1. Find \( \frac{dy}{dx} \) of the given functions.

(a) \( y = x^{4/3} - x^{3/4} \)

\[
\frac{dy}{dx} = \frac{4}{3} x^{\frac{1}{3}} - \frac{3}{4} x^{-\frac{1}{4}}
\]

(b) \( y = x \left( x + \frac{1}{x} \right) = x^2 + 1 \)

\[
\frac{dy}{dx} = 2x
\]

OR

Product rule

\[
\frac{dy}{dx} = \left( x + \frac{1}{x} \right) + x \left( 1 - x^2 \right)
\]

\[
= x + \frac{1}{x} + x - \frac{x^3}{x}
\]

\[
= 2x
\]
(c) \( y = \frac{1}{\ln x} = (\ln x)^{-1} \)

\[ \frac{dy}{dx} = - (\ln x)^{-2} \cdot \frac{1}{x} = - \frac{1}{x \ln^2 x} \]

OR

Quotient rule

\[ \frac{dy}{dx} = \frac{0 \cdot \ln x - (1 - \frac{1}{x})}{\ln^2 x} = - \frac{1}{x \ln^2 x} \]

(d) \( y = \sin^2 x + \sin x^2 \)

\[ \frac{dy}{dx} = 2 \sin x \cos x + (\cos x^2) (2x) \]

\[ = 2 \sin x \cos x + 2x \cos x \]

\[ = 2 \cos x (\sin x + x \cos x) \]
(e) \( y = \frac{\sin(\cos x)}{\sin x} \)

\[
\frac{dy}{dx} = \frac{(\cos(\cos x))(-\sin x)(\sin x) - (\sin(\cos x)) \cos x}{\sin^2 x}
\]

\[
= \frac{\sin^2 x \cdot \cos(\cos x) - \cos x \cdot \sin(\cos x)}{\sin^2 x}
\]

(f) \( xe^y = x - y \)

\[
e^y + xe^y \frac{dy}{dx} = 1 - \frac{dy}{dx}
\]

\[
(xe^y + 1) \frac{dy}{dx} = 1 - e^y
\]

\[
\frac{dy}{dx} = \frac{1 - e^y}{xe^y + 1}
\]
2. A particle moves along the curve given by \( f(t) = e^{3t} - 1 \) for \( t \geq 0 \).

(a) Find the velocity at time \( t = 0 \).

\[
\begin{align*}
\mathbf{v}(t) &= f'(t) = 3e^{3t} \\
\mathbf{v}(0) &= 3e^{3 \cdot 0} = 3
\end{align*}
\]

(b) Find the acceleration at time \( t = 0 \).

\[
\begin{align*}
\mathbf{a}(t) &= \mathbf{v}'(t) = 9e^{3t} \\
\mathbf{a}(0) &= 9e^{3 \cdot 0} = 9
\end{align*}
\]
3. Find an equation of the tangent line to the curve given by

\[ f(x) = \frac{x + 1}{x} \]

at the point \((1, 2)\).

\[ f(x) = 1 + \frac{1}{x} = 1 + x^{-1} \]

\[ f'(x) = -x^{-2} \]

\[ m = f'(1) = -1^{-2} = -1 \]

\[ y - 2 = -(x - 1) \]

\[ y = -x + 3 \]
4. Use differentials to estimate the change of $\sqrt{x}$ when $x$ changes from 1 to 1.04. Simplify your answer.

$$f(x) = \sqrt{x} = x^{\frac{1}{2}} \Rightarrow f'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$dy = f'(x)dx$$

$$dy = \frac{1}{2\sqrt{x}} dx$$

$$dx = 1.04 - 1 = 0.04$$

$$dy = \frac{1}{2\sqrt{1}} (0.04) = 0.02$$
5. The area of a square is decreasing at a rate of 20 cm$^2$/h. How fast is the length of an edge decreasing when the area is 25 cm$^2$?

(a) Picture and Parameters.

![A square with side length $x$]

(b) Known rate(s)

\[
\frac{dA}{dt} = -20 \text{ cm}^2/\text{h}
\]

(c) Unknown rate to find out

\[
\frac{dx}{dt} \bigg|_{A=25 \text{ cm}^2}
\]

(d) An equation that relates parameters in (a)

\[ A = x^2 \]

\[ 25 = x^2 \Rightarrow x = 5 \]

(e) Differentiate the equation in (d) with respect to an appropriate variable

\[
\frac{dA}{dt} = 2x \frac{dx}{dt}
\]

(f) Find the unknown rate. (Write the unit also.)

\[
\frac{dx}{dt} \bigg|_{A=25} = \frac{1}{2(5)} (-20) = -2 \text{ cm/h}
\]