

# On Generalized Van der Waerden Triples

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## Abstract

Van der Waerden's classical theorem on arithmetic progressions states that for any positive integers  $k$  and  $r$ , there exists a least positive integer,  $w(k, r)$ , such that any  $r$ -coloring of  $\{1, 2, \dots, w(k, r)\}$  must contain a monochromatic  $k$ -term arithmetic progression  $\{x, x+d, x+2d, \dots, x+(k-1)d\}$ . We investigate the following generalization of  $w(3, r)$ . For fixed positive integers  $a$  and  $b$  with  $a \leq b$ , define  $N(a, b; r)$  to be the least positive integer, if it exists, such that any  $r$ -coloring of  $\{1, 2, \dots, N(a, b; r)\}$  must contain a monochromatic set of the form  $\{x, ax+d, bx+2d\}$ . We show that  $N(a, b; 2)$  exists if and only if  $b \neq 2a$ , and provide upper and lower bounds for it. We then show that for a large class of pairs  $(a, b)$ ,  $N(a, b; r)$  does not exist for  $r$  sufficiently large. We also give a result on sets of the form  $\{x, ax+d, ax+2d, \dots, ax+(k-1)d\}$ .