

# INTEGERS CONFERENCE 2003

On the Occasion of the 65<sup>th</sup>  
Birthday of Tom Brown

October 31 - November 2  
State University of West Georgia  
Carrollton, Georgia

## Abstracts

Friday, October 31

Morning talks held in Education Center, Room 1; Afternoon talks held in TLC 1301 and 1303.

9:15-9:30     **Welcome and Opening Remarks** – Richard Miller, Dean College of Arts and Sciences; Bruce Landman

9:30-9:50     **Jaclyn Anderson**, University of Wisconsin-Madison

### On the Existence of Rook Equivalent $t$ -core Partitions

For a positive integer  $t$ , a partition is said to be a  $t$ -core if each of the hook numbers from its Ferrers-Young diagram is not divisible by  $t$ . In 1998, Haglund, Ono, and Sze proved that if  $t = 2, 3$ , or  $4$ , then two distinct  $t$ -core partitions are rook equivalent if and only if they are conjugates. In contrast to this theorem, they conjectured that if  $t \geq 5$ , then there exists a constant  $N(t)$  such that for every positive integer  $n \geq N(t)$ , there exist two distinct rook equivalent  $t$ -core partitions of  $n$  which are not conjugate. I prove this conjecture for  $t \geq 12$  with  $N(t) = 4$  in all cases.

10:00-10:20   **Timothy B. Flowers**, Clemson University; Neil J. Calkin; and Steven R. Finch

### Difference Density and Aperiodic Sum-Free Sets

Cameron has introduced a natural one-to-one correspondence between infinite binary sequences and sets of positive integers with the property that no two elements add up to a third. He observed that, if a sum-free set is ultimately periodic, so is the corresponding binary sequence, and asked if the converse also holds. We introduce the concept of difference density and show how this can be used to test specific sets. These tests produce further evidence of a positive nature that certain sets are, in fact, not ultimately periodic.

10:30-10:50   **Alex Iosevich**, University of Missouri-Columbia

### Distribution of Lattice Points in Convex Domains: Mean Square Estimates

We obtain sharp mean square estimates for the error term in the classical lattice point problem. We will also see how combinatorial geometry sometimes yields estimates previously obtained via Fourier analysis.

11:00-11:20 **Clay Culver** and Bruce Landman, State University of West Georgia

**On Some New Mixed Van Der Waerden Numbers**

The van der Waerden number  $w(k_1, k_2, \dots, k_r; r)$  is defined as the least positive integer  $w$  such that, for every  $r$ -coloring  $\chi : [1, w] \rightarrow \{1, 2, \dots, r\}$ , for some color  $i$  there is a  $k_i$ -term arithmetic progression of color  $i$ . We present some new exact values of  $w(k_1, k_2, \dots, k_r; r)$ . Some observations and conjectures are given concerning the magnitude of this function. For each new exact value of  $w$ , we describe those colorings of  $[1, w - 1]$  that avoid all corresponding monochromatic arithmetic progressions. The algorithm is also discussed.

11:30-11:50 **Gretchen L. Matthews**, Clemson University

**Numerical Semigroups Generated by Generalized Arithmetic Sequences**

In this talk, we consider numerical semigroups generated by generalized arithmetic sequences. We characterize numerical semigroups generated by generalized arithmetic sequences that satisfy other semigroup properties, such as symmetric, pseudo-symmetric, and Arf. This is a consequence of a study of two chains of numerical semigroups obtained from a numerical semigroup  $S$  by iterating the constructions for the dual of  $S$ ,  $B(S) := \{x \in \mathbb{N}_0 : x + S \setminus \{0\} \subseteq S\}$ , and the Lipman semigroup of  $S$ ,  $L(S) := \cup_{l=1}^{\infty} \{x \in \mathbb{N}_0 : x + l(S \setminus \{0\}) \subseteq l(S \setminus \{0\})\}$ .

12:00-1:20 **Lunch** – Executive Dining Room, 2<sup>nd</sup> floor Z-6 Building

1:30-1:50 **James Sellers**, The Pennsylvania State University  
TLC 1301

**A Generalization of Overpartitions: Preliminary Results**

In recent work, various authors (Cortee, Lovejoy, Yee) have extensively studied overpartitions as a means of better understanding and interpreting various  $q$ -series identities. In this paper, we consider a generalization of overpartitions which we call “ $k$ -overpartitions”. The generating function for these objects is provided, and then numerous arithmetic relations are proven via elementary means.

1:30-1:50 **Miklos Bona**, University of Florida  
TLC 1303

**A Simple Proof For the Exponential Upper Bound for Some Tenacious Patterns.**

We present a new way of decomposing permutations that enables us to find simple proofs for some important results in the theory of pattern avoidance.

2:00-2:50 **Plenary Talk: Melvyn Nathanson**, Lehman College, City University of New  
TLC 1301 York

### **Representation Functions of Additive Bases for the Integers**

3:00-3:20 **Florian Luca**, National Autonomous University of Mexico; and Martin Klazar  
TLC 1301

### **Arithmetic Properties of Motzkin Numbers**

For a given positive integer  $n$ , the  $n$ th Motzkin number counts the number of paths in the  $xy$ -plane starting at  $(0, 0)$ , ending at  $(0, n)$ , which use only steps parallel to  $(1, 1)$  (up step),  $(1, 0)$  (horizontal step), or  $(1, -1)$  (down step), and which never pass below the  $x$ -axis. The Motzkin numbers satisfy the recurrence  $(n + 2)m_n = (2n + 1)m_{n-1} + 3(n - 1)m_{n-2}$  for  $n > 2$  with the starting values  $m_1 = 1$  and  $m_2 = 2$ . In my talk, I will report about some arithmetic properties of the Motzkin numbers  $(m_n)_{n \geq 1}$ . Some of the results have been obtained jointly with Martin Klazar.

3:00-3:20 **David Wolfe**, Gustavus Adolphus College; William Fraser; Susan Hirshberg; and  
TLC 1303 Marc Paulhus

### **Putting the Combinatorics in Combinatorial Game Theory**

Under Conway's simple but powerful game theory axioms, games form a group with a partial order. While a great deal has been known about the group structure of large subsets of games, surprisingly little was known about the overall partial order. We prove that games lasting a fixed number of turns form a distributive lattice, but that the collection of all finite games does not form a lattice. We are also able to give stronger bounds on the number of games born on day  $n$  than those known previously.

We will also present theorems about the structure of this lattice. A direct corollary of these theorems is that all maximal chains in the day  $n$  lattice are of the same length, that length being exactly one plus twice the number of games born by day  $n-1$ .

3:20-3:40 **Break**

3:40-4:00  
TLC 1301 **Dennis Eichhorn**, University of Arizona

### **A Classical Treatment of the Divisibility Properties of Partition Functions**

There has been an explosion of results in recent years that has provided divisibility information for partition functions using the theory of modular forms. Are classical methods obsolete when it comes to divisibility? In this talk, we try to demonstrate that the answer to this question is “no”. We prove several theorems regarding the divisibility and indivisibility of a large class of functions that includes  $p(n)$  and many other traditionally studied partition functions. We will cover ground that is often off-limits to modular forms techniques, and we will even provide a new result modulo 3.

3:40-4:00  
TLC 1303 **Veselin Jungic**, Simon Fraser University; and Rados Radoicic, Massachusetts Institute of Technology

### **About 3-term Arithmetic Rainbow Progressions**

An arithmetic progression is rainbow if all terms are colored in distinct colors. A finite coloring of  $[n]$  is equinumerous if all color classes are of the same cardinality. R. Radoicic (MIT - Cambridge) conjectured that for every equinumerous 3-coloring of  $[n]$  there exists a rainbow 3-term arithmetic progression. We answer the conjecture in the affirmative.

4:10-4:30  
TLC 1301 **Pantelimon Stănică**, Auburn University Montgomery

### **Cholesky Factorizations of Matrices Associated with $r$ -Order Recurrent Sequences**

In this paper we generalize many existent results on the factorization of matrices associated to Fibonacci, Pascal, Stirling sequences to matrices constructed using any  $r$ -order recurrent sequence  $U_n$  (having  $U_0 = 0$ ). The Cholesky factorization for the associated symmetric matrix is obtained and we provide explicit factorization of *any* matrix by the matrix associated to an  $r$ -order recurrent sequence. As applications we derive some combinatorial identities.

4:10-4:30  
TLC 1303 **Sarah McCurdy**, University of New Brunswick

### **Cutthroat: An All Small Game on Graphs**

In the game of Cutthroat, each vertex of a graph is colored either red or blue. Left removes blue vertices and Right removes red vertices. Any monochromatic component that results is also deleted. The last player to move wins. Clearly, either both players have a move or neither does, so no player can gain a big advantage. We examine the strategies and values that arise for complete graphs, alternating paths and (most interestingly) for the stars  $K_{1,n}$ .

4:40-5:00  
TLC 1301

**Karl Mahlburg**, University of Wisconsin-Madison; and Jayce Getz

### **Partition Identities and a Theorem of Zagier**

In the study of partition theory and  $q$ -series, identities that relate series to infinite products are of great interest (such as the famous Rogers-Ramanujan identities). Using a recent result of Zagier, we obtain an infinite family of such identities that is indexed by the positive integers. For example, if  $m = 1$ , then we obtain the classical Eisenstein series identity

$$\sum_{\lambda \geq 1 \text{ odd}} \frac{(-1)^{(\lambda-1)/2} q^\lambda}{(1 - q^{2\lambda})} = q \prod_{n=1}^{\infty} \frac{(1 - q^{8n})^4}{(1 - q^{4n})^2}.$$

We describe some of the partition theoretic consequences of these identities. In particular, we find simple formulas that solve the well-known problem of counting the number of representations of an integer as a sum of an arbitrary number of triangular numbers.

4:40-5:00  
TLC 1303

**Todd Will**, University of Wisconsin-La Crosse

### **Generating Maximum Size Shadows**

A non-negative integer triple  $(a, b, c)$  has size  $a + b + c$  and generates the three triples  $(a + 1, b, c)$ ,  $(a, b + 1, c)$ , and  $(a, b, c + 1)$ . A linear programming technique is used to determine the fewest number of size  $d - 1$  triples required to generate all size  $d$  triples. A generalization of the technique provides partial results on the analogous problem for integer 4-tuples.

5:10-5:30  
TLC 1301

**Omar Kihel**, Brock University

### **Brocard-Ramanujan Diophantine Equations**

The study of the diophantine equation  $n! + 1 = m^2$  was first initiated by H. Brocard in 1876 and again in 1885. S. Ramanujan in 1913 unaware about Brocard's problem asked the same question in the following way: The number  $1 + n!$  is a perfect square for the values 4, 5, 7 of  $n$ . Find other values? In this talk after presenting what is known about this equation, we will present new results on a more general equation  $n! + A = m^q$  and discuss a related conjecture.

5:10-5:30 **Paul T. Ottaway** and Richard J. Nowakowski, Dalhousie University  
TLC 1303

### **A Parity-Rules Vertex Deletion Game**

Given a graph  $G$ , Left and Right take turns deleting vertices of  $G$ . Left deletes vertices of even degree and Right those of odd degree—the degrees are recalculated after each move. We show that there are no graphs where Right can win going first and second. A collapsing lemma allows some classes of graphs, e.g. complete graphs, to be evaluated quickly. Although the (surreal) values for co-graphs seem difficult to describe, the collapsing lemma together with some parity considerations allow us to find these values quickly. We also show that the value of  $G + P_k$  is equal to 1 plus the value of  $G + P_{k-3}$  where  $P_k$  is a path of length  $k > 6$ .

5:30-7:30 **Dinner** – Executive Dining Room, 2<sup>nd</sup> floor Z-6 Building

Saturday, November 1

Talks held in Education Center, Room 3

9:00-9:20 **Aaron Robertson**, Colgate University

### **Permutations from Catalan to Fine and Back**

We give some history and recent results in the area of pattern restricted permutations. We also present a new bijection between certain pattern restricted permutations.

9:30-9:50 **Tom Brown**, Simon Fraser University

### **On the Canonical Version of a Theorem in Ramsey Theory.**

The constant colorings and the one-to-one colorings are insufficient for a canonical version of a certain theorem in Ramsey theory. The theorem: If  $\mathbf{N}$  is finitely colored, there exist a fixed  $d \geq 1$  ( $d$  depends only on the coloring) and arbitrarily large monochromatic sets  $A = \{a_1 < a_2 < \dots < a_n\}$  with  $\max\{a_{j+1} - a_j : 1 \leq j \leq n - 1\} = d$ .

10:30-11:20 **E. Rodney Canfield**, University of Georgia; Carla D. Savage, North Carolina State University; and Herbert S. Wilf, University of Pennsylvania

### Regularly Spaced Subsums of Integer Partitions

For integer partitions  $\lambda : n = a_1 + \dots + a_k$ , where  $a_1 \geq a_2 \geq \dots \geq a_k \geq 1$ , we study the sum  $a_1 + a_3 + \dots$  of the parts of odd index. We show that the average of this sum, over all partitions  $\lambda$  of  $n$ , is of the form  $n/2 + (\sqrt{6}/(8\pi))\sqrt{n} \log n + c_{2,1}\sqrt{n} + O(\log n)$ . More generally, we study the sum  $a_i + a_{m+i} + a_{2m+i} + \dots$  of the parts whose indices lie in a given arithmetic progression and we show that the average of this sum, over all partitions of  $n$ , is of the form  $n/m + b_{m,i}\sqrt{n} \log n + c_{m,i}\sqrt{n} + O(\log n)$ , with explicitly given constants  $b_{m,i}, c_{m,i}$ . Interestingly, for  $m$  odd and  $i = (m + 1)/2$  we have  $b_{m,i} = 0$ , so in this case the error term is of lower order. The methods used involve asymptotic formulas for the behavior of Lambert series and the Zeta function of Hurwitz.

We also show that if  $f(n, j)$  is the number of partitions of  $n$  the sum of whose parts of even index is  $j$ , then for every  $n$ ,  $f(n, j)$  agrees with a certain universal sequence, Sloane's sequence #A000712, for  $j \leq n/3$  but not for any larger  $j$ .

10:30-11:20 **Plenary Talk: Ronald Graham**, University of California at San Diego

### Euclidean Ramsey Theory

11:30-11:50 **Ernie Croot**, Georgia Institute of Technology

### Critical Sets for Arithmetic Progressions

Given an integer  $N$ , and a number  $r$  in  $(0,1)$ , we say that a subset of  $\mathbb{Z}/N\mathbb{Z}$  is a critical set for the density  $r$  if the subset has at least  $rN$  elements, and has the minimal number of solutions to  $x+y=2z \pmod{N}$ , among all other sets with at least  $rN$  elements. In this talk I will discuss some recent work by myself which shows that for sufficiently large primes  $N$ , these critical sets have an unexpected amount of arithmetic structure.

11:50-1:30 **Break/Lunch**

1:30-1:50 **Richard Nowakowski**, Dalhousie University; M. Albert; J.P. Grossman; and D. Wolfe.

### The Game of Clobber

'Clobber' was invented in 2002. (Take a  $6 \times 7$  checkerboard with black pieces on all the black squares and white on all the white squares. A piece can only be moved one square horizontally or vertically provided there is an opponent's piece on the other square which is now 'clobbered' and removed.) The game was introduced to the 'public' at the 2002 Dagstuhl Games Workshop. I will present what little is known about this game and many conjectures.

2:00-2:20 **Kevin O’Bryant** and Ron Graham, University of California at San Diego

### Fraenkel’s Conjecture

The set  $S(\alpha, \beta) = \{\lfloor n\alpha + \beta \rfloor : n \in \mathbb{Z}\}$  is called a Beatty set. For example, if  $\alpha$  is an integer, then  $S(\alpha, \beta)$  is a congruence class with modulus  $\alpha$ . An old chestnut of generating functionology is that if  $m > 1$  congruence classes are disjoint and cover the integers, then two of the moduli are equal. Around 1970, Fraenkel conjectured that if  $m > 2$  Beatty sets are disjoint and cover the integers, then either two of the  $\alpha$  are equal or  $\{\alpha_1, \alpha_2, \dots, \alpha_m\} = \{2^m - 1, (2^m - 1)/2, (2^m - 1)/4, \dots, (2^m - 1)/2^{m-1}\}$ . Fraenkel quickly proved some special cases, and Ron Graham proved the conjecture under the additional hypothesis that some  $\alpha$  is irrational. While the integer case and irrational case have long been resolved, the rational case remains open. Recently, R. Tijdeman proved the conjecture for  $m < 7$ . We discuss recent progress (joint work with Graham) towards a solution.

2:30-3:20 **Plenary Talk: Jaroslav Nešetřil**, Charles University, Prague

### Characterization of Ramsey Classes

3:20-3:40 **Break**

3:40-4:00 **Neil Hindman**, Howard University; Tim Carlson; and Dona Strauss

### Ramsey Theoretic Consequences of Some New Results about the Algebra of $\beta\mathbb{N}$ (and Other Semigroups).

Recently we have obtained some new algebraic results about  $\beta\mathbb{N}$ , the Stone-Čech compactification of the discrete set of positive integers and about  $\beta W$ , where  $W$  is the free semigroup over a nonempty alphabet with infinitely many variables adjoined. (The results about  $\beta W$  extend the Graham-Rothschild Parameter Sets Theorem.) In this paper we derive some Ramsey Theoretic consequences of these results. Among these is the following, which extends the Finite Sums Theorem.

**Theorem.** Let  $\mathbb{N}$  be finitely colored and let  $k \in \mathbb{N}$ . Then for each  $i, j \in \{1, \dots, k\}$  there exists a monochrome central set  $E_{i,j}$  and for each  $i \in \{1, \dots, k\}$  there exists a sequence  $\langle x_{i,n} \rangle_{n=1}^\infty$  in  $E_{i,i}$  such that

- (a)  $E_{i,j} \cap E_{l,m} = \emptyset$  if  $(i, j) \neq (l, m)$  and
- (b) whenever  $F$  is a finite nonempty subset of  $\mathbb{N}$ ,  $f : F \rightarrow \{1, \dots, k\}$ ,  $i = f(\min F)$ , and  $j = f(\max F)$ , one has  $\sum_{n \in F} x_{f(n), n} \in E_{i,j}$ .

4:05-4:25 **Matthew Boylan**, University of Illinois at Urbana-Champaign

### **Arithmetic Properties of the Partition Function**

Many interesting arithmetic properties of the partition function were conjectured and proved by Ramanujan. Of these properties, perhaps the most famous are the celebrated Ramanujan congruences

$$\begin{aligned}p(5n + 4) &\equiv 0 \pmod{5} \\p(7n + 5) &\equiv 0 \pmod{7}, \\p(11n + 6) &\equiv 0 \pmod{11}.\end{aligned}$$

In the eighty years since Ramanujan's work, the problem of determining whether these are the only congruences of this type has generated a large amount of interest. In this paper, we use modular forms modulo  $\ell$  to show that these congruences are indeed, the only ones of their kind. As a corollary, we prove a famous conjecture of Newman on the distribution of values of  $p(n)$  modulo positive integers.

4:30-4:50 **Donald Mills**, Southern Illinois University, and P. Mitchell

### **Avoidability of Sets of Positive Integers**

The topic of avoidable sets of positive integers has received its fair share of attention since Erdős, Alladi, and Hoggatt wrote a paper on the avoidability of the Fibonacci sequence in 1978. In a recent paper, the speaker and P. Mitchell approach this topic by investigating the avoidability of sets of specified size, using methods that employ both combinatorial and geometric notions. In this talk, the speaker will present the results from that paper, relating the problem in so doing to the classic problem of counting the number of lattice points in a convex set in  $n$ -dimensional space over the reals. This connection, in turn, may lead to some very intriguing future directions for research.

4:55-5:15 **Joshua Cooper**, Courant Institute of Mathematical Sciences, New York University

### **Quasirandomness and Continued Fractions**

A quasirandom permutation is a bijection of a finite cyclic group to itself that exhibits low discrepancy on intervals. We consider, in particular, the permutation whose order is induced by the ordering of the fractional parts of  $\{j\alpha\}$ ,  $j = 1$  to  $n$ , for some  $\alpha$  irrational. This permutation is well studied, and the magnitude of its discrepancy is intimately connected with the theory of diophantine approximation. We discuss these connections, suggest several intriguing conjectures, and present a result on the number of multiplicative automorphisms of  $Z/nZ$  giving rise to "optimally" quasirandom permutations.

5:20-5:40 **Jerrold Griggs**, University of South Carolina; Julie Emery; Katherine Heller; and Teresa Xiaohua Jin

### **Real Number Channel Assignments with Distance Conditions**

We discuss a generalized graph coloring problem, “lambda-labelling”, motivated by the goal of efficiently assigning channels to a network of transmitters while avoiding interference that depends on the distance between them. We extend previous work, which considered integer vertex labellings, to real number labellings. In this talk, we will study the symmetry properties of optimal labellings of the infinite triangular lattice (which corresponds to a planar network of transmitters with hexagonal coverage regions).

6:30-? **Dinner** – Little Hawaiian Restaurant, Carrollton

Sunday, November 2

Talks held in Education Center, Room 3

9:00-9:20 **Renling Jin**, College of Charleston, and Perna Bihani

### **Inverse Problems for Upper Banach Density**

Given a set of natural numbers  $A$ , let  $A(a,b)$  denote the number of elements in  $A \cap [a, b]$ . The upper Banach density of  $A$  is the largest real number  $\alpha$  such that there is a sequence of intervals  $[a_n, b_n]$  of natural numbers with  $b_n - a_n \rightarrow \infty$  and  $A(a, b)/(b - a + 1) \rightarrow \alpha$ . In the talk, we describe the structure of  $A$  when the upper Banach density of  $2A$  is less than 2 times the upper Banach density of  $A$ , where  $2A$  is the set  $\{a + b : a, b \in A\}$ .

9:30-9:50 **Brendan Nagle**, University of Nevada Reno; V. Rodl; and M. Schacht

### **Hypergraph Regularity and an Application to Integers**

In this talk, we focus to recent developments in the area of hypergraph regularity and discuss connections these developments have in combinatorial number theory. In particular, our main result concerns the development of a so-called “Counting Lemma” (for  $k$ -graphs) which is a companion statement to a recent hypergraph regularity lemma due to V. Rodl and J. Skokan. Both of these results were known for 3-graphs and proved to be useful tools in handling several extremal hypergraph problems and a couple problems on the integers.

This is a joint work with V. Rodl and M. Schacht. The results we discuss were independently (and alternatively) obtained by W.T. Gowers.

10:00-10:20 **Rong Luo**, Middle Tennessee State University

### On the Degree Sequences of Simple Graphs

An  $n$ -term nonincreasing positive integer sequence  $\pi = (d_1, d_2, \dots, d_n)$  is said to be *graphic* if it is the degree sequence of a simple graph  $G$  of order  $n$  and such a graph  $G$  is referred to as a *realization* of  $\pi$ . Let  $H$  be a simple graph. A graphic sequence  $\pi$  is said to be *potentially  $H$ -graphic* if it has a realization  $G$  containing  $H$  as a subgraph (not . In this talk, I will first give a survey on potentially  $H$ -graph sequences and then present the characterizations of potentially  $H$ -graphic sequences for some special graphs  $H$ .

10:30-11:20 **Plenary Talk: Carl Pomerance**, Dartmouth College; and Hendrik Lenstra

### Recent Developments in Primality Testing

In August, 2002, Manindra Agrawal, Neeraj Kayal, and Nitin Saxena, all from the Indian Institute of Technology in Kanpur, announced a new algorithm to distinguish between prime numbers and composite numbers. Unlike earlier methods, their test is completely rigorous, deterministic, and runs in polynomial time. The heart of the procedure for testing  $n$  involves verifying some identities  $(x + a)^n = x^n + a$  in the ring  $(\mathbf{Z}/n\mathbf{Z})[x]/(f(x))$ , where  $a$  runs over a small set of integers, and  $f(x)$  is a (craftily chosen) polynomial. In the original paper  $f(x)$  is of the form  $x^r - 1$ , where  $r$  is a prime with some additional properties. Two weaknesses in this construction: the argument for the complexity is ineffective, and it falls short of achieving what is likely to be the true complexity for the algorithm. We have found a way to instead use the polynomial for an appropriate subfield of a possibly large cyclotomic field. It is important that the degree of  $f(x)$  be large enough so that the primality test is valid, but not so large that the running time suffers. We are able to choose the degree fairly precisely using some tools from analytic number theory and a new result, due to Daniel Bleichenbacher and Vsevolod Lev, from combinatorial number theory. We thus achieve a rigorous and effective running time of about  $(\log n)^6$ , the heuristic complexity of the original test.

11:20-12:20 **Break/Lunch**

12:20-12:40 **David Gunderson**, University of Manitoba; Imre Leader; Hans Jürgen Prömel; and Vojtech Rödl

### Integers and Ramsey Theory

Ramsey theory for colouring integers is well developed, whereas the theory for colouring pairs of integers is less so. We give a single Ramsey type theorem for 2-colouring pairs of integers that implies many Ramsey type theorems for colouring integers, for example, Hilbert's affine cube lemma, Schur's theorem, and van der Waerden's theorem. This result also extends Ramsey's theorem for graphs and partition regularity theorems of Rado and Deuber.

12:45-1:05 **Peter Shiue** University of Nevada at Las Vegas; Tom Brown, Simon Fraser University; and W.S. Chou

### **On the Partition Function of a Finite Set**

Let  $A$  be a set of  $k$  relatively prime positive integers. Let  $P(A,n)$  denote the number of partitions of  $n$  with parts belonging to  $A$ . This paper is devoted to the study of  $P(A,n)$  when  $k=3$ .

1:10-1:30 **Zhongshan Li**, Georgia State University; Frank Hall, Georgia State University; and Jeffrey Stuart, Pacific Lutheran University

### **Reducible Powerful Ray Pattern Matrices**

A ray pattern is a matrix each of whose entries is either 0 or a ray in the complex plane originating from 0 (but not including 0). A ray pattern is a natural generalization of the concept of a sign pattern, whose entries are from the set  $\{+, -, 0\}$ . Powers of sign patterns and ray patterns, especially patterns whose powers are periodic, have been studied in several recent papers. A ray pattern  $A$  is said to be powerful if  $A^k$  is unambiguously defined for all positive integers  $k$ . Irreducible powerful ray patterns have been characterized recently. In this paper, reducible powerful ray patterns are investigated. In particular, for a powerful ray pattern in Frobenius normal form, it is shown that the existence of a nonzero entry in an off diagonal block implies that the corresponding irreducible components are related in a certain way. Reducible powerful ray patterns all of whose irreducible components are nonzero are characterized. Further, the structure of each of the off diagonal blocks is characterized. The techniques used in the paper involve matrix theory, graph theory and number theory.

1:35-1:55 **Steve Edwards**, Southern Polytechnic State University

### **Lucas Numbers in the Regular Pentagon**

The regular pentagon can be dissected in many ways into golden triangles. Certain dissections result in the numbers of triangles always being Lucas numbers. We use these dissections to generate Lucas number identities and examine the geometry of the regular pentagon.

1:55-2:05 **Closing Remarks**