

# Practice Exam for Exam 3, Math 3303 (ODE), Spring 2007

Print Your Name:

**Direction:** Show all your work. Work without sufficient detail will not get full credits.

1. (15 points) Use the Laplace transform to solve the initial-value problem  $y'' - y' - 6y = 0$ ,  $y(0) = 4$ ,  $y'(0) = 1$ . **No credit if the Laplace transform is not used.**
2. (10 points) Use the Laplace transform to solve the initial-value problem  $y' - y = 2te^t$ ,  $y(0) = 1$ . **No credit if the Laplace transform is not used.**
3. (15 points) Find  $\mathcal{L}^{-1} \left\{ \frac{2s}{s^2 - 4s + 5} \right\}$ .
4. (10 points) Find  $\mathcal{L}^{-1} \left\{ \frac{2se^{-s}}{(s-2)^2 + 1} \right\}$ .
5. (15 points) Find  $\mathcal{L} \{t^2 e^{2t} \sin 3t\}$ .
6. (10 points) Suppose that  $|f(t)| \leq 10$  for all  $t \geq 0$ . Prove that  $\mathcal{L}\{f'(t)\} = sF(s) - f(0)$ ,  $s > 0$ , where  $\mathcal{L}\{f(t)\} = F(s)$ ,  $s > 0$ .
7. Find the following: You do not need to justify your answer in these problems.
  - a) (5 points)  $\mathcal{L} \{t^3 + 6t\}$
  - b) (5 points)  $\mathcal{L} \{\sin(2t) - \cos t\}$
  - c) (5 points)  $\mathcal{L} \{t^4 e^{\pi t}\}$
  - d) (5 points)  $\mathcal{L} \{\cos(t - \pi)\mathcal{U}(t - \pi)\}$
8. Write  $\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} + \sum_{n=0}^{\infty} c_n x^{n+1}$  as a single power series whose general term involves  $x^n$ .