MATH 1634 The cases when computing the limit $\lim_{x\to a} f(x)$
(and f is not continuous at $x = a$)

Show your each step **accurately**.

1. $\frac{\text{polynomial}}{\text{polynomial}} \Rightarrow$ Factor out and cancel the common factors. §**2.3**
2. $\sqrt{A} \pm \sqrt{B} \Rightarrow$ Multiply the conjugate to the top & and bottom. §**2.3**
3. Piecewise function.
   (a) Piecewise function, and f takes the same function at the left and the right near $x = a$.
   $\Rightarrow$ Replace $f(x)$ by the function. §**2.3**
   (b) Piecewise function, and f takes the different functions at the left and the right near $x = a$.
   $\Rightarrow$ Evaluate the Left-hand limit (LHL) and Right-hand limit (RHL) separately; replace $f(x)$ by appropriate functions *depending on the directions to approach.* §**2.3**
4. Absolute value $|\cdot|$ (This is basically the same as the previous case 3b, but disguised in $|\cdot|$)
   $\Rightarrow$ Use the fact that $|A| = A$ when $A > 0$, and $|A| = -A$ when $A < 0$ to remove $|\cdot|$ and rewrite f into a piecewise function. Then follow the case 3b. §**2.3**
5. Squeeze theorem.
   $\Rightarrow$
   (a) Find a lower bound function $g(x)$ and an upper bound function $h(x)$ for $f(x)$, i.e., $g(x) \leq f(x) \leq h(x)$,
   (b) These two functions must have the same limit, which is also the limit for $f(x)$, i.e., $\lim g(x) = \lim h(x) = L$ exists so that $\lim f(x) = L$.

   You will see two types of problems in exams; easy ones that provide the lower and upper bound functions for $f(x)$; the ones in intermediate level that typically have the sine or cosine functions in which you can start with the inequality $-1 \leq \sin x, \cos x \leq 1$ to obtain the "useful" lower and upper bound functions for $f(x)$. §**2.3**
6. Using $\displaystyle\lim_{x\to 0} \frac{\sin\theta}{\theta} = 1$ §**3.3**