1. Let $y = x^3$.

   (a) Find the differential $dy$ in terms of the differential $dx$.

   (b) Use differentials to estimate the change of $x^3$ when $x$ changes from 2 to 2.01.
2. Find the critical points of \( f(x) = \frac{\ln x}{x} \).

3. For the given function \( f(x) = x^3 - 3x \), find \( x = c \) of Mean Value Theorem on the interval \([0, \sqrt{3}]\).
4. Use L’Hospital’s rule to find the limit. Determine first whether L’Hospital’s rule is appropriate.

(a) \( \lim_{x \to 0} \frac{1 - \cos x}{x^2} \)

(b) \( \lim_{x \to \infty} \frac{\ln x}{\sqrt{x}} \)
5. Let

\[ f(x) = \frac{x^2}{x^2 + 3}. \]

Use

\[ f'(x) = \frac{6x}{(x^2 + 3)^2}, \quad f''(x) = \frac{-18(x + 1)(x - 1)}{(x^2 + 3)^3} \]

to answer the followings.

(a) Find the local maxima or minima.

(b) Find the inflection points.
(c) Sketch the curve. Your reason should be clear if it’s your own method, or as we did in class such as Big table.

(d) Find the horizontal asymptote(s). Give the asymptote(s) in equation(s), and draw the asymptote(s) on your picture in (c).
6. A farmer with 10,000 feet of fencing wants to enclose a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. Find the dimension to maximize the area.

(a) Picture and Variables.

(b) Set up, but do not evaluate, the function and the equation in terms of two variables as we did in class. That is, choose MAXIMIZE or MINIMIZE, find appropriate expressions at underlines ___ in the followings;

\[
\begin{align*}
\text{MAXIMIZE/MINIMIZE} & \quad \underline{\text{ }} \\
\text{Restricted to} & \quad \underline{\text{ }}
\end{align*}
\]

(The object to optimize using two variables from (a))
(The equation type of restriction on your two variables)

(c) Set up, but do not evaluate, the single variable function and the interval to be solved as we did in class.

\[
\begin{align*}
\text{MAXIMIZE/MINIMIZE} & \quad (\ ) = \underline{\text{ }} \\
\text{Restricted to} & \quad \underline{\text{ }}
\end{align*}
\]

(The same object to optimize as in (b), rephrased as a single variable function)
(A valid (open or closed) interval on your variable)
(d) Find the optimum points of the function in (c). Show the reason why your critical point yields the optimum value.

(e) Interpret your answer. That is, state clearly the dimensions.