1. Evaluate indefinite integrals and definite integrals. Show your substitution clearly if you use any.

(a) \[ \int \frac{x^2 + 1}{x^2} \, dx = \int (1 + x^{-2}) \, dx \]
\[ = x + \frac{1}{x} + C \]
\[ = x - \frac{1}{x} + C \]

(b) \[ \int \tan x \sec^2 x \, dx \]
\[ u = \tan x \]
\[ du = \sec^2 x \, dx \]
\[ \int u \, du = \frac{u^2}{2} + C \]
\[ = \frac{\tan^2 x}{2} + C \]
(c) \( \int (x + 2) \sqrt{x^2 + 4x} \, dx = \int \sqrt{u} \cdot \frac{du}{2} \)

\[ u = x^2 + 4x \]
\[ du = (2x + 4) \, dx \]
\[ = 2(x + 2) \, dx \]
\[ \Rightarrow (x + 2) \, dx = \frac{du}{2} \]
\[ = \frac{1}{2} \int u^{1/2} \, du \]
\[ = \frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C \]
\[ = \frac{1}{3} (x^2 + 4x)^{3/2} + C \]

(d) \( \int_1^2 \frac{1}{x} (x - 1) \, dx \) Simplify your answer as much as possible.

\[ = \int_1^2 \left( 1 - \frac{1}{x} \right) \, dx \]
\[ = \left[ x - \ln x \right]_1^2 \]
\[ = (2 - \ln 2) - (1 - \ln 1) \]
\[ = 1 - \ln 2 \]
(e) \[ \int_0^1 \frac{1}{\sqrt{4x + 1}} \, dx = \frac{5}{4} \ln \left( \frac{1 + \sqrt{5}}{2} \right) \]

\[ u = 4x + 1 \]
\[ du = 4 \, dx \]
\[ dx = \frac{du}{4} \]

\[ \frac{5}{4} \int_1^5 \frac{1}{\sqrt{u}} \, du = \frac{5}{4} \cdot 2 \left[ u^\frac{1}{2} \right]_1^5 = \frac{1}{2} \left( \sqrt{5} - 1 \right) \]

(f) \[ \int_0^1 e^{3x} \, dx = \frac{1}{3} \int_0^3 e^u \, du \]

\[ u = 3x \]
\[ du = 3 \, dx \]

\[ \frac{1}{3} \left[ e^u \right]_0^3 = \frac{1}{3} \left( e^3 - e^0 \right) = \frac{1}{3} \left( e^3 - 1 \right) \]
2. Find \( \frac{d}{dx} \int_{100}^{x} \sqrt{t^6 - 1} \, dt \).

\[ \int_{100}^{x} \sqrt{t^6 - 1} \, dt = \int \left( \sqrt{x} - 1 \right) \, dx \]

3. Evaluate the integral by interpreting it in terms of area. Sketch and shade the appropriate area. Show your work clearly. Simplify your answer as much as possible.

\[ \int_{-3}^{3} \sqrt{9 - x^2} \, dx \]

\[ y = \sqrt{9 - x^2} \]
\[ \Rightarrow y^2 = 9 - x^2 \]
\[ \Rightarrow x^2 + y^2 = 9 \]
\[ \Rightarrow x + y = 3 \]

\[ A = \frac{1}{2} \cdot \pi \cdot 3^2 = \frac{9\pi}{2} \]
4. Find the area of the region between the $x$-axis, the curve $y = \cos x$ over the interval $-\pi \leq x \leq \pi$.

(a) Sketch the curves and shade the region. Mark the intercepts of the curves clearly if there are any.

(b) Set up, but do NOT evaluate, the integral.

$$\int_{-\frac{\pi}{2}}^{0} \cos x \, dx + \int_{0}^{\frac{\pi}{2}} \cos x \, dx - \int_{\frac{\pi}{2}}^{\pi} \cos x \, dx \quad \text{OR} \quad 2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \, dx$$

(c) Evaluate the integral in (b) to find the area. Simplify your answer as much as possible.

$$4 \int_{0}^{\frac{\pi}{2}} \cos x \, dx = 4 \left[ \sin x \right]_{0}^{\frac{\pi}{2}} = 4 \left( \sin \frac{\pi}{2} - \sin 0 \right) = 4 \left( 1 - 0 \right) = 4$$
5. Consider the area $A$ of the region that lies under the graph $f(x) = 3x^2 + 1$ over the interval $[0, 2]$.

(a) Estimate the area $A$ using four approximating rectangles, i.e., $n = 4$, and by taking the sample points to be right-most points. You don't need to evaluate your estimate; you can leave your answer in a raw form such as $9/10(0.5)^3 + 10/10(0.7)^6$.

\[
A \approx \frac{1}{2} \left( \frac{1}{2} \right)^2 + \frac{1}{2} \left( \frac{1}{2} \right)^2 + \frac{1}{2} \left( \frac{3}{2} \right)^2 + \frac{1}{2} \left( \frac{3}{2} \right)^2
\]

(b) Find the exact area $A$ using definite integrals.

\[
A = \int_{0}^{2} (3x^2 + 1) \, dx
\]

\[
= \left[ x + x \right]_{0}^{2}
\]

\[
= (8 + 2) - 0
\]

\[
= 10
\]