Limit laws as $x \to \infty$

The Limit Laws with $x \to \infty$ are listed below. It is important to know whether each step is legal or illegal while computing limits with $x \to \infty$. (In fact, there are Limit Laws with $x \to a$ for a finite number (numerical value) a in section 2.3 of the textbook, which are similar to the followings.)

•Suppose that c is a constant and **BOTH LIMITS**

$$\lim_{x\to\infty} f(x) \quad \text{and} \quad \lim_{x\to\infty} g(x)$$

EXIST. Then

1. $\lim\limits_{x\to\infty} \Big(f(x)+g(x)\Big) = \lim\limits_{x\to\infty} f(x) + \lim\limits_{x\to\infty} g(x)$
2. $\lim\limits_{x\to\infty} \Big(f(x)-g(x)\Big) = \lim\limits_{x\to\infty} f(x) - \lim\limits_{x\to\infty} g(x)$
3. $\lim\limits_{x\to\infty} \Big(cf(x)\Big) = c\lim\limits_{x\to\infty} f(x)$
4. $\lim\limits_{x\to\infty} \Big(f(x)g(x)\Big) = \lim\limits_{x\to\infty} f(x) \cdot \lim\limits_{x\to\infty} g(x)$
5. $\lim\limits_{x\to\infty} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to\infty} f(x)}{\lim_{x\to\infty} g(x)}$
6. $\lim\limits_{x\to\infty} \Big(f(x)\Big)^n = \Big(\lim\limits_{x\to\infty} f(x)\Big)^n$ where n is a positive integer
7. $\lim\limits_{x\to\infty} c = c$
8. $\lim\limits_{x\to\infty} x = \infty$ (too trivial !)

   Laws 9 and 10 in Section 2.3 don't make much sense for the case of $x \to \infty$.

11. $\lim\limits_{x\to\infty} \sqrt[n]{f(x)} = \sqrt[n]{\lim\limits_{x\to\infty} f(x)}$ where n is a positive integer

•Easy facts that we will use without proof:

$$\lim_{x\to\infty} \frac{1}{x} = 0$$

Furthermore, if $r > 0$ is a rational number, then

$$\lim_{x\to\infty} \frac{1}{x^r} = 0$$

Turn over please.

•Strictly speaking, the followings are not the Limit Laws. But they are **right conclusions**:

$\infty + \infty = \infty$ or $-\infty - \infty = -\infty$

$\infty \cdot \infty = \infty$ or $(-\infty) \cdot (-\infty) = \infty$

$\infty \pm c = \infty$ or $(-\infty) \pm c = -\infty$ for a constant c

$c \cdot \infty = \infty$ for a constant $c > 0$, $\quad c \cdot \infty = -\infty$ for a constant $c < 0$

$\dfrac{c}{\infty} = 0$ for a constant c

$\dfrac{\infty}{c} = \infty$ for a constant $c > 0$, $\quad \dfrac{\infty}{c} = -\infty$ for a constant $c < 0$

•**Wrong conclusions**:

$\infty - \infty = 0$ or ∞

$0 \cdot \infty = 0$ or ∞

$\dfrac{\infty}{\infty} = 0, 1$ or ∞

$\dfrac{0}{0} = 0, 1,$ or ∞