• 100 points for total.
• No calculator. Show all your work clearly.
• Presentation counts !!! Bad presentations are such as wrong notations or omitting $dx$, etc.

1. Evaluate indefinite integrals and definite integrals. Show your substitution clearly if you use any.

(a) $\int \frac{x^2 + 1}{x^2} \, dx = \int \left(1 + \frac{1}{x^2}\right) \, dx$

$= x + \frac{x^{-1}}{-1}$

$= x - \frac{1}{x} + C$

(b) $\int \tan x \sec^2 x \, dx$

$u = \tan x$

$du = \sec^2 x \, dx$

$\int u \, du = \frac{u^2}{2} + C$

$= \frac{\tan^2 x}{2} + C$
(c) \[ \int (x+2)\sqrt{x^2+4x} \, dx \]

\[ u = x + 4 \]
\[ du = (2x + 4) \, dx \]
\[ = 2(x + 2) \, dx \]

\[ = \frac{1}{2} \int u^{\frac{3}{2}} \, du \]
\[ = \frac{1}{3} u^\frac{5}{2} + C \]
\[ = \frac{(x + 4)^{\frac{5}{2}}}{3} + C \]

(d) \[ \int_1^2 \frac{1}{x(x - 1)} \, dx \]

Simplify your answer as much as possible.

\[ = \int_1^2 \left( 1 - \frac{1}{x} \right) \, dx \]
\[ = \left[ x - \ln|x| \right]_1^2 \]
\[ = (2 - \ln 2) - (1 - \ln 1) \]
\[ = 1 - \ln 2 \]
(e) \[ \int_0^1 \frac{1}{\sqrt{4x+1}} \, dx \]

\[ = \int_1^{5} \frac{1}{\sqrt{u}} \cdot \frac{du}{4} \]

\[ = \frac{1}{4} \int_1^{5} u^{-\frac{1}{2}} \, du \]

\[ = \frac{1}{4} \left[ \frac{2}{u^{\frac{1}{2}}} \right]_1^5 \]

\[ = \frac{1}{2} \left( \sqrt{5} - 1 \right) \]

\[ u = 4x + 1 \]

\[ du = 4 \, dx \]

2. Find \( \frac{d}{dx} \int_{100}^{x} \sqrt{t^5 - 1} \, dt \).

\[ = 3 \sqrt{x^5 - 1} \] by FTC I.
3. Evaluate the integral by interpreting it in terms of area. Sketch and shade the appropriate area. Show your work clearly. Simplify your answer as much as possible.

\[ \int_{-2}^{2} (1 + \sqrt{4 - x^2}) \, dx \]

\[ y = 1 + \sqrt{4 - x^2} \]

\[ \Rightarrow y - 1 = \sqrt{4 - x^2} \]

\[ \Rightarrow (y - 1)^2 = 4 - x^2 \]

\[ \Rightarrow x^2 + (y - 1)^2 = 2^2 \]

That is,

\[ A = \frac{1}{2} \cdot \pi \cdot 2^2 + 4 \]

\[ = 2\pi + 4 \]
4. Find the area of the region enclosed by the \( x \)-axis, the curve \( y = 4 - x^2 \).

(a) Sketch the curves and shade the region. *Mark the intercepts of the curves clearly if there are any.*

(b) Set up, but do NOT evaluate, the integral.

\[
\int_{-2}^{2} (4 - x^2) \, dx = 2 \int_{0}^{2} (4 - x^2) \, dx
\]

(c) Evaluate the integral in (b) to find the area. *Simplify your answer as much as possible.*

\[
2 \left[ 4x - \frac{x^3}{3} \right]_{0}^{2} = 2 \left( \frac{16}{3} \right)= \frac{32}{3}
\]
5. Consider the area $A$ of the region that lies under the graph $f(x) = \sin x$ over the interval $[0, \pi/2]$.

(a) Estimate the area $A$ using four approximating rectangles, i.e., $n = 2$, and by taking the sample points to be right-most points.

\[ A \approx \frac{\pi}{4} \cdot \sin \frac{\pi}{4} + \frac{\pi}{4} \cdot \sin \frac{\pi}{2} \]

(b) Find the exact area $A$ using definite integrals.

\[ A = \int_{0}^{\pi/2} \sin x \, dx \]

\[ = \left[ -\cos x \right]_{0}^{\pi/2} \]

\[ = -\cos \frac{\pi}{2} + \cos 0 \]

\[ = 1 \]
6. For the given integrals \( \int_1^e \frac{(\ln x)^2}{x} \, dx \), answer the followings.

(a) Evaluate the integral by substitution.

\[
u = \ln x
\]
\[
du = \frac{1}{x} \, dx
\]
\[
\int u^2 \, du = \left[ \frac{u^3}{3} \right]_0^1 = \frac{1}{3}
\]

(b) Sketch the region that is represented by the definite integral in (a) after the \( u \)-substitution on the \( uv \)-plane.

(c) What is the area of the region that you draw in (b)?

\[
\frac{1}{3}
\]

(d) What is the area of the region that is represented by the given integral?

\[
\frac{1}{3}
\]