MATH 1634  Hour Exam 2  Spring 2017

- 100 points for total.
- No calculator. Show all your work clearly. Correct final answers without appropriate steps won’t get credit.
- Don’t give multiple solutions that contradict each other.

1. Find $\frac{dy}{dx}$ of the given functions.

(a) $y = \sqrt{x} + 7 = x^{\frac{1}{2}} + 7$

\[
\frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}}
\]

(b) $y = (x + \frac{1}{x})^2$

Method 1 (Algebra first)

\[
y = x^2 + 2 + \frac{1}{x^2}
\]

\[
\frac{dy}{dx} = 2x - 2x^{-3}
\]

Method 2 (Chain Rule)

\[
\frac{dy}{dx} = 2(x + \frac{1}{x})(1 - x^2)
\]

\{ simplify \}

Should be the same as
(c) \( y = \frac{1 + 5x^3}{x^3} \)

\[
\frac{dy}{dx} = \frac{(15x^2) \cdot x^3 - (1 + 5x^3) \cdot 3x^2}{(x^3)^2}
\]

\[
= \frac{15x^5 - 3x^2 - 15x^5}{x^6}
\]

\[
= -\frac{3}{x^4}
\]

You must give

\((x^3) = x^6\)

(d) \( y = \ln^2 x + \ln x^2 \)

\[
\frac{dy}{dx} = 2(\ln x) \cdot \frac{1}{x} + \frac{1}{x^2} \cdot 2x
\]

\[
= \frac{2\ln x}{x} + \frac{2}{x}
\]

\[
= \frac{2(\ln x + 1)}{x}
\]

Comment: No tolerance, if the given eqn is interpreted into \((\ln^2 x) \cdot x + \ln(x^2)\) and apply product rule to one or both terms.
(e) \( y = e^{2x} \sin x \)

\[
\frac{dy}{dx} = e^{2x} \cdot 2 \cdot \sin x + e^{2x} \cdot \cos x \\
= e^{2x} \left( 2 \sin x + \cos x \right)
\]

(f) \( x^3y + xy^3 = 5 \)

\[
3x^2 \cdot y + x^3 \frac{dy}{dx} + y^3 + x \cdot 3y^2 \cdot \frac{dy}{dx} = 0 \\
\Rightarrow \left(x + 3xy^2\right) \frac{dy}{dx} = -3x^2y - y^3 \\
\Rightarrow \frac{dy}{dx} = -\frac{3x^2y + y^3}{x^3 + 3xy^2}
\]
2. Find \( \frac{d^2y}{dx^2} \) (i.e., the second derivative) of

\[ y = \tan x. \]

\[ \frac{dy}{dx} = \sec^2 x \]

\[ \frac{d^2y}{dx^2} = 2 \sec x \cdot (\sec x \cdot \tan x) \]

\[ = 2 \sec^2 x \cdot \tan x \]
3. The position of a particle is given by the equation $s(t) = \cos t$

(a) Find the velocity at time $t = 3\pi/4$.

\[
\begin{align*}
\dot{v}(t) &= \frac{ds}{dt} = -\sin t \\
\dot{v}\left(\frac{3\pi}{4}\right) &= -\sin \frac{3\pi}{4} = -\frac{1}{\sqrt{2}}
\end{align*}
\]

(b) Find the acceleration at time $t = 3\pi/4$.

\[
\begin{align*}
a(t) &= \frac{d\dot{v}}{dt} = -\cos t \\
a\left(\frac{3\pi}{4}\right) &= -\cos \frac{3\pi}{4} \\
&= -\left(-\frac{1}{\sqrt{2}}\right) \\
&= \frac{1}{\sqrt{2}}
\end{align*}
\]
4. Find an equation of the tangent line to the curve given by

\[ f(x) = \frac{1}{x+1}, \]

at the point \((1, 1/2)\)

\[ f'(x) = \frac{1}{(x+1)^2} \]

\[ f'(1) = \frac{1}{2^2} = \frac{1}{4} \]

\[ y - \frac{1}{2} = -\frac{1}{4} (x-1) \]

OR \[ y = -\frac{1}{4} x + \frac{1}{4} + \frac{1}{2} = -\frac{1}{4} x + \frac{3}{4} \]
5. The volume of a cube is decreasing at a rate of 30 cm³/min. How fast is the length of an edge decreasing when the length of an edge is 10 cm?

(a) Picture and Parameters. (Don’t forget the parameters!)

(b) Known rate(s). (Write in equation rather than only numbers.)

\[
\frac{dV}{dt} = -30 \text{ cm}^3/\text{min}
\]

(c) Unknown rate to find out.

\[
\left. \frac{dx}{dt} \right|_{x=10} \text{ cm}
\]

(d) An equation that relates parameters in (a).

\[
3 = x
\]

(e) Differentiate the equation in (d) with respect to an appropriate variable.

\[
\frac{dV}{dt} = 3 \cdot 10^2 \cdot \frac{dx}{dt}
\]

(f) Find the unknown rate. (Write the unit also.)

\[
-30 = 3 \cdot 10^2 \cdot \frac{dx}{dt} \implies \frac{dx}{dt} = -\frac{1}{10} \text{ cm/ min} = -0.1 \text{ cm/ min}
\]