MATH 1634  Sketching curves in §4.1, §4.3, and §4.5.

• Most of the following problems appear in the textbook as example or exercise in Sections 4.1, 4.3, or 4.5. Locate them!!!

1. Example. Let $f(x) = x^3 - 2x^2$.
   (a) Find the critical points (i.e., possible local max/min points).
   (b) Find the local maxima or local minima. Your reason for your answer must be clear. Give your answers in cartesian coordinates.
(c) Find the possible inflection points.

(d) Find the inflection points. Your reason for your answer must be clear. Write your answers in cartesian coordinates.
2. **Practice.** Let $f(x) = x^3 - 3x^2$.
   Do (a)–(e).

(e) Sketch the curve. *Your reason for your answers must be clear.*
3. \textbf{Example.} Let $f(x) = x^4 - 4x^3$.

(a) Find the critical points (i.e., possible local max/min points).

(b) Find the local maxima or local minima. \textit{Your reason for your answer must be clear. Give your answers in cartesian coordinates.}
(c) Find the possible inflection points.

(d) Find the inflection points. Your reason for your answer must be clear. Write your answers in cartesian coordinates.
(e) Sketch the curve. Your reason for your answers must be clear.

4. Practice. Let \( f(x) = x^4 - 2x^3 \).
   Do (a)–(e).
5. Example. Let \( f(x) = x^{2/3}(6 - x)^{1/3} \).

(a) Find the critical points (i.e., possible local max/min points).

(b) Find the local maxima or local minima. Your reason for your answer must be clear. Give your answers in cartesian coordinates.
(c) Find the possible inflection points.

(d) Find the inflection points. Your reason for your answer must be clear. Write your answers in cartesian coordinates.
(e) Sketch the curve. Your reason for your answers must be clear.

6. **Practice.** Let $f(x) = x^{3/5}(4 - x)$.
   Do (a)–(d).
Example. Let \( f(x) = xe^{-3x} \).

(a) Find the critical points (i.e., possible local max/min points).

(b) Find the local maxima or local minima. Your reason for your answer must be clear. Give your answers in cartesian coordinates.
(c) Find the possible inflection points.

(d) Find the inflection points. Your reason for your answer must be clear. Write your answers in cartesian coordinates.
(e) Sketch the curve. Your reason for your answers must be clear.

(f) Find horizontal asymptotes if exist, and draw them back on (d).

8. Practice. Let \( f(x) = x^2 e^{-3x} \).
    Do (a)–(f).
9. **Example.** Let \( f(x) = \frac{x^2}{x^2 + 3} \).

Hint:

\[
f'(x) = \frac{6x}{(x^2 + 3)^2}, \quad f''(x) = \frac{-18(x + 1)(x - 1)}{(x^2 + 3)^3}.
\]

(a) Find the critical points (i.e., possible local max/min points).

(b) Find the local maxima or local minima. *Your reason for your answer must be clear. Give your answers in cartesian coordinates.*
(c) Find the possible inflection points.

(d) Find the inflection points. Your reason for your answer must be clear. Write your answers in cartesian coordinates.
(e) Sketch the curve. Your reason for your answers must be clear.

(f) Find horizontal asymptotes if exist, and draw them back on (d).

10. **Practice.** Let \( f(x) = \frac{x^2}{x^2 + 9} \).
    Do (a)–(f)
11. **Example.** Let \( f(x) = \frac{x}{x^2 + 1} \).

Hint:

\[
f'(x) = \frac{-(x + 1)(x - 1)}{(x^2 + 1)^2}, \quad f''(x) = \frac{2x(x + \sqrt{3})(x - \sqrt{3})}{(x^2 + 1)^3}.
\]

(a) Find the critical points (i.e., possible local max/min points).

(b) Find the local maxima or local minima. *Your reason for your answer must be clear. Give your answers in cartesian coordinates.*
(c) Find the possible inflection points.

(d) Find the inflection points. Your reason for your answer must be clear. Write your answers in cartesian coordinates.
(e) Sketch the curve. Your reason for your answers must be clear.

(f) Find horizontal asymptotes if exist, and draw them back on (d).

12. Practice. Let $f(x) = \frac{x}{x^2 + 9}$.

Do (a)–(f).
13. **Example.** Let $g(\theta) = 4\theta - \tan \theta$.
   Do (a)–(e).

14. **Example.** Let $f(x) = \frac{x^2 + 4}{x^2 - 1}$.
    Do (a)–(f).