CHAPTER QUESTIONS

6-1  a. Stand-alone risk is only a part of total risk and pertains to the risk an investor takes by holding only one asset. Risk is the chance that some unfavorable event will occur. For instance, the risk of an asset is essentially the chance that the asset’s cash flows will be unfavorable or less than expected. A probability distribution is a listing, chart or graph of all possible outcomes, such as expected rates of return, with a probability assigned to each outcome. When in graph form, the tighter the probability distribution, the less uncertain the outcome.

b. The expected rate of return ($\hat{r}$) is the expected value of a probability distribution of expected returns.

c. A continuous probability distribution contains an infinite number of outcomes and is graphed from $-\infty$ and $+\infty$.

d. The standard deviation ($\sigma$) is a statistical measure of the variability of a set of observations. The variance ($\sigma^2$) of the probability distribution is the sum of the squared deviations about the expected value adjusted for deviation. The coefficient of variation (CV) is equal to the standard deviation divided by the expected return; it is a standardized risk measure which allows comparisons between investments having different expected returns and standard deviations.

e. A risk averse investor dislikes risk and requires a higher rate of return as an inducement to buy riskier securities. A realized return is the actual return an investor receives on their investment. It can be quite different than their expected return.

f. A risk premium is the difference between the rate of return on a risk-free asset and the expected return on Stock i which has higher risk. The market risk premium is the difference between the expected return on the market and the risk-free rate.

g. CAPM is a model based upon the proposition that any stock’s required rate of return is equal to the risk free rate of return plus a risk premium reflecting only the risk remaining after diversification.
h. The expected return on a portfolio, \( \hat{r}_p \), is simply the weighted-average expected return of the individual stocks in the portfolio, with the weights being the fraction of total portfolio value invested in each stock. The market portfolio is a portfolio consisting of all stocks.

i. Correlation is the tendency of two variables to move together. A correlation coefficient (\( \rho \)) of +1.0 means that the two variables move up and down in perfect synchronization, while a coefficient of -1.0 means the variables always move in opposite directions. A correlation coefficient of zero suggests that the two variables are not related to one another; that is, they are independent.

j. Market risk is that part of a security’s total risk that cannot be eliminated by diversification. It is measured by the beta coefficient. Diversifiable risk is also known as company specific risk, that part of a security’s total risk associated with random events not affecting the market as a whole. This risk can be eliminated by proper diversification. The relevant risk of a stock is its contribution to the riskiness of a well-diversified portfolio.

k. The beta coefficient is a measure of a stock’s market risk, or the extent to which the returns on a given stock move with the stock market. The average stock’s beta would move on average with the market so it would have a beta of 1.0.

l. The security market line (SML) represents in a graphical form, the relationship between the risk of an asset as measured by its beta and the required rates of return for individual securities. The SML equation is essentially the CAPM, \( r_i = r_{RF} + b_i(r_M - r_{RF}) \).

m. The slope of the SML equation is \( (r_M - r_{RF}) \), the market risk premium. The slope of the SML reflects the degree of risk aversion in the economy. The greater the average investors aversion to risk, then the steeper the slope, the higher the risk premium for all stocks, and the higher the required return.

6-2 a. The probability distribution for complete certainty is a vertical line.

b. The probability distribution for total uncertainty is the X axis from \(-\infty\) to \(+\infty\).

6-3 Security A is less risky if held in a diversified portfolio because of its lower beta and negative correlation with other stocks. In a single-asset portfolio, Security A would be more risky because \( \sigma_A > \sigma_B \) and \( CV_A > CV_B \).
6-4  a. No, it is not riskless. The portfolio would be free of default risk and liquidity risk, but inflation could erode the portfolio’s purchasing power. If the actual inflation rate is greater than that expected, interest rates in general will rise to incorporate a larger inflation premium (IP) and the value of the portfolio would decline.

b. No, you would be subject to reinvestment rate risk. You might expect to “roll over” the Treasury bills at a constant (or even increasing) rate of interest, but if interest rates fall, your investment income will decrease.

c. A U.S. government-backed bond that provided interest with constant purchasing power (that is, an indexed bond) would be close to riskless.

6-5 The risk premium on a high beta stock would increase more.

\[ \text{RP}_j = \text{Risk Premium for Stock } j = (r_M - r_{RF})b_j. \]

If risk aversion increases, the slope of the SML will increase, and so will the market risk premium \((r_M - r_{RF})\). The product \((r_M - r_{RF})b_j\) is the risk premium of the jth stock. If \(b_j\) is low (say, 0.5), then the product will be small; \(\text{RP}_j\) will increase by only half the increase in \(\text{RP}_M\). However, if \(b_j\) is large (say, 2.0), then its risk premium will rise by twice the increase in \(\text{RP}_M\).

6-6 According to the Security Market Line (SML) equation, an increase in beta will increase a company’s expected return by an amount equal to the market risk premium times the change in beta. For example, assume that the risk-free rate is 6 percent, and the market risk premium is 5 percent. If the company’s beta doubles from 0.8 to 1.6 its expected return increases from 10 percent to 14 percent. Therefore, in general, a company’s expected return will not double when its beta doubles.

6-7 Yes, if the portfolio’s beta is equal to zero. In practice, however, it may be impossible to find individual stocks that have a nonpositive beta. In this case it would also be impossible to have a stock portfolio with a zero beta. Even if such a portfolio could be constructed, investors would probably be better off just purchasing Treasury bills, or other zero beta investments.
SOLUTIONS TO END-OF-CHAPTER PROBLEMS

6-1

<table>
<thead>
<tr>
<th>Investment</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>$35,000</td>
<td>0.8</td>
</tr>
<tr>
<td>$40,000</td>
<td>1.4</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$75,000</strong></td>
</tr>
</tbody>
</table>

($35,000/$75,000)(0.8) + ($40,000/$75,000)(1.4) = 1.12.

6-2

$r_{RF} = 6\%; \ r_M = 13\%; \ b = 0.7; \ r_s = ?$

$r_s = r_{RF} + (r_M - r_{RF})b$

= 6\% + (13\% - 6\%)0.7

= 10.9\%.$

6-3

$r_{RF} = 5\%; \ R_P = 6\%; \ r_M = ?$

$r_M = 5\% + (6\%)1 = 11\%.$

$r_s$ when $b = 1.2 = ?$

$r_s = 5\% + 6\%(1.2) = 12.2\%.$

6-4

$r = (0.1)(-50\%) + (0.2)(-5\%) + (0.4)(16\%) + (0.2)(25\%) + (0.1)(60\%)$

= 11.40\%.$

$\sigma^2 = (-50\% - 11.40\%)^2(0.1) + (-5\% - 11.40\%)^2(0.2) + (16\% - 11.40\%)^2(0.4) + (25\% - 11.40\%)^2(0.2) + (60\% - 11.40\%)^2(0.1)$

$\sigma^2 = 712.44; \ \sigma = 26.69\%.$

$CV = \frac{26.69\%}{11.40\%} = 2.34.$
6-5  

a. \( \hat{r}_m = (0.3)(15\%) + (0.4)(9\%) + (0.3)(18\%) = 13.5\% \).

\[ \hat{r}_j = (0.3)(20\%) + (0.4)(5\%) + (0.3)(12\%) = 11.6\% . \]

b. \( \sigma_M = \sqrt{[ (0.3)(15\% - 13.5\%)^2 + (0.4)(9\% - 13.5\%)^2 + (0.3)(18\% - 13.5\%)^2 ]} = 3.85\% . \)

\( \sigma_J = \sqrt{[ (0.3)(20\% - 11.6\%)^2 + (0.4)(5\% - 11.6\%)^2 + (0.3)(12\% - 11.6\%)^2 ]} = 6.22\% . \)

c. \( CV_M = \frac{3.85\%}{13.5\%} = 0.29 . \)

\( CV_J = \frac{6.22\%}{11.6\%} = 0.54 . \)

6-6  

a. \( r_A = r_{RF} + (r_M - r_{RF})b_A \)

\[ 12\% = 5\% + (10\% - 5\%)b_A \]

\[ 12\% = 5\% + 5\%(b_A) \]

\[ 7\% = 5\%(b_A) \]

\[ 1.4 = b_A . \]

b. \( r_A = 5\% + 5\%(b_A) \)

\[ r_A = 5\% + 5\%(2) \]

\[ r_A = 15\% . \]
6-7  

a. \[ r_i = r_{RF} + (r_M - r_{RF})b_i = 9\% + (14\% - 9\%)1.3 = 15.5\%. \]

b. 1. \( r_{RF} \) increases to 10%:

\[ r_M \text{ increases by 1 percentage point, from 14\% to 15\%.} \]
\[ r_i = r_{RF} + (r_M - r_{RF})b_i = 10\% + (15\% - 10\%)1.3 = 16.5\%. \]

2. \( r_{RF} \) decreases to 8%:

\[ r_M \text{ decreases by 1%, from 14\% to 13\%.} \]
\[ r_i = r_{RF} + (r_M - r_{RF})b_i = 8\% + (13\% - 8\%)1.3 = 14.5\%. \]

c. 1. \( r_M \) increases to 16%:

\[ r_i = r_{RF} + (r_M - r_{RF})b_i = 9\% + (16\% - 9\%)1.3 = 18.1\%. \]

2. \( r_M \) decreases to 13%:

\[ r_i = r_{RF} + (r_M - r_{RF})b_i = 9\% + (13\% - 9\%)1.3 = 14.2\%. \]

6-8  

Old portfolio beta = \[ \frac{142,500}{150,000} (b) + \frac{7,500}{150,000} (1.00) \]
\[ = 1.12 = 0.95b + 0.05 \]
\[ = 1.07 = 0.95b \]
\[ = 1.13 = b. \]

New portfolio beta = 0.95(1.13) + 0.05(1.75) = 1.16.

Alternative Solutions:

1. Old portfolio beta = 1.12 = (0.05)b_1 + (0.05)b_2 +...+(0.05)b_{20}

\[ 1.12 = (\Sigma b_i)(0.05) \]
\[ \Sigma b_i = 1.12/0.05 = 22.4. \]

New portfolio beta = (22.4 - 1.0 + 1.75)(0.05) = 1.1575 = 1.16.

2. \( \Sigma b_i \) excluding the stock with the beta equal to 1.0 is 22.4 - 1.0 = 21.4, so the beta of the portfolio excluding this stock is \( b = 21.4/19 = 1.1263 \). The beta of the new portfolio is:

\[ 1.1263(0.95) + 1.75(0.05) = 1.1575 = 1.16. \]

Answers and Solutions: 6 - 6
6-9 Portfolio beta = \[ \frac{400,000}{4,000,000} \times 1.50 + \frac{600,000}{4,000,000} \times (-0.50) + \frac{1,000,000}{4,000,000} \times 1.25 + \frac{2,000,000}{4,000,000} \times 0.75 \]

= 0.1 \times 1.50 + (0.15) \times (-0.50) + (0.25) \times 1.25 + (0.5) \times 0.75

= 0.15 - 0.075 + 0.3125 + 0.375 = 0.7625.

\[ r_p = r_{RF} + (r_M - r_{RF})b_p = 6\% + (14\% - 6\%) \times 0.7625 = 12.1\%. \]

Alternative solution: First compute the return for each stock using the CAPM equation \[ r_i = r_{RF} + (r_M - r_{RF})b_i \] and then compute the weighted average of these returns.

\[ r_{RF} = 6\% \text{ and } r_M - r_{RF} = 8\%. \]

<table>
<thead>
<tr>
<th>Stock</th>
<th>Investment</th>
<th>Beta</th>
<th>( r = r_{RF} + (r_M - r_{RF})b )</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$400,000</td>
<td>1.50</td>
<td>18%</td>
<td>0.10</td>
</tr>
<tr>
<td>B</td>
<td>600,000</td>
<td>(0.50)</td>
<td>2</td>
<td>0.15</td>
</tr>
<tr>
<td>C</td>
<td>1,000,000</td>
<td>1.25</td>
<td>16</td>
<td>0.25</td>
</tr>
<tr>
<td>D</td>
<td>2,000,000</td>
<td>0.75</td>
<td>12</td>
<td>0.50</td>
</tr>
<tr>
<td>Total</td>
<td>$4,000,000</td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>

\[ r_p = 18\%(0.10) + 2\%(0.15) + 16\%(0.25) + 12\%(0.50) = 12.1\%. \]

6-10 First, calculate the beta of what remains after selling the stock:

\[ b_p = 1.1 = \frac{100,000}{2,000,000} \times 0.9 + \frac{1,900,000}{2,000,000}b_R \]

\[ b_R = 1.105. \]

\[ b_N = (0.95) \times 1.105 + (0.05) \times 1.4 = 1.125. \]

6-11 We know that \( b_R = 1.50, b_S = 0.75, r_M = 13\%, r_{RF} = 7\%. \)

\[ r_i = r_{RF} + (r_M - r_{RF})b_i = 7\% + (13\% - 7\%)b_i. \]

\[ r_R = 7\% + 6\%(1.50) = 16.0\% \]

\[ r_S = 7\% + 6\%(0.75) = 11.5 \]

\[ 4.5\% \]
6-12  The answers to a, b, c, and d are given below:

<table>
<thead>
<tr>
<th>Year</th>
<th>$\bar{r}_A$</th>
<th>$\bar{r}_B$</th>
<th>Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>(18.00%)</td>
<td>(14.50%)</td>
<td>(16.25%)</td>
</tr>
<tr>
<td>2005</td>
<td>33.00</td>
<td>21.80</td>
<td>27.40</td>
</tr>
<tr>
<td>2006</td>
<td>15.00</td>
<td>30.50</td>
<td>22.75</td>
</tr>
<tr>
<td>2007</td>
<td>(0.50)</td>
<td>(7.60)</td>
<td>(4.05)</td>
</tr>
<tr>
<td>2008</td>
<td>27.00</td>
<td>26.30</td>
<td>26.65</td>
</tr>
</tbody>
</table>

Mean: 11.30  11.30  11.30  
Std Dev: 20.79  20.78  20.13  
CV: 1.84  1.84  1.78

e. A risk-averse investor would choose the portfolio over either Stock A or Stock B alone, since the portfolio offers the same expected return but with less risk. This result occurs because returns on A and B are not perfectly positively correlated ($\rho_{AB} = 0.88$).

6-13  
a. $b_X = 1.3471$; $b_Y = 0.6508$. These can be calculated with a spreadsheet.

b. $r_X = 6\% + (5\%)1.3471 = 12.7355\%$.  
   $r_Y = 6\% + (5\%)0.6508 = 9.2540\%$.

c. $b_p = 0.8(1.3471) + 0.2(0.6508) = 1.2078$.  
   $r_p = 6\% + (5\%)1.2078 = 12.04\%$.  
   Alternatively,  
   $r_p = 0.8(12.7355\%) + 0.2(9.254\%) = 12.04\%$.

d. Stock X is undervalued, because its expected return exceeds its required rate of return.
6-14 The detailed solution for the spreadsheet problem is available in the file *Solution for CF3 Ch 06 P14 Build a Model.xls* at the textbook’s Web site.
Assume that you recently graduated with a major in finance, and you just landed a job as a financial planner with Barney Smith Inc., a large financial services corporation. Your first assignment is to invest $100,000 for a client. Because the funds are to be invested in a business at the end of one year, you have been instructed to plan for a one-year holding period. Further, your boss has restricted you to the following investment alternatives, shown with their probabilities and associated outcomes. (Disregard for now the items at the bottom of the data; you will fill in the blanks later.)

Barney Smith’s economic forecasting staff has developed probability estimates for the state of the economy, and its security analysts have developed a sophisticated computer program which was used to estimate the rate of return on each alternative under each state of the economy. Alta Industries is an electronics firm; Repo Men collects past-due debts; and American Foam manufactures mattresses and other foam products. Barney Smith also maintains an “index fund” which owns a market-weighted fraction of all publicly traded stocks; you can invest in that fund, and thus obtain average stock market results. Given the situation as described, answer the following questions.

*Note that the estimated returns of American Foam do not always move in the same direction as the overall economy. For example, when the economy is below average, consumers purchase fewer mattresses than they would if the economy was stronger. However, if the economy is in a flat-out recession, a large number of consumers who were planning to purchase a more expensive inner spring mattress may purchase, instead, a cheaper foam mattress. Under these circumstances, we would expect American Foam’s stock price to be higher if there is a recession than if the economy was just below average.*

### Returns On Alternative Investments

<table>
<thead>
<tr>
<th>State of the economy</th>
<th>T- Bills</th>
<th>Alta Inds</th>
<th>Repo Men</th>
<th>Am. Foam</th>
<th>Market portfolio</th>
<th>2-stock portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>0.1</td>
<td>8.0%</td>
<td>-22.0%</td>
<td>28.0%</td>
<td>10.0%*</td>
<td>-13.0%</td>
</tr>
<tr>
<td>Below avg</td>
<td>0.2</td>
<td>8.0</td>
<td>-2.0</td>
<td>14.7</td>
<td>-10.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Average</td>
<td>0.4</td>
<td>8.0</td>
<td>2.0</td>
<td>0.0</td>
<td>7.0</td>
<td>15.0</td>
</tr>
<tr>
<td>Above avg</td>
<td>0.2</td>
<td>8.0</td>
<td>35.0</td>
<td>-10.0</td>
<td>45.0</td>
<td>29.0</td>
</tr>
<tr>
<td>Boom</td>
<td>0.1</td>
<td>8.0</td>
<td>50.0</td>
<td>-20.0</td>
<td>30.0</td>
<td>43.0</td>
</tr>
</tbody>
</table>

\[ r-hat (\hat{r}) \]

<p>| |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Recession</td>
</tr>
<tr>
<td>Below avg</td>
</tr>
<tr>
<td>Average</td>
</tr>
<tr>
<td>Above avg</td>
</tr>
<tr>
<td>Boom</td>
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</tbody>
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\[ \text{Std dev (}\sigma) \]

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<tbody>
<tr>
<td>Recession</td>
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<tr>
<td>Below avg</td>
</tr>
<tr>
<td>Average</td>
</tr>
<tr>
<td>Above avg</td>
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<tr>
<td>Boom</td>
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</table>

\[ \text{Coef of var (cv) } \]

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<tbody>
<tr>
<td>Recession</td>
</tr>
<tr>
<td>Below avg</td>
</tr>
<tr>
<td>Average</td>
</tr>
<tr>
<td>Above avg</td>
</tr>
<tr>
<td>Boom</td>
</tr>
</tbody>
</table>

\[ \text{beta (b) } \]

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<tbody>
<tr>
<td>Recession</td>
</tr>
<tr>
<td>Below avg</td>
</tr>
<tr>
<td>Average</td>
</tr>
<tr>
<td>Above avg</td>
</tr>
<tr>
<td>Boom</td>
</tr>
</tbody>
</table>

*Answers and Solutions: 6 - 10*
a. What are investment returns? What is the return on an investment that costs $1,000 and is sold after one year for $1,100?

Answer: Investment return measures the financial results of an investment. They may be expressed in either dollar terms or percentage terms.

The dollar return is $1,100 - $1,000 = $100. The percentage return is $100/$1,000 = 0.10 = 10%.

b. 1. Why is the t-bill’s return independent of the state of the economy? Do t-bills promise a completely risk-free return?

Answer: The 8 percent t-bill return does not depend on the state of the economy because the treasury must (and will) redeem the bills at par regardless of the state of the economy.

The t-bills are risk-free in the default risk sense because the 8 percent return will be realized in all possible economic states. However, remember that this return is composed of the real risk-free rate, say 3 percent, plus an inflation premium, say 5 percent. Since there is uncertainty about inflation, it is unlikely that the realized real rate of return would equal the expected 3 percent. For example, if inflation averaged 6 percent over the year, then the realized real return would only be 8% - 6% = 2%, not the expected 3%. Thus, in terms of purchasing power, t-bills are not riskless.

Also, if you invested in a portfolio of T-bills, and rates then declined, your nominal income would fall; that is, t-bills are exposed to reinvestment rate risk. So, we conclude that there are no truly risk-free securities in the United States. If the treasury sold inflation-indexed, tax-exempt bonds, they would be truly riskless, but all actual securities are exposed to some type of risk.

b. 2. Why are Alta Ind.’s returns expected to move with the economy whereas Repo Men’s are expected to move counter to the economy?

Answer: Alta Industries’ returns move with, hence are positively correlated with, the economy, because the firm’s sales, and hence profits, will generally experience the same type of ups and downs as the economy. If the economy is booming, so will Alta. On the other hand, Repo Men is considered by many investors to be a hedge against both bad times and high inflation, so if the stock market crashes, investors in this stock should do relatively well. Stocks such as Repo Men are thus negatively correlated with (move counter to) the economy. (note: in actuality, it is almost impossible to find stocks that are expected to move counter to the economy. Even Repo Men shares have positive (but low) correlation with the market.)
c. Calculate the expected rate of return on each alternative and fill in the blanks on the row for $r$ in the table above.

**Answer:** The expected rate of return, $r$, is expressed as follows:

$$r = \sum_{i=1}^{n} p_i r_i.$$  

Here $p_i$ is the probability of occurrence of the $i$th state, $r_i$ is the estimated rate of return for that state, and $n$ is the number of states. Here is the calculation for Alta Inds.:

$$r_{Alta\ Inds} = 0.1(-22.0\%) + 0.2(-2.0\%) + 0.4(20.0\%) + 0.2(35.0\%) + 0.1(50.0\%) = 17.4\%.$$  

We use the same formula to calculate $r$’s for the other alternatives:

$$r_{T-bills} = 8.0\%.$$  

$$r_{Repo\ Men} = 1.7\%.$$  

$$r_{Am\ Foam} = 13.8\%.$$  

$$r_{M} = 15.0\%.$$
d. You should recognize that basing a decision solely on expected returns is only appropriate for risk-neutral individuals. Since your client, like virtually everyone, is risk averse, the riskiness of each alternative is an important aspect of the decision. One possible measure of risk is the standard deviation of returns.

1. Calculate this value for each alternative, and fill in the blank on the row for $\sigma$ in the table above.

**Answer:** The standard deviation is calculated as follows:

$$\sigma = \sqrt{\sum_{i=1}^{n} (r_i - \hat{r}_i)^2 p_i} .$$

$$\sigma_{\text{Alta}} = [(-22.0 - 17.4)^2(0.1) + (-2.0 - 17.4)^2(0.2) + (20.0 - 17.4)^2(0.4) + (35.0 - 17.4)^2(0.2) + (50.0 - 17.4)^2(0.1)]^{0.5}$$

$$= \sqrt{401.4} = 20.0\% .$$

Here are the standard deviations for the other alternatives:

$\sigma_{\text{T-bills}} = 0.0\% .$

$\sigma_{\text{Repo}} = 13.4\% .$

$\sigma_{\text{Am Foam}} = 18.8\% .$

$\sigma_{M} = 15.3\% .$

**d. 2. What type of risk is measured by the standard deviation?**

**Answer:** The standard deviation is a measure of a security’s (or a portfolio’s) stand-alone risk. The larger the standard deviation, the higher the probability that actual realized returns will fall far below the expected return, and that losses rather than profits will be incurred.
d. 3. Draw a graph which shows *roughly* the shape of the probability distributions for Alta Inds, Am Foam, and T-bills.

Answer:

![Graph showing probability distributions](image)

Based on these data, Alta Inds is the most risky investment, t-bills the least risky.
e. Suppose you suddenly remembered that the coefficient of variation (CV) is generally regarded as being a better measure of stand-alone risk than the standard deviation when the alternatives being considered have widely differing expected returns. Calculate the missing CVs, and fill in the blanks on the row for CV in the table above. Does the CV produce the same risk rankings as the standard deviation?

Answer: The coefficient of variation (CV) is a standardized measure of dispersion about the expected value; it shows the amount of risk per unit of return.

\[
CV = \frac{\sigma}{\hat{r}}.
\]

\[CV_{T\text{-bills}} = 0.0\%/8.0\% = 0.0.\]

\[CV_{Alta\ Inds} = 20.0\%/17.4\% = 1.1.\]

\[CV_{Repo\ Men} = 13.4\%/1.7\% = 7.9.\]

\[CV_{Am\ Foam} = 18.8\%/13.8\% = 1.4.\]

\[CV_M = 15.3\%/15.0\% = 1.0.\]

When we measure risk per unit of return, Repo Men, with its low expected return, becomes the most risky stock. The CV is a better measure of an asset’s stand-alone risk than \(\sigma\) because CV considers both the expected value and the dispersion of a distribution—a security with a low expected return and a low standard deviation could have a higher chance of a loss than one with a high \(\sigma\) but a high \(\hat{r}\).

f. Suppose you created a 2-stock portfolio by investing $50,000 in Alta Inds and $50,000 in Repo Men.

1. Calculate the expected return (\(\hat{r_p}\)), the standard deviation (\(\sigma_p\)), and the coefficient of variation (\(cv_p\)) for this portfolio and fill in the appropriate blanks in the table above.
**Answer:** To find the expected rate of return on the two-stock portfolio, we first calculate the rate of return on the portfolio in each state of the economy. Since we have half of our money in each stock, the portfolio’s return will be a weighted average in each type of economy. For a recession, we have: \( r_p = 0.5(-22\%) + 0.5(28\%) = 3\% \). We would do similar calculations for the other states of the economy, and get these results:

<table>
<thead>
<tr>
<th>State</th>
<th>Portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>3.0%</td>
</tr>
<tr>
<td>Below Average</td>
<td>6.4</td>
</tr>
<tr>
<td>Average</td>
<td>10.0</td>
</tr>
<tr>
<td>Above Average</td>
<td>12.5</td>
</tr>
<tr>
<td>Boom</td>
<td>15.0</td>
</tr>
</tbody>
</table>

Now we can multiply probabilities times outcomes in each state to get the expected return on this two-stock portfolio, 9.6%.

Alternatively, we could apply this formula,

\[
R = w_i \times r_i = 0.5(17.4\%) + 0.5(1.7\%) = 9.6\%,
\]

Which finds \( r \) as the weighted average of the expected returns of the individual securities in the portfolio.

It is tempting to find the standard deviation of the portfolio as the weighted average of the standard deviations of the individual securities, as follows:

\[
\sigma_p = w_i (\sigma_i) + w_j (\sigma_j) = 0.5(20\%) + 0.5(13.4\%) = 16.7\%.
\]

However, this is not correct—it is necessary to use a different formula, the one for \( \sigma \) that we used earlier, applied to the two-stock portfolio’s returns.

The portfolio’s \( \sigma \) depends jointly on (1) each security’s \( \sigma \) and (2) the correlation between the securities’ returns. The best way to approach the problem is to estimate the portfolio’s risk and return in each state of the economy, and then to estimate \( \sigma_p \) with the \( \sigma \) formula. Given the distribution of returns for the portfolio, we can calculate the portfolio’s \( \sigma \) and CV as shown below:

\[
\sigma_p = [(3.0 - 9.6)^2(0.1) + (6.4 - 9.6)^2(0.2) + (10.0 - 9.6)^2(0.4) + (12.5 - 9.6)^2(0.2) + (15.0 - 9.6)^2(0.1)]^{0.5} = 3.3\%.
\]

\[
CV_p = 3.3\%/9.6\% = 0.3.
\]
f. How does the riskiness of this 2-stock portfolio compare with the riskiness of the individual stocks if they were held in isolation?

Answer: Using either $\sigma$ or CV as our stand-alone risk measure, the stand-alone risk of the portfolio is significantly less than the stand-alone risk of the individual stocks. This is because the two stocks are negatively correlated--when Alta Inds is doing poorly, Repo Men is doing well, and vice versa. Combining the two stocks diversifies away some of the risk inherent in each stock if it were held in isolation, i.e., in a 1-stock portfolio.

g. Suppose an investor starts with a portfolio consisting of one randomly selected stock. What would happen (1) to the riskiness and (2) to the expected return of the portfolio as more and more randomly selected stocks were added to the portfolio? What is the implication for investors? Draw a graph of the two portfolios to illustrate your answer.

Answer:

![Graph of stock returns]

The standard deviation gets smaller as more stocks are combined in the portfolio, while $r_p$ (the portfolio’s return) remains constant. Thus, by adding stocks to your portfolio, which initially started as a 1-stock portfolio, risk has been reduced.
In the real world, stocks are positively correlated with one another--if the economy does well, so do stocks in general, and vice versa. Correlation coefficients between stocks generally range from +0.5 to +0.7. The average correlation between stocks is about 0.35. A single stock selected at random would on average have a standard deviation of about 35 percent. As additional stocks are added to the portfolio, the portfolio’s standard deviation decreases because the added stocks are not perfectly positively correlated. However, as more and more stocks are added, each new stock has less of a risk-reducing impact, and eventually adding additional stocks has virtually no effect on the portfolio’s risk as measured by $\sigma$. In fact, $\sigma$ stabilizes at about 20 percent when 40 or more randomly selected stocks are added. Thus, by combining stocks into well-diversified portfolios, investors can eliminate almost one-half the riskiness of holding individual stocks. (Note: it is not completely costless to diversify, so even the largest institutional investors hold less than all stocks. Even index funds generally hold a smaller portfolio which is highly correlated with an index such as the S&P 500 rather than hold all the stocks in the index.)

The implication is clear: investors should hold well-diversified portfolios of stocks rather than individual stocks. (In fact, individuals can hold diversified portfolios through mutual fund investments.) By doing so, they can eliminate about half of the riskiness inherent in individual stocks.

**h. 1. Should portfolio effects impact the way investors think about the riskiness of individual stocks?**

**Answer:** Portfolio diversification does affect investors’ views of risk. A stock’s stand-alone risk as measured by its $\sigma$ or CV, may be important to an undiversified investor, but it is not relevant to a well-diversified investor. A rational, risk-averse investor is more interested in the impact that the stock has on the riskiness of his or her portfolio than on the stock’s stand-alone risk. Stand-alone risk is composed of diversifiable risk, which can be eliminated by holding the stock in a well-diversified portfolio, and the risk that remains is called market risk because it is present even when the entire market portfolio is held.
h. If you decided to hold a 1-stock portfolio, and consequently were exposed to more risk than diversified investors, could you expect to be compensated for all of your risk; that is, could you earn a risk premium on that part of your risk that you could have eliminated by diversifying?

Answer: If you hold a one-stock portfolio, you will be exposed to a high degree of risk, but you won’t be compensated for it. If the return were high enough to compensate you for your high risk, it would be a bargain for more rational, diversified investors. They would start buying it, and these buy orders would drive the price up and the return down. Thus, you simply could not find stocks in the market with returns high enough to compensate you for the stock’s diversifiable risk.

i. How is market risk measured for individual securities? How are beta coefficients calculated?

Answer: Market risk, which is relevant for stocks held in well-diversified portfolios, is defined as the contribution of a security to the overall riskiness of the portfolio. It is measured by a stock’s beta coefficient, which measures the stock’s volatility relative to the market.

Run a regression with returns on the stock in question plotted on the y axis and returns on the market portfolio plotted on the x axis.

The slope of the regression line, which measures relative volatility, is defined as the stock’s beta coefficient, or b.

j. Suppose you have the following historical returns for the stock market and for another company, P.Q. Unlimited. Explain how to calculate beta, and use the historical stock returns to calculate the beta for PQU. Interpret your results.

<table>
<thead>
<tr>
<th>YEAR</th>
<th>MARKET</th>
<th>PQU</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25.7%</td>
<td>40.0%</td>
</tr>
<tr>
<td>2</td>
<td>8.0%</td>
<td>-15.0%</td>
</tr>
<tr>
<td>3</td>
<td>-11.0%</td>
<td>-15.0%</td>
</tr>
<tr>
<td>4</td>
<td>15.0%</td>
<td>35.0%</td>
</tr>
<tr>
<td>5</td>
<td>32.5%</td>
<td>10.0%</td>
</tr>
<tr>
<td>6</td>
<td>13.7%</td>
<td>30.0%</td>
</tr>
<tr>
<td>7</td>
<td>40.0%</td>
<td>42.0%</td>
</tr>
<tr>
<td>8</td>
<td>10.0%</td>
<td>-10.0%</td>
</tr>
<tr>
<td>9</td>
<td>-10.8%</td>
<td>-25.0%</td>
</tr>
<tr>
<td>10</td>
<td>-13.1%</td>
<td>25.0%</td>
</tr>
</tbody>
</table>
**Answer:** Betas are calculated as the slope of the “characteristic” line, which is the regression line showing the relationship between a given stock and the general stock market.

Show the graph with the regression results. Point out that the beta is the slope coefficient, which is 0.83. State that an average stock, by definition, moves with the market. Beta coefficients measure the relative volatility of a given stock relative to the stock market. The average stock’s beta is 1.0. Most stocks have betas in the range of 0.5 to 1.5. Theoretically, betas can be negative, but in the real world they are generally positive.

In practice, 4 or 5 years of monthly data, with 60 observations, would generally be used. Some analysts use 52 weeks of weekly data. Point out that the $r^2$ of 0.36 is slightly higher than the typical value of about 0.29. A portfolio would have an $r^2$ greater than 0.9.
k. The expected rates of return and the beta coefficients of the alternatives as supplied by Barney Smith’s computer program are as follows:

<table>
<thead>
<tr>
<th>Security</th>
<th>Return ((\bar{r}))</th>
<th>Risk (Beta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alta Inds</td>
<td>17.4%</td>
<td>1.29</td>
</tr>
<tr>
<td>Market</td>
<td>15.0</td>
<td>1.00</td>
</tr>
<tr>
<td>Am. Foam</td>
<td>13.8</td>
<td>0.68</td>
</tr>
<tr>
<td>T-Bills</td>
<td>8.0</td>
<td>0.00</td>
</tr>
<tr>
<td>Repo Men</td>
<td>1.7</td>
<td>(0.86)</td>
</tr>
</tbody>
</table>

(1) Do the expected returns appear to be related to each alternative’s market risk?  
(2) Is it possible to choose among the alternatives on the basis of the information developed thus far?

Answer: The expected returns are related to each alternative’s market risk—that is, the higher the alternative’s rate of return the higher its beta. Also, note that t-bills have 0 risk.

We do not yet have enough information to choose among the various alternatives. We need to know the required rates of return on these alternatives and compare them with their expected returns.
1. Write out the security market line (SML) equation, use it to calculate the required rate of return on each alternative, and then graph the relationship between the expected and required rates of return.

**Answer:** Here is the SML equation:

\[ r_i = r_{RF} + (r_m - r_{RF})b_i. \]

If we use the t-bill yield as a proxy for the risk-free rate, then \( r_{RF} = 8\% \). Further, our estimate of \( r_m = \hat{r}_m \) is 15\%. Thus, the required rates of return for the alternatives are as follows:

- **Alta Inds:** \( 8\% + (15\% - 8\%)1.29 = 17.03\% \approx 17.0\% \).
- **Market:** \( 8\% + (15\% - 8\%)1.00 = 15.0\% \).
- **Am Foam:** \( 8\% + (15\% - 8\%)0.68 = 12.76\% \approx 12.8\% \).
- **T-Bills:** \( 8\% + (15\% - 8\%)1.29 = 17.03\% \approx 17.0\% \).
- **Repo Men:** \( 8\% + (15\% - 8\%) -0.86 = 1.98\% \approx 2\% \).

1. How do the expected rates of return compare with the required rates of return?

**Answer:** We have the following relationships:

<table>
<thead>
<tr>
<th>SECURITY</th>
<th>Expected Return ( (\hat{r}) )</th>
<th>Required Return ( (r) )</th>
<th>CONDITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alta Inds</td>
<td>17.4%</td>
<td>17.0%</td>
<td>Undervalued: ( \hat{r} &gt; R )</td>
</tr>
<tr>
<td>Market</td>
<td>15.0</td>
<td>15.0</td>
<td>Fairly Valued (Market Equilibrium)</td>
</tr>
<tr>
<td>Am Foam</td>
<td>13.8</td>
<td>12.8</td>
<td>Undervalued: ( \hat{r} &gt; R )</td>
</tr>
<tr>
<td>T-Bills</td>
<td>8.0</td>
<td>8.0</td>
<td>Fairly Valued</td>
</tr>
<tr>
<td>Repo Men</td>
<td>1.7</td>
<td>2.0</td>
<td>Overvalued: ( R &gt; \hat{r} )</td>
</tr>
</tbody>
</table>
Mini Case: 6 - 23

(Note: the plot looks somewhat unusual in that the x axis extends to the left of zero. We have a negative beta stock, hence a required return that is less than the risk-free rate.) The t-bills and market portfolio plot on the SML, Alta Inds. And Am. Foam plot above it, and Repo Men plots below it. Thus, the t-bills and the market portfolio promise a fair return, Alta Inds and Am. Foam are good deals because they have expected returns above their required returns, and Repo Men has an expected return below its required return.

1. Does the fact that Repo Men has an expected return which is less than the t-bill rate make any sense?

Answer: Repo Men is an interesting stock. Its negative beta indicates negative market risk—including it in a portfolio of “normal” stocks will lower the portfolio’s risk. Therefore, its required rate of return is below the risk-free rate. Basically, this means that Repo Men is a valuable security to rational, well-diversified investors. To see why, consider this question: would any rational investor ever make an investment which has a negative expected return? The answer is “yes”—just think of the purchase of a life or fire insurance policy. The fire insurance policy has a negative expected return because of commissions and insurance company profits, but businesses buy fire insurance because they pay off at a time when normal operations are in bad shape. Life insurance is similar—it has a high return when work income ceases. A negative beta stock is conceptually similar to an insurance policy.
1. 4. What would be the market risk and the required return of a 50-50 portfolio of Alta Inds and Repo Men? Of Alta Inds and Am. Foam?

**Answer:** Note that the beta of a portfolio is simply the weighted average of the betas of the stocks in the portfolio. Thus, the beta of a portfolio with 50 percent Alta Inds and 50 percent Repo Men is:

\[ b_p = \sum_{i=1}^{n} w_i b_i. \]

\[ b_p = 0.5(b_{\text{Alta}}) + 0.5(b_{\text{Repo}}) = 0.5(1.29) + 0.5(-0.86) = 0.215, \]

\[ r_p = r_{RF} + (r_M - r_{RF})b_p = 8.0\% + (15.0\% - 8.0\%)(0.215) = 8.0\% + 7\%(0.215) = 9.51\% \approx 9.5\%. \]

For a portfolio consisting of 50% Alta Inds plus 50% Am. Foam, the required return would be 14.9%:

\[ b_p = 0.5(1.29) + 0.5(0.68) = 0.985. \]

\[ r_p = 8.0\% + 7\%(0.985) = 14.9\%. \]
m. 1. Suppose investors raised their inflation expectations by 3 percentage points over current estimates as reflected in the 8 percent t-bill rate. What effect would higher inflation have on the SML and on the returns required on high- and low-risk securities?

Answer:

Here we have plotted the SML for betas ranging from 0 to 2.0. The base case SML is based on $r_{RF} = 8\%$ and $r_M = 15\%$. If inflation expectations increase by 3 percentage points, with no change in risk aversion, then the entire SML is shifted upward (parallel to the base case SML) by 3 percentage points. Now, $r_{RF} = 11\%$, $r_M = 18\%$, and all securities’ required returns rise by 3 percentage points. Note that the market risk premium, $r_m - r_{RF}$, remains at 7 percentage points.

m. 2. Suppose instead that investors’ risk aversion increased enough to cause the market risk premium to increase by 3 percentage points. (Inflation remains constant.) What effect would this have on the SML and on returns of high- and low-risk securities?

Answer: When investors’ risk aversion increases, the SML is rotated upward about the y-intercept ($r_{RF}$). $r_{RF}$ remains at 8 percent, but now $r_M$ increases to 18 percent, so the market risk premium increases to 10 percent. The required rate of return will rise sharply on high-risk (high-beta) stocks, but not much on low-beta securities.
Web Appendix 6B
Calculating Beta Coefficients With a Financial Calculator

Solutions to Problems

6A-1  a. 

b = Slope = 0.62. However, b will depend on students’ freehand line. Using a calculator, we find b = 0.6171 ≈ 0.62.

b. Because b = 0.62, Stock Y is about 62% as volatile as the market; thus, its relative risk is about 62% that of an average stock.

c. 1. Stand-alone risk as measured by σ would be greater, but beta and hence systematic (relevant) risk would remain unchanged. However, in a 1-stock portfolio, Stock Y would be riskier under the new conditions.

2. CAPM assumes that company-specific risk will be eliminated in a portfolio, so the risk premium under the CAPM would not be affected. However, if the scatter were wide, we would not have as much confidence in the beta, and this could increase the stock's risk and thus its risk premium.

d. 1. The stock's variance and σ would not change, but the risk of the stock to an investor holding a diversified portfolio would be greatly reduced, because it would now have a negative correlation with \( r_M \).

2. Because of a relative scarcity of such stocks and the beneficial net effect on portfolios that include it, its “risk premium” is likely to be very low or even negative. Theoretically, it should be negative.
e. The following figure shows a possible set of probability distributions. We can be reasonably sure that the 100-stock portfolio comprised of b = 0.62 stocks as described in Condition 2 will be less risky than the “market.” Hence, the distribution for Condition 2 will be more peaked than that of Condition 3.

We can also say on the basis of the available information that \( \sigma_Y \) is smaller than \( \sigma_M \); Stock Y’s market risk is only 62% of the “market,” but it does have company-specific risk, while the market portfolio does not, because it has been diversified away. However, we know from the given data that \( \sigma_Y = 13.8\% \), while \( \sigma_M = 19.6\% \). Thus, we have drawn the distribution for the single stock portfolio more peaked than that of the market. The relative rates of return are not reasonable. The return for any stock should be

\[
    r_i = r_{RF} + (r_M - r_{RF})b_i.
\]

Stock Y has \( b = 0.62 \), while the average stock (M) has \( b = 1.0 \); therefore,

\[
    r_Y = r_{RF} + (r_M - r_{RF})0.62 < r_M = r_{RF} + (r_M - r_{RF})1.0.
\]

A disequilibrium exists—Stock Y should be bid up to drive its yield down. More likely, however, the data simply reflect the fact that past returns are not an exact basis for expectations of future returns.
f. The expected return could not be predicted with the historical characteristic line because the increased risk should change the beta used in the characteristic line.

g. The beta would decline to 0.53. A decline indicates that the stock has become less risky; however, with the change in the debt ratio the stock has actually become more risky. In periods of transition, when the risk of the firm is changing, the beta can yield conclusions that are exactly opposite to the actual facts. Once the company's risk stabilizes, the calculated beta should rise and should again approximate the true beta.
6B-1 a. 

b = Slope = 0.62. However, b will depend on students’ freehand line. Using a calculator, we find b = 0.6171 ≈ 0.62.

b. Because b = 0.62, Stock Y is about 62% as volatile as the market; thus, its relative risk is about 62% that of an average stock.

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e. The following figure shows a possible set of probability distributions. We can be reasonably sure that the 100-stock portfolio comprised of \( b = 0.62 \) stocks as described in Condition 2 will be less risky than the “market.” Hence, the distribution for Condition 2 will be more peaked than that of Condition 3.

We can also say on the basis of the available information that \( \sigma_Y \) is smaller than \( \sigma_M \); Stock Y’s *market* risk is only 62% of the “market,” but it does have company-specific risk, while the market portfolio does not, because it has been diversified away. However, we know from the given data that \( \sigma_Y = 13.8\% \), while \( \sigma_M = 19.6\% \). Thus, we have drawn the distribution for the single stock portfolio more peaked than that of the market. The relative rates of return are *not* reasonable. The return for any stock should be

\[
i = r_{RF} + (r_M - r_{RF})b_i.
\]

Stock Y has \( b = 0.62 \), while the average stock (M) has \( b = 1.0 \); therefore,

\[
r_Y = r_{RF} + (r_M - r_{RF})0.62 < r_M = r_{RF} + (r_M - r_{RF})1.0.
\]

A disequilibrium exists—Stock Y should be bid up to drive its yield down. More likely, however, the data simply reflect the fact that past returns are not an exact basis for expectations of future returns.

f. The expected return could not be predicted with the historical characteristic line because the increased risk should change the beta used in the characteristic line.
g. The beta would decline to 0.53. A decline indicates that the stock has become less risky; however, with the change in the debt ratio the stock has actually become more risky. In periods of transition, when the risk of the firm is changing, the beta can yield conclusions that are exactly opposite to the actual facts. Once the company's risk stabilizes, the calculated beta should rise and should again approximate the true beta.

6B-2 a. The slope of the characteristic line is the stock’s beta coefficient.

\[
\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta r_i}{\Delta \bar{r}_m}.
\]

\[
\text{Slope}_A = \text{Beta}_A = \frac{29.00 - 15.20}{29.00 - 15.20} = 1.0.
\]

\[
\text{Slope}_B = \text{Beta}_B = \frac{20.00 - 13.10}{29.00 - 15.20} = 0.5.
\]

b. \( \bar{r}_m = 0.1(-14\%) + 0.2(0\%) + 0.4(15\%) + 0.2(25\%) + 0.1(44\%) \)

\[= -1.4\% + 0\% + 6\% + 5\% + 4.4\% = 14\%.
\]
The graph of the SML is as follows:

\[ r_i = r_{RF} + (r_M - r_{RF})b_i = 9\% + (14\% - 9\%)b_i. \]

c. Required rate of return on Stock A:
\[ r_A = r_{RF} + (r_M - r_{RF})b_A = 9\% + (14\% - 9\%)1.0 = 14\%. \]

Required rate of return on Stock B:
\[ r_B = 9\% + (14\% - 9\%)0.5 = 11.50\%. \]

d. Expected return on Stock C = \( \hat{r}_C \) = 18\%.

Return on Stock C if it is in equilibrium:
\[ r_C = r_{RF} + (r_M - r_{RF})b_C = 9\% + (14\% - 9\%)2 = 19\% \neq 18\% = \hat{r}_C. \]

A stock is in equilibrium when its required return is equal to its expected return. Stock C’s required return is greater than its expected return; therefore, Stock C is not in equilibrium. Equilibrium will be restored when the expected return on Stock C is driven up to 19\%. With an expected return of 18\% on Stock C, investors should sell it, driving its price down and its yield up.