MATH 1634 The cases when computing the limit \( \lim_{x \to a} f(x) \)
(and \( f \) is not continuous at \( x = a \))

Show your each step accurately.

1. \( \frac{\text{polynomial}}{\text{polynomial}} \Rightarrow \) Factor out and cancel the common factors. §2.3

2. \( \sqrt{A} \pm \sqrt{B} \Rightarrow \) Multiply the conjugate to the top & and bottom. §2.3

3. Piecewise function.
   
   (a) Piecewise function, and \( f \) takes the same function at the left and the right near \( x = a \).
   
   \( \Rightarrow \) Replace \( f(x) \) by the function. §2.3

   (b) Piecewise function, and \( f \) takes the different functions at the left and the right near \( x = a \).
   
   \( \Rightarrow \) Evaluate the Left-hand limit (LHL) and Right-hand limit (RHL) separately; replace \( f(x) \) by appropriate functions depending on the directions to approach. §2.3

4. Squeeze theorem.
   
   \( \Rightarrow \)

   (a) Find a lower bound function \( g(x) \) and an upper bound function \( h(x) \) for \( f(x) \), i.e., \( g(x) \leq f(x) \leq h(x) \).

   (b) These two functions must have the same limit, which is also the limit for \( f(x) \), i.e., \( \lim g(x) = \lim h(x) = L \) exists so that \( \lim f(x) = L \).

You will see two types of problems in exams; easy ones that provide the lower and upper bound functions for \( f(x) \); the ones in intermediate level that typically have the sine or cosine functions in which you can start with the inequality \(-1 \leq \sin x, \cos x \leq 1\) to obtain the “useful” lower and upper bound functions for \( f(x) \). §2.3