ABSTRACTS

Thursday, October 27

10:30-10:50 **Kennan Shelton,** University of Central Arkansas
Room 2

The Game of Take Turn

We introduce the game of Take Turn, a vertex-deletion game played with coins placed on vertices of a graph. A move is to remove a heads-up coin and turn over any coins adjacent from the removed coin; the loser is the player who cannot remove a coin. We will relate the Sprague-Grundy values for games played on directed paths and cycles to the octal game .37 and show that the game played on arbitrary directed graphs is PSPACE-Complete.

10:30-10:50 **Christian Ballot,** Caen University
Room 3

An Elementary Method to Compute Prime Densities in $\mathbb{F}_q[X]$

Fixing a prime l and a power q of a prime p, we compute the Dirichlet density of the set S of primes P of $\mathbb{F}_q[X]$ defined by

$$S = \{P \in \mathbb{F}_q[X]; l \text{ divides the order of } X (\text{mod } P)\}$$

Our method is fully elementary in that it does not use any Kummer theory, or the Chebotarev density theorem in any form.

11:00-11:20 Room 2

Shalom Eliahou, Université du Littoral Côte d'Opale

Steinhaus Matrices

A Steinhaus matrix is a zero-diagonal, symmetric matrix $M = (a_{i,j})$ with entries in the field GF(2), satisfying the relation $a_{i,j} = a_{i-1,j-1} + a_{i-1,j}$ in its strict upper triangular part. Such a matrix is thus completely determined by its first row. In this talk, we shall survey a few problems and results about Steinhaus matrices, with the purpose of showing how tricky the behavior of so simply defined objects can be.

11:00-11:20 Room 3

Matthias Beck, San Francisco State University

The Enumeration of Nowhere-Zero Integral Flows on Graphs

A *nowhere-zero* k-flow on a graph G is a mapping from the edges of G to the set $\{-k+1, -k+2, \ldots, -1, 1, 2, \ldots, k-1\}$ such that, given a fixed orientation of G, at each node the sum of the edge labels pointing towards the node equals the sum of the edge labels pointing away from the node. Some classic problems are connected to flows; for example, the four-color theorem is equivalent to every planar bridgeless graph admitting a 4-flow, and one of the major open questions in graph theory is whether every bridgeless graph admits a 5-flow.

Kochol proved recently that the number $f_G(k)$ of nowhere-zero k-flows of a given graph G is a polynomial in the positive integer variable k. We will show how one can interpret the flow polynomial f_G geometrically by means of *Ehrhart theory*, i.e., lattice-point enumeration in polytopes. Our interpretation gives a simple geometric reason why f_G is a polynomial.

Going a step further, we use the central reciprocity theorem of *inside-out polytopes* to obtain an interpretation of f_G at negative integers. Namely, $f_G(-k)$ counts totally cyclic orientations of G that are compatible with a k-flow of G. This result parallels Stanley's theorem that the chromatic polynomial of G evaluated at negative integers counts acyclic orientations compatible with a coloring of G.

This is joint work with Thomas Zaslavsky (Binghamton University, SUNY).

1:00-1:20 Room 2

Mark Kozek, University of South Carolina; Michael Filaseta; and Carrie Finch

On Two Conjectures of Chen

Yong-Gao Chen conjectured that for each positive integer r, there exist infinitely many positive odd numbers k such that $k^r 2^n + 1$ has at least two distinct prime divisors for all natural numbers n. Chen further made the analogous conjecture for numbers of the form $k^r - 2^n$. In making these conjectures, Chen (2003) gave proofs to both these claims for the particular cases: $r = m$, where m is an odd natural number, and $r = 2m'$, where m' is an odd natural number not divisible by 3, and posited $r = 4$ and $r = 6$ as the next interesting cases for study. We establish the existence of infinitely many such odd k for any fixed r for numbers of the form $k^r 2^n + 1$, and we establish the existence of infinitely many odd k for the aforementioned interesting choices for r for numbers of the form $k^r - 2^n$.

1:00-1:20 Room 3

Donald Mills, Southern Illinois University Carbondale

Polynomial Sequences Generated by Fibonacci & Lucas Numbers

Given the Fibonacci and Lucas sequences, defined as $\{f_1, f_2, f_3, ...\} = \{1, 1, 2, ...\}$ and $\{l_1, l_2, l_3, ...\} = \{1, 3, 4, ...\}$ respectively, we define corresponding polynomial sequences $\mathbf{p}_f$ and $\mathbf{p}_l$. Specifically, for $\mathbf{p}_f$ we set $p_{f,0}(x) = 1$ and $p_{f,i}(x) = xp_{f,i-1}(x) + f_{i+1}$ for $i \geq 1$, with $p_{f,j}(x) = \sum_{k=0}^{j} f_{k+1}x^{j-k}$. The sequence $\mathbf{p}_l$ is similarly defined. Then $p_{f,j}(x)$ (respectively, $p_{l,j}(x)$) is called the *Fibonacci-coefficient polynomial*, or *FCP*, of order j (respectively, the *Lucas-coefficient polynomial*, or *LCP*, of order j), and we observe that the elements of said sequences are distinct from the well-known Fibonacci and Lucas polynomial sequences, respectively. We address several properties of these polynomials, including the number of real roots of each polynomial, the number of rational roots of each, and Mahler measures of various reduced forms of said polynomials.

This is joint work with Patrick Mitchell of Midwestern State University, and James Sellers of The Pennsylvania State University.

1:30-1:50 Room 2

Stephen C. Milne, The Ohio State University

C_ℓ Mock Theta Functions

We use the techniques of Andrews' (1986) along with the C_ℓ Bailey Lemma established by Milne and Lilly (1992) to derive the corresponding multivariable extension of a transformation of Watson for one of the classical third order mock theta functions of Ramanujan. This yields the definition of our C_ℓ mock theta function in the setting of multivariable basic symplectic hypergeometric series associated to root systems. Motivated by Milne's recent sums of squares work, we then establish explicit general formulas for this new multivariable mock theta function in terms of Schur symmetric functions, Littlewood-Richardson coefficients, and Hall-Littlewood vertex operators. A Schur function "positivity" conjecture analogous to theorems for Macdonald polynomials also arises from some of our computed examples. All of this work provides more combinatorial insight into the classical mock theta functions of Ramanujan. This is joint work with Jon W. Breitenbucher.

1:30-1:50 Room 3

David Leach, University of West Georgia

Lattice-Ordered Graphs

The set of unlabeled induced subgraphs of a graph form a partially ordered set, with $H \leq D$ if and only if H is an induced subgraph of D. If this poset is a lattice, we say that the graph is *lattice-ordered*. In this talk we characterize the lattice-ordered graphs and determine when two graphs to have isomorphic lattices.

This is joint work with Matt Walsh of Indiana-Purdue University, Fort Wayne.

2:00-2:20 Room 2

Melkamu Zeleke, William Paterson University

K-Trees, Catalan Identities, and their Applications

A k-tree is constructed from a single distinguished k-cycle by repeatedly gluing other k-cycles to existing ones along an edge. If K is any nonempty subset of $\{2, 3, 4, \cdots\}$, then a K-tree is obtained as above using k-cycles, where $k \in K$. In this talk, we look at the enumeration of K-trees, show that the ratio of terminal edges to total number of edges in k-trees is $\frac{k-1}{k}$, and use K-trees as models to enumerate planted plane cacti. We also use these objects to obtain generating function identities involving generalizations of Catalan Numbers, Central Binomial Numbers, and Fine Numbers and show that the ratio of generalized Fine numbers to Catalan numbers is asymptotic to $\frac{2k}{(k+1)^2}$.

2:00-2:20 Room 3

Elnatan Reisner, Brandeis University; Aviezri S. Fraenkel,Weizmann Institute of Science

The Game of End-Wythoff

Given a row of finitely many piles of finitely many tokens. In End-Wythoff, two players alternate in taking a positive number of tokens from either end-pile, or taking the *same* positive number of tokens from both ends. The player first unable to move loses and the opponent wins. We characterize the P-positions (a_i, K, b_i) of the game as a function of the row K of middle piles, where a_i, b_i denote the sizes of the end piles. When K itself is a P-position, the recursive characterization becomes simpler. The case where K is a palindrome leads to a more succinct characterization. For this case the (noisy) initial behavior of the P-positions is described precisely. Beyond the initial behavior, we have $b_i - a_i = i$, as in the normal 2-pile Wythoff game.

2:30-2:50 Room 2

John H. Jaroma, Austin College

A Generalization of the Lucas-Lehmer Test

The *Lucas-Lehmer* test states that $N = 2^n - 1$ is prime if and only if N divides the $(n-1)$st term of the sequence $4, 14, 194, 37634, \ldots$. In this talk, we utilize the underlying principles of the given theorem, as well as incorporate the notion of a Somer-Lucas pseudoprime in order to establish a similar primality result for an arbitrary odd integer.

2:30-2:50 Room 3

Angela Siegel, Dalhousie University

All-But Geography

All-But Geography (or All-But Modular Nim) allows for all moves except for a finite set. The main results presented are for the impartial and partizan versions when the game is played on the cartesian product of K_2 and K_m.

3:10-3:30 Room 2

James Sellers, Penn State University

An Infinite Family of Overpartition Congruences Modulo 12

In recent years, the subject of overpartitions has become very popular. In particular, a number of arithmetic properties of the function $\overline{p}(n)$ (the function which counts the number of overpartitions of the integer n) have been proven recently. However, all of these congruences have involved moduli which are powers of 2. In this paper, which is joint work with Michael Hirschhorn, we use elementary generating function manipulations to yield an infinite family of congruences with a modulus that is not a power of 2 by proving that, for all $n \geq 0$ and all $\alpha \geq 0$,

$$\overline{p}(9^\alpha(27n+18)) \equiv 0 \pmod{12}.$$

3:10-3:30 Room 3

Veselin Jungic, Simon Fraser University

On the Existence of Rainbow 4-term Arithmetic Progressions

The following question will be answered.
Does any equinumerous 4-coloring of $[1, 4n]$ contain a rainbow 4-term arithmetic progression?
Some related results and open problems will be mentioned.

3:40-4:00 Room 2

Pantelimon Stănică, Auburn University Montgomery

Avalanche Features and Walsh Spectrum of Boolean Cayley Graph

Let f be a Boolean function on V_n (vector space of dimension n over the two-element field F_2). We define the *Cayley graph* of f to be the graph $G_f = (V_n, E_f)$ whose vertices set is V_n and the set of edges is defined by

$$E_f = \{(w, u) \in V_n \times V_n \mid f(w \oplus u) = 1\}.$$

We investigate the cryptographic avalanche features of f in terms of its associated Cayley graph.

3:40-4:00 Room 3

Georges Grekos, Universite de Saint-Etienne

Extremal Problems about Additive Bases

This talk is a short report on works by Alain Plagne, Melvyn B. Nathanson, Xing-De Jia, John C. M. Nash, Julien Cassaigne, Bruno Deschamps and myself, inspired and arisen from the paper "On bases with an exact order" by Paul Erdős and Ronald L. Graham, published in Acta Arith. 37 (1980), 201–207, MR0598875 (82e:10093).

Let $h \geq 2$ be an integer. A set A of non negative integers is called an $h-$**basis** if every sufficiently large integer n can be written as $n = a_1 + \cdots + a_h$, $a_j \in A, 1 \leq j \leq h$. Let $\mathcal{B}(h)$ be the set of $h-$bases. We say that A is a **basis** if A belongs to $\mathcal{B}(k)$ for some k. The least integer g such that A belongs to $\mathcal{B}(g)$ is the **order** $G(A)$ of A.

I shall present results on the quantities

$$\mathbf{X}(h) = \max_{A \in \mathcal{B}(h)} \mathbf{x}(A), \quad \mathbf{S}(h) = \max_{A \in \mathcal{B}(h)} \mathbf{s}(A),$$

where

$$\mathbf{x}(A) = \max_{x \in A} G(A \setminus \{x\}), \quad \mathbf{s}(A) = \limsup_{\substack{x \in A \\ x \to +\infty}} G(A \setminus \{x\})$$

and x runs only through elements of A such that $A \setminus \{x\}$ is a basis.
Actually, it is known that $\mathbf{X}(h)$ roughly lies within $h^2/3$ and $h^2/2$, and that $\mathbf{S}(h)$ is less than or equal to $2h$.

4:10-4:30 Room 2

Peter Shiue, University of Nevada Las Vegas

A Symbolic Operator Approach to Several Summation Formulas for Power Series and Combinatorial Identities

Here expounded is a kind of symbolic operator method that can be used to construct many transformation formulas and summation formulas for various types of power series and combinatorial identities including some old ones and more new ones.

4:10-4:30 Room 3

Melkamu Zeleke, William Paterson University

K-Trees, Catalan Identities, and their Applications

A k-tree is constructed from a single distinguished k-cycle by repeatedly gluing other k-cycles to existing ones along an edge. If K is any nonempty subset of $\{2, 3, 4, \cdots\}$, then a K-tree is obtained as above using k-cycles, where $k \in K$. In this talk, we look at the enumeration of K-trees, show that the ratio of terminal edges to total number of edges in k-trees is $\frac{k-1}{k}$, and use K-trees as models to enumerate planted plane cacti. We also use these objects to obtain generating function identities involving generalizations of Catalan Numbers, Central Binomial Numbers, and Fine Numbers and show that the ratio of generalized Fine numbers to Catalan numbers is asymptotic to $\frac{2k}{(k+1)^2}$.

4:40-5:00 **Vladimir Bozovic,** Florida Atlantic University
Room 2

The Distribution of the Size of the Intersection of a k-Tuple of Intervals

Let $(I_1, I_2, \ldots, I_k)$ be a random k-tuple of subintervals of the discrete interval $[1, n]$ and L_n the random variable that measures the size of their intersection. We derive the exact and asymptotic distribution of L_n under the assumption of equally likely drawn k-tuples. The enumeration of such k-tuples and refinements of the given statistic lead to interesting relations to other topics, like octahedral numbers and bipartite graphs.

4:40-5:00 **Xinyu Sun,** Temple University
Room 3

WZ-algorithm and Jones Polynomial

The noncommutative C-polynomial of a knot is the characteristic polynomial of the cyclotomic function of the knot, and it is entirely determined by the colored Jones function of the knot. We verified that the C-polynomial of all twist knots are related by an explicit rational map of degree 2. Our computation of the noncommutative C-polynomial of twist knots utilizes explicit single-sum formulas for the cyclotomic function, and Zeilberger's theory of recursion relations for sums of q-hypergeometric terms.

Friday, October 28

9:00-9:20
Room 1

David Wolfe, Gustavus Adolphus College

Games Played in Tall Warehouses

Partizan End Nim is a variation of the classical game known as Nim. Partizan End Nim is played by two players called Left and Right. The game begins with stacks of boxes lined up in a row, each stack containing at least one box. Players (with forklifts) take turns removing boxes from the stacks on their respective sides. (Left removes from the leftmost stack, while Right removes from the rightmost stack.) The player removing the last box wins. One added twist is that the piles can be of any ordinal height.
We give an efficient recursive method to compute the outcome of (and winning moves from) any position.
This research is joint work with two undergraduates, Adam Duffy and Garrett Kolpin, and extends the work of Michael Albert and Richard Nowakowski.

9:00-9:20
Room 5

Wolfgang A. Schmid, University of Graz; A. Plagne, École polytechnique, Palaiseau

Half-factorial Sets in Cyclic Groups

A monoid H (as a domain) is called factorial if each element $a \in H$ has an essentially unique factorization into irreducible elements (atoms); if for each $a \in H$ all factorizations into atoms have the same number of factors, then H is called half-factorial.
For $(G, +)$ a finite abelian group and $G_0 \subset G$, let $\mathcal{B}(G_0)$ denote the monoid of all zero-sum sequences in G_0, i.e., the set of all finite unordered sequences $g_1 \dots g_l$, with $g_i \in G_0$, such that

$$g_1 + \cdots + g_l = 0 \in G$$

with juxtaposition as operation.
The set $G_0 \subset G$ is called a half-factorial set, if $\mathcal{B}(G_0)$ is a half-factorial monoid; denote $\mu(G) = \max\{|G_0| \colon G_0 \subset G \text{ half-factorial}\}$. The problem of determining $\mu(G)$ has been posed by W. Narkiewicz (Colloq. Math., 1979), since this constant occurs in asypmtotic formulas for certain counting functions of algebraic integers, where G is the class group of the number field. Among others, P. Erdős and A. Zaks (J. Number Theory, 1990) investigated half-factorial subsets of cyclic groups (using different terminology).
In this talk, we present recent results on $\mu(\mathbb{Z}/n\mathbb{Z})$: in particular, we show

$$\mu(\mathbb{Z}/n\mathbb{Z}) \asymp \tau(n),$$

where τ denotes the number of divisors, and give the precise value of $\mu(\mathbb{Z}/n\mathbb{Z})$ in (further) special cases.

9:30-9:50 Room 1

Arie Bialostocki, University of Idaho

Some Problems in View of Recent Developments of the Erdös Ginzburg Ziv Theorem

Two conjectures concerning the Erdös Ginzburg Ziv theorem were recently confirmed. Reiher proved that the two dimension analogue of the EGZ theorem, as conjectured by Kemnitz's and Grynkiewicz proved the weighed generalization of the EGZ theorem as conjectured by Caro. These developments trigger some further problems. We will present computer experiments that at least for small numbers reveal very simple phenomena that seem to be difficult to prove in general.

9:30-9:50 Room 5

John Brillhart, University of Arizona; Richard Blecksmith, and John Brillhart

Algorithms For Finding and Proving Balanced Q^2 Identities

In this talk we will give a brief overview of two algorithms; one to find and one to prove balanced T^2 and Q^2 identities, where T and Q are respectively the Jacobi triple product and quintuple product in one variable. The terminology T^2 (resp. Q^2) means that each term of an identity is a product of two T (resp. Q) factors and a possible power of x.
The search algorithm is an elementary method using mod 2 power series and Gaussian elimination in a bit matrix. The proving algorithm is based on a combinatorial method in which the power series for the positive terms of the identity are partitioned into sub-series that are then added together in a different way to form the power series for the negative terms of the identity.
We will also discuss some of the results we obtained from a massive computing project we carried out using these two algorithms, results that contain many new identities and some conjectures based on the statistics we gathered. Some identities are interestingly the "seed" of an infinite family of identities.
This work is in large part the culmination of developments that appeared earlier in a series of papers published by two of us (RḂand JḂ) and the late Irving Gerst. We should also mention that significant work beyond what we present here has been carried out by one of us (GȦ) using modular forms. This will be published elsewhere.

10:00-10:20 **Jaroslav Nesetril,** Charles University
Room 1

On an Erdos Turan Problem

9:30-9:50 **Alex Iosevich,** University of Missouri-Columbia
Room 5

Erdos Distance Problem in Vector Spaces over Finite Fields

The classical Erdos distance conjecture says that N points in $\mathbb{R}^d$ determine at least $N^{\frac{2}{d}-\delta}$ Euclidean distances. We shall discuss this problem in vector spaces over finite fields. Estimates for classical Kloosterman sums play an important role.

10:30-11:20 **Doron Zeilberger,** Rutgers University
Room 1

Why is Ramsey Theory Sooooo Eeeenormsly Hard?

The short answer is that Ron Graham, one of the leaders of Ramsey theory, co-author of the definitive book (with Rothschild and Spencer) on the subject and co-prover of one of its Super-Six Theorems (with Leeb and Rothschild), would not choose to work on an easy subject.
A longer answer, from my enumerator's perspective is that Ramsey theory, that according to Motzkin, proves that complete disorder is impossible is equivalent to proving that for sufficiently large universes we are guaranteed islands of order. More precisely, if X is the random variable, "number of orderly islands", we have to find (or bound) the number of universes with X=0. If we knew all the moments of X, we would be done. Already the first moment, the expectation E[X], gives us some information (as first observed by Erdos). The second moment is harder, but still tractable, even for humans, but for the third and fourth moments we need computers. Beyond that, even computers seem to get stumped.

11:30-11:50 Room 1

Doug Iannucci, University of the Virgin Islands

Aliquot Squares

Let us call $n \in \mathbb{Z}^+$ an *aliquot square* if $\sigma(n^2) - n^2$ is itself a square. Only four such integers are known. One of them has the form pq for distinct odd primes p and q. We haven't found any more of these, but we do have necessary conditions for their existence. The main result, however, is that the sum $\sum 1/m$, taken over all aliquot squares m, is finite.

11:30-11:50 Room 5

Eric Duchêne, Grenoble Institute of Informatics and Applied Mathematics; P. Dorbec; L. Faria; S. Gravier.

Solitaire Clobber Played on Graphs

Clobber is a two-player combinatorial game that was introduced in Dalhousie by Albert et al. Demaine et al worked on a solitaire version of it. Here is the description of their one-player game: on a rectangular checkerboard, black and white stones are placed on some sets of squares. A black (respectively white) move consists in picking a black (resp. white) stone and clobbering a white (resp. black) one on a horizontally or vertically adjacent square. The clobbered stone is removed from the checkerboard and is replaced by the picked one. The goal is to minimize the number of remaining stones, by alternating black and white moves. A game configuration of solitaire clobber is said *k-reducible* if there exists a succession of moves that leaves at most k stones on the board. In our version of solitaire clobber (call it *SC2*), the player is not forced to alternate black and white moves.

In this talk, we will consider *SC2* played on graphs with an arbitrary arrangement of the stones. Given a game configuration G, the objective consists in finding the minimum value k such that G is k-reducible. This value is called the *reducibility value*. Demaine et al proved that this problem is NP-complete when G is a grid (a simple adaptation of their proof gives this result). We investigate this game on other kinds of graphs, such that paths, cycles, trees, and Hamming graphs. In the cases of paths, cycles, and trees, we give a way to compute polynomially the reducibilty value. We then prove that we can reduce configurations on Hamming graphs to one or two stones (in particular, hypercubes are 2-reducible).

1:30-1:50 **Sidney Graham,** Central Michigan University
Room 1

Small Gaps Between Products of Two Primes

The techniques that Goldston, Pintz, and Yıldırım recently used to prove the existence of short gaps between primes can be applied to other sequences. For example, one can apply these techniques to the sequence of numbers that are products of exactly two primes. Using this, we can prove that there are infinitely many integers n such that at least two of the numbers $n, n+2, n+6$ are products of exactly two primes. The same can be done for more general linear forms; e.g., there are infintely many n such at least two of $42n+1, 44n+1, 45n+1$ are products of exactly two primes. This in turn leads to new proofs of Heath-Brown's theorem that $\tau(n) = \tau(n+1)$ infinitely often and of Schlage-Puchta's theorem that $\omega(n) = \omega(n+1)$ infinitely often.
This is joint work with D. Goldston, J. Pintz, and C. Yıldırım.

1:30-1:50 **Gautami Bhowmik,** University of Science and Technology of Lille
Room 5

An Upper Bound for the Davenport Constant

Let G be an abelian group of n elements. The Davenport constant $D(G)$ is the smallest integer such that every sequence of $D(G)$ elements of G contains a subsequence with a zero sum. We prove that except for a finite number of cases (which we characterise), $D(G)$ is atmost $n/5+4$.
(Joint work with R. Balasubramaniam).

2:00-2:50 Room 1

Dan Goldston, San Jose State University

Small Gaps Between Primes

I will describe my recent joint work with Janos Pintz and Cem Yildirim on small gaps between primes. One surprising result is that the level of distribution of primes in arithmetic progressions can have dramatic consequences for the local distribution of primes. For example, if the Elliott-Halberstam conjecture is true (level of distribution equal to 1), then there are infinitely many pairs of primes with difference 16 or less. Unconditionally we prove that there are pairs of primes much closer together than the average distance between consecutive primes.
This work has generated some attention in the media. For me there have been three stages so far: the enjoyment of small-time public fame for proving the result two years ago, followed by the ignominy of having the proof crash and burn, and now (and I hope this is the end) the redemption of a new proof strangely emerging. After Wiles the press takes this as normal in mathematics, but I would not recommend this as a model for you to follow.

3:00-3:20 Room 1

Renling Jin, College of Charleston

Nudging Forward on Freiman's $3k-3+b$ Conjecture

Let A be a set of integers of size k. Freiman conjectured that for sufficiently large k, if $|A+A| = 3k-3+b$ for $0 < b < (k/3)-2$, then A is either a subset of an arithmetic progression of length $2k-1+2b$ or a subset of the union of two arithmetic progressions of the same difference with a combined length $k+b$. A weak version of the conjecture was recently confirmed with the condition $0 < b < (k/3)-2$ being replaced by $0 < b < ek$ for some small positive real number e. In order to prove that the weak version is also true for any positive $e < 1/3$, one has to improve a key lemma. In the talk we present an improved key lemma for that purpose.

3:00-3:20 Room 5

Richard Nowakowski, Dalhousie University

Regularities in Taking and Breaking Games

In Taking (from a heap) and Breaking (the heap into several) Games the search is for "easily" understood strategies. In impartial octal games, the hope is that all games are periodic. This talk reports on new regularities found in impartial hexadecimal, poctal and partizan octal games.

3:40-4:00 **Fan Chung,** University of California, San Diego
Room 1

de Bruijn Sequences and Covering Codes

What is the length of a shortest sequence of integers S so that so that every q-ary string is at most R symbol changes from some n-word appearing consecutively in S? We consider these covering codes and prove that they can have size close to the smallest possible covering code. We also examine covering codes for permutations and in general covering codes for rooted hypergraphs. Many related problems and conjectures remain unresolved.
This is a joint work with Joshua Cooper.

4:10-4:30 **Zhi-Wei Sun,** Nanjing University
Room 1

Some Congruences Motivated by Algebraic Topology

In July 2005, D. M. Davis posted several original number-theoretic conjectures arising from his investigation of homotopy exponents of the special unitary group SU(n). The recent study of these conjectures and their generalizations by the author and Davis led to a strong lower bound for the homotopy p-exponent of SU(n), as well as a number of new combinatorial congruences some of which extend vastly some classical congruences (such as Lucas' theorem) concerning binomial coefficients. We talk about these results and propose some further conjectures.

4:10-4:30 **Gang Yu,** University of South Carolina
Room 5

A New Upper Bound for $B_2[g]$ Sets

For an integer $g \geq 1$, a set $\mathcal{A} \subset \mathbb{Z}$ is called a $B_2[g]$ set if, for every integer n, the equation $n = a + b$ has at most g solutions with $a, b \in \mathcal{A}$ and $a \leq b$. For a large real number N, let $\mathcal{A}(N)$ be any finite $B_2[g]$ set satisfying $\mathcal{A}(N) \subset [1, N] \bigcap \mathbb{Z}$. In the talk, I will give a new upper bound for $|\mathcal{A}(N)|$.

4:40-5:00 Room 1

Omar Kihel, Brock University

On Some Variants of Lucas' Square Pyramid Problem

The problem of finding integers k such that the sum of k squares is a square has been initiated by Lucas. A first complete solution to Lucas' problem was given by Watson who showed that Lucas' problem has only two solutions, namely $k = 1$ and 24. It was natural to consider a generalisation of Lucas' problem and ask whether this phenomenon keeps occurring when the initial square is shifted. This is in fact equivalent to solving the following diophantine equation:

$$n^2 + (n+1)^2 + \cdots + (n+k-1)^2 = y^2 \tag{1}$$

This problem has been treated by many authors from different points of view. For instance, Beeckmans determined all values $1 \leq k \leq 1000$ for which equation (1) has solutions (n, y). Other people studied equation (1) by taking n as a fixed integer and ask for corresponding k and y. Bremner, Stroeker and Tzanakis found all solutions k and y to equation (1) when n is a fixed integer and $1 \leq n \leq k$. Another generalisation of Lucas' problem has been considered by Stroeker who asked when does a sum of consecutive cubes a perfect square. In this talk we consider a similar question for shifted consecutive cubes.

4:40-5:00 Room 5

Heinrich Niederhausen, Florida Atlantic University

The Number of Contacts with the Diagonal Made by Random Walks in the First Octant

Our random walks take unit steps in the four directions of the compass, N, E, S, and W. We derive a closed form for the number of random walks from the origin to some point (n, n) on the diagonal in k steps inside the first octant $0 \leq y \leq x$, touching the diagonal exactly c times.

5:10-5:30 Room 1

Stephan Baier, Queen's University

The Large Sieve With Sparse Sets of Moduli

Motivated by a problem of Erdös and Sárközy in combinatorial number theory, we investigate the large sieve with characters to moduli q running over a sparse set S. We obtain an improvement of the classical large sieve inequality if S is in a sense well-distributed in arithmetic progressions. We then use this general result together with Fourier techniques to derive large sieve bounds for the special case when $\mathcal{S}$ consists of sqares.

5:10-5:30 Room 5

Peter Floodstrand Blanchard, Miami University, Hamilton, Ohio.

Some 'Some-sum' Consequences and Variations of the Folkman Theorem

The Folkman-Sanders theorem gurantees the existence of set whose sum-set is monochromatic. Asking for the entire sum-set is asking a lot. We ask what may happen if you ask for less, and present some variations with computations of some 'Some-sum' Folkman type numbers for up to five colors.

Saturday, October 29

9:00-9:20
Room 1

Kevin O'Bryant, University of California, San Diego.

Reciprocals of Binary Power Series

If A is a set of nonnegative integers containing 0, then there is a unique nonempty set B of nonnegative integers such that every positive integer can be written in the form $a+b$, where $a \in A$ and $b \in B$, in an even number of ways. We compute the natural density of B for several specific sets A, including the Prouhet-Thue-Morse sequence, $\{0\} \cup \{2^n \colon n \in N\}$, and random sets, and we also study the distribution of densities of B for finite sets A. This problem is motivated by Euler's observation that if A is the set of n that have an odd number of partitions, then B is the set of pentagonal numbers $\{n(3n+1)/2 \colon n \in Z\}$. Joint work with Joshua N. Cooper and Dennis Eichhorn.

9:00-9:20
Room 5

Tom Brown, Simon Fraser University; Allen Freedman, Simon Fraser University; Peter Shiue, University of Nevada Las Vegas

Progressions of Squares

We consider the occurrence of certain generalizations of arithmetic progressions, amongst the squares of integers. More specifically, let $a_1, a_2, \ldots, a_n$ be an increasing sequence of squares, let $d_i = a_{i+1} - a_i$, and let $D = \{d_1, d_2, \ldots, d_{n-1}\}$. If the given sequence of squares is an arithmetic progression (impossible if $n > 3$) then D has size 1 and diameter 0. When $n > 3$, we seek to minimize either the size of D or the diameter of D.

9:30-9:50 Room 1

Bruce Landman, University of West Georgia

Recent Results in Ramsey Theory on the Integers

I will discuss some problems from the area of Ramsey theory on the set of Integers, and briefly survey recent progress on these problems. Topics will include mixed van der Waerden numbers, the regularity of subfamilies of the set of arithmetic progressions, and generalized arithmetic progressions.

9:30-9:50 Room 5

Silvia Heubach, California State University Los Angeles; Toufik Mansour, Haifa University

Enumeration of 3-letter Patterns in Compositions

Let A be any set of positive integers and $n \in \mathbb{N}$. A composition of n with parts in A is an ordered collection of one or more elements in A whose sum is n. We derive generating functions for the number of compositions of n with m parts in A that have r occurrences of 3-letter patterns formed by two (adjacent) instances of levels (= number followed by itself), rises (= number followed by bigger number) and drops (= number followed by a smaller number). We also derive asymptotics for the number of compositions of n that avoid a given pattern. Finally, we obtain the generating function for the number of k-ary words of length m which contain a prescribed number of occurrences of a given pattern as a special case of our results.

10:00-10:20 Room 1

Ognian Trifonov, University of South Carolina; Sharon Sholz

On a Problem of Ore

In 1961 Ore posed the following problem: Show that for each positive integer a there exists a positive integer k_a such that the equation $\varphi(x) = 2^a k_a$ has no solution (here $\varphi(x)$ is Euler's function). Two solutions of Ore's problem where published - one by J. Selfridge and one by P. Bateman. In his solution Bateman formulated certain conjecture. We show that Bateman's conjecture is false and prove that a revised version of Bateman's conjecture holds. Also, we consider the related problem when one replaces Euler's φ-function by σ - the sum of divisors function.

10:00-10:20 Room 5

Glenn Hurlbert, Arizona State University

An Application of Graph Pebbling to Zero-Sum Sequences in Abelian Groups

A sequence of elements of a finite group G is called a zero-sum sequence if it sums to the identity of G. The study of zero-sum sequences has a long history with many important applications in number theory and group theory. In 1989 Kleitman and Lemke, and independently Chung, proved a strengthening of a number theoretic conjecture of Erdős and Lemke. Kleitman and Lemke then made more general conjectures for finite groups, strengthening the requirements of zero-sum sequences. In this paper we prove their conjecture (first obtained by Geroldinger) in the case of abelian groups. Namely, we use graph pebbling to prove that for every sequence $(g_k)_{k=1}^{|G|}$ of $|G|$ elements of a finite abelian group G there is a nonempty subsequence $(g_k)_{k\in K}$ such that $\sum_{k\in K} g_k = 0_G$ and $\sum_{k\in K} 1/|g_k| \leq 1$, where $|g|$ is the order of the element $g \in G$. This is joint work with Shawn Elledge.

10:30-10:50 Room 1

Ron Graham, University of California, San Diego

Some of My Favorite Problems in Ramsey Theory

I will discuss a number of my favorite old and new problems in the area of Ramsey theory.

11:00-11:50 **Neil Hindman,** Howard University
Room 1

Partition Regularity of Matrices

In honor of Ron Graham's long association with the field of Ramsey Theory, I will discuss the following general problem. Given a $u \times v$ matrix A with rational entries, when can one guarantee that for any finite coloring of $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, or $\mathbb{R}$

(1) there must exist $\vec{x}$ with monochromatic entries such that $A\vec{x} = \vec{0}$ or

(2) there must exist $\vec{x}$ such that the entries of $A\vec{x}$ are monochromatic?

I will also discuss the problems, given A as above and $\vec{b} \in \mathbb{Q}^u$, of determining when one can guarantee for any finite coloring of $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, or $\mathbb{R}$ the existence of $\vec{x}$ with monochromatic entries, or distinct monochromatic entries, such that $A\vec{x} = \vec{b}$. The results discussed will range from the very old (Rado, 1933) to the very new (Hindman and Leader, to appear). And, of course, they will include a result of Graham.

1:30-1:50 Room 1

Aaron Meyerowitz, Florida Atlantic University

The 392 Problem

Let $n = rs^2$ with $r > 1$ and square-free. The smallest $m > n$ with nm square is clearly $m = r\,(s+1)^2$. It has been conjectured that for all such n except $n = 8$ and $n = 392$ there are integers x, y such that $rs^2 = n < x, y < r\,(s+1)^2$ and nxy is a square. It is known that an exception must be of the form $n = 2s_k^2$ where s_k is the kth term of the sequence $0, 2, 14, 84, 492, 2870, 16730\ldots$. These numbers are related to convergents to $\sqrt{2}$ and have recurrence

$$s_{k+2} = 6s_{k+1} - s_k + 2.$$

We investigate these remaining open cases. The conjecture is still open but we will show among other things that there are such x, y if $n = 2s_k^2$ for $3 \le k \le 20,000$ or if $k \bmod 21 \in \{0, 7, 8, 10, 11, 15, 16, 20\}$. There is no difficulty in principle in extending the first result well past $k = 20,000$. Many congruences like the second result can be given. This is joint work with John Selfridge.

1:30-1:50 Room 5

Aaron Robertson, Colgate University

On Monochromatic Ascending Waves

A sequence of positive integers $w_1, w_2, \ldots, w_n$ is called an ascending wave if $w_{i+1} - w_i \ge w_i - w_{i-1}$ for $2 \le i \le n-1$. For integers $k, r \ge 1$, let $AW(k;r)$ be the least positive integer such that under any r-coloring of $[1, AW(k;r)]$ there exists a k-term monochromatic ascending wave.
The existence of $AW(k;r)$, although guaranteed by van der Waerden's theorem on arithmetic progressions, can be proved directly. Brown, Erdős, and Freedman originally defined such sequences and proved that $k^2 - k + 1 \le AW(k;2) \le \frac{1}{3}(k^3 - 4k + 9)$. Alon and Spencer then showed that $AW(k;2) = O(k^3)$.
We will show that $AW(k;3) = O(k^5)$ and that for any $\epsilon > 0$ and $r \ge 1$, $O(k^{2r-\epsilon}) \le AW(k;r) \le O(k^{2r})$
Our results improve upon the best known upper bound for $AW(k;2)$ and also give us $AW(k;r) \le \frac{2^{k-2}}{(k-1)!}\,r^{k-1}(1+o(1))$ for any fixed $k \ge 3$.
This talk is based on joint work that led to a high honor's thesis in mathematics for Tim LeSaulnier (who is now in his first year of graduate school).

2:00-2:50 Room 1

Jozsef Solymosi, University of British Colombia

Squares on the Integer Lattice

Szemerédi's theorem states that any dense subset of the integers contains arbitrary long arithmetic progressions. Around 1970, Ron Graham asked the following related question. Is it true that every dense subset of the two dimensional integer grid contains a square? Ten years later Fürstenberg and Katznelson proved that for every $\delta > 0$, every positive integer r and every finite subset $X \subset \mathbf{Z}^r$ there is a positive integer N such that every subset A of the grid $\{1, 2, \ldots, N\}^r$ of size at least δN^r has a subset of the form $a + dX$ for some positive integer d. In this talk we show a combinatorial proof for the original question of Graham, the special case, when $X = \{(0,0), (0,1), (1,0), (1,1)\}$. In the second part of the talk we show how to find squares in very sparse subsets of the integer grid using the density version of the Hales-Jewett theorem.

3:00-3:20 Room 1

Carl Pomerance, Dartmouth College

Covering Congruences

A famous old problem of Paul Erdős is whether for each number B the set of integers may be covered with a finite collection of congruence classes with distinct moduli each at least B. In fact, Erdős wrote of this as his "favorite problem." In recent work with Filaseta, Ford, Konyagin, and Yu, we have found some new results in this area; for example, if such finite collections should exist, the largest modulus cannot be $O(B)$. These new results settle some conjectures of Erdős, Graham, and Selfridge.

3:40-4:00
Room 1

David S. Gunderson, University of Manitoba; Christian Elsholtz, Royal Holloway University of London

Congruence Properties of Multiplicative Functions on Sumsets

Rivat, Sárközy and Stewart (1999) proved a result regarding maximal cardinalities of sets $\mathcal{A}, \mathcal{B} \subset \{1, \ldots, N\}$ so that for every $a \in \mathcal{A}$ and $b \in \mathcal{B}$, $\Omega(a+b)$ is even, where $\Omega(n)$ denotes the number of prime factors of n counted with multiplicity. This paper extends their work in several directions, both in qualitative and quantitative aspects. The role of $\lambda(n) = (-1)^{\Omega(n)}$ is generalized to all non-constant completely multiplicative functions from the natural numbers to $\{-1, 1\}$, and for all possible parities of Ω on $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{A}+\mathcal{B}$. Also, in each case, the pairs $(\mathcal{A}, \mathcal{B})$ occur with positive density.
The proof uses combinatorial counting arguments from number theory, arithmetic Ramsey theory, and extremal graph theory.

3:40-4:00
Room 5

Carla D. Savage, North Carolina State University

Five Guidelines for Partition Analysis

Five simple rules are proposed that can be applied to enumerate the nonnegative integer solutions to any set of homogeous linear inequalities. We demonstrate their application in the theory of partitions and compositions, using them to *derive recurrences* for several families. The set of rules can be regarded as a simplification of MacMahon's partition analysis or as an extension of Elliott reduction and "adding a slice". This is joint work with Sylvie Corteel and Sunyoung Lee.

4:10-4:30 Room 1

Vsevolod F. Lev, University of Haifa

Powers of 2 by Five Distinct Summands

Answering a question by Erdos and Freud, we have recently shown that for any (sufficiently large) integer l and any set $A \subset [1, l]$ of density, larger than $1/3$, there is a power of 2 representable as a sum of at most five pairwise distinct elements of A. In our talk we will review the history of the problem, discuss related results, and outline the proof.

4:10-4:30 Room 5

Michael Z. Spivey, University of Puget Sound Tacoma; Laura L. Steil, Samford University

The k-Binomial Transform and the Hankel Transform

We give a new proof of the invariance of the Hankel transform under the binomial transform of a sequence. Our method of proof leads to three variations of the binomial transform; we call these the k-binomial transforms. We give a simple means of constructing these transforms via a triangle of numbers. We also show how the exponential generating function of a sequence changes after our transforms are applied. Addressing a question of Layman, we then show that the Hankel transform of a sequence is invariant under one of our transforms, and we show how the Hankel transform changes after the other two transforms are applied. Finally, we use these results to determine the Hankel transform of several integer sequences.

4:40-5:00 **Jacob Fox,** Massachusetts Institute of Technology
Room 1

Recent Developments on Partition Regularity of Linear Equations

I will briefly discuss the following results and conjectures on partition regularity of systems of linear homogeneous equations.
1. A linear equation is called r-*regular* if, for every r-coloring of the positive integers, there is a monochromatic solution to that linear equation. A linear equation is called *regular* if it is r-regular for all positive integers r. Daniel Kleitman and I proved that if a linear equation in three variables is 36-regular, then it is regular. This establishes the first nontrivial case of Rado's Boundedness Conjecture (1933).
2. Let $E(n)$ denote the following equation in $n+1$ variables: $x_0 + 2x_1 + \ldots + 2^{n-1}x_{n-1} = 2^n x_n$. Ron Graham and I conjecture that for every integer $n > 1$, there is a unique $(n+1)$-coloring of the positive rational numbers without a monochromatic solution to $E(n)$. We have established the conjecture for $n = 2$ and $n = 3$. The conjecture implies a 1933 conjecture of Rado that for every positive integer n, there is a linear homogeneous equation that is n-regular but not $(n+1)$-regular.
3. Rados Radoičić and I proved that the following statement is independent of Zermelo-Fraenkel set theory: for every 4-coloring of the nonzero real numbers, there is a monochromatic solution to $E(3)$.
4. A system $\mathcal{L}$ of linear homogeneous equations is called *countably regular* if, for every countable coloring of the real numbers, there is a monochromatic solution to the system in distinct variables. In ZFC, we give a classification of those systems of linear equations that are countably regular. This classification depends on the cardinality of the continuum. In fact, we have the following combinatorial interpretation of the cardinality of the continuum: the cardinality of the continuum is greater than $\aleph_n$ if and only if the following equation is countably regular: $x_0 + \cdots + x_n = x_{n+1} + nx_{n+2}$. The case $n = 1$ is due to Erdős and Kakutani.

4:40-5:00 **Florian Luca,** National Autonomous University of Mexico
Room 5

On the Number of Ordered Factorizations of a Positive Integer

Let $m(n)$ be the function which counts the number of ordered factorizations of n in factors larger than 1. In 1941, P. Erdős claimed that there exist two constants c_1 and c_2 such that the inequality $m(n) < \frac{n^\rho}{\exp((\log n)^{c_1})}$ holds for all sufficiently large positive integers n while the inequality $m(n) > \frac{n^\rho}{\exp((\log n)^{c_2})}$ holds for infinitely many positive integers n, where $\rho = 1.72864\ldots$ is the real solution to $\zeta(\rho) = 2$. In my talk, I will sketch a proof of these inequalities with $c_1 = 1/\rho - \varepsilon$ and $c_2 = \rho/(\rho^2 - 1) + \varepsilon$, where $\varepsilon > 0$ is arbitrary. I will also survey some other properties of the function $m(n)$.

This is joint work with M. Klazar.

5:10-5:30 Room 1

Takao Komatsu, Hirosaki University

Hurwitz and Tasoev Continued Fractions with Very Long Period

Two quasi-periodic continued fractions with regular pattern on the sequence of partial quotients are Hurwitz continued fraction and Tasoev continued fraction. Some generalizations of these continued fractions have been studied. The period of some typical general cases was restricted to less than 5 though many particular numbers do have longer period. In this talk many generalized Hurwitz and Tasoev continued fractions with longer period are shown. As one of applications, the continued fraction expansion of e^2 is generalized.

5:10-5:30 Room 5

Sarah Mason, University of Pennsylvania

Non-symmetric Schur Functions and Standard Bases

The Schur functions, s_μ, form a basis for the ring of symmetric functions. Macdonald polynomials are symmetric functions $P_\mu(x;q,t)$ in variables $x = x_1, x_2, ...$, with coefficients which are rational functions of two parameters q and t. The Schur functions are obtained from Macdonald polynomials by setting $q = t = 0$. Recently Haglund, Haiman, and Loehr derived a combinatorial formula for non-symmetric Macdonald polynomials, which gives a new decomposition of the Macdonald polynomial into nonsymmetric components and provides a combinatorial description of the nonsymmetric Schur functions, NS_λ. Letting $q = t = 0$ in this identity implies $s_\mu(x) = \sum_\lambda NS_\lambda(x)$, where the sum is over all rearrangements λ of the partition μ. We exhibit a weight-preserving bijection between semi-standard Young tableaux and semi-standard skyline fillings to give a combinatorial proof of the formula. The bijection involves an analogue of the Robinson-Schensted-Knuth Algorithm. We also provide a non-inductive combinatorial interpretation of the standard bases of Lascoux and Schützenberger.

Sunday, October 30

9:00-9:20
Room 4

E. Rodney Canfield, University of Georgia; Brendan D. McKay, Australian National University

The Number of Biregular Bipartite Graphs

How many 4×6 $(0, 1)$-matrices are there with three 1's in every row and two 1's in every column? (This is also the number of labeled bipartite graphs of a certain type.) In this talk we'll look at this question for $m \times n$ matrices required to have s 1's in every row and t 1's in every column. We have proven an asymptotic formula via an intricate calculation. The formula simplifies to a succinct form in need of an explanation.

9:00-9:20
Room 5

Radoš Radoičić, Rutgers University

Ramsey-type Results for the Hypercube

We consider the question of existence of monochromatic cycles for edge colorings of the hypercube, raised by Fan Chung in 1992. She proved that for any fixed $k \geq 2$ and an *even* $l \geq 4$, for a sufficiently large hypercube, any k-coloring of the edges contains a monochromatic cycle of length $2l$. On the other hand, there is a 2-coloring of any hypercube which avoids monochromatic cycles of length 4, and a 3-coloring which avoids monochromatic cycles of length 6. Fan Chung asked what happens for *odd* $l \geq 5$, i.e. whether it is possible to avoid monochromatic cycles of length $10, 14, 18, \ldots$
We answer this question by proving that for any fixed $k \geq 2$ and $l \geq 5$, any k-coloring of a sufficiently large hypercube contains a monochromatic cycle of length $2l$. More generally, we provide a characterization of all subgraphs of the hypercube with this Ramsey property. In addition, we show the existence of subgraphs H_k such that for a sufficiently large hypercube, any k-coloring of the edges contains a monochromatic copy of H_k but this is not the case for all $(k+1)$-colorings.
This is a joint work with N. Alon, B. Sudakov, and J. Vondrák.

9:30-9:50
Room 4

Aviezri S. Fraenkel, Weizmann Institute of Science

The Raleigh Game

We present a game on 3 piles of tokens, which is neither a generalization of Nim, nor of Wythoff's game. Three winning strategies are given and validated. They are, respectively, recursive, algebraic and arithmetic in nature, and differ in their time and space requirements. The game is a birthday present for Ron Graham, but it would take too much time and space to explain why.

10:00-10:20 **Melvin Nathanson,** CUNY (Lehman College and the Graduate Center)
Room 4

Addition Rules for Quantum Integers

The quantum integer $[n]_q$ is the polynomial $1+q+q^2+\cdots+q^{n-1}$. (This polynomial was known to Euler, of course, but not with this name.) The talk will give a complete classification of linear and quadratic addition rules for the quantum integers.

10:30-10:50 **Steven Butler**, University of California, San Diego.
Room 4

Constructing Balanced Strategies for the Hats Game

The hats game consists of n players with one adversary. The adversary will place hats of two colors on the heads of the players, the players will then be able to see what hat everyone (not including themselves) is wearing, no communication is allowed. Each player must then "guess" their hat color. The goal is for the n players to find a strategy to maximize the minimum number of correct guesses that are guaranteed.
We will show that there exists strategies which guarantee at least $\lfloor n/2 \rfloor$ correct guesses, this bound is sharp. We will also give a hypergraph interpretation of the game which can be used to derive various results. In particular, we will construct a "balanced" strategy where, for instance, if there are k blue hats and ℓ red hats placed on the players then there are at least $\lfloor k/2 \rfloor$ people wearing blue who guess blue and $\lfloor \ell/2 \rfloor$ people wearing red who guess red.

10:30-10:50 **Linyuan Lu,** University of South Carolina
Room 5

Hexagon-free Subgraphs in Hypercube Q_n

Let hypercube Q_n be the graph with a vertex set of all binary strings of length n. Two binary strings are adjacent in Q_n if and only if they differ exactly at one coordinate. For any integer k, let $f(n, 2k)$ be the maximum number of edges, which a subgraph of Q_n can have without containing an even cycle C_{2k}. It can be shown that the limit of the maximum edge-density $\lim_{n\to\infty} \frac{f(n,2k)}{n2^{n-1}}$ always exists. We denote this limit by σ_{2k}. The problem of determining σ_{2k} is proposed by Erdős. In 1992, Chung proved

$$\frac{1}{4} \le \sigma_6 \le \sqrt{2} - 1.$$

The low bound is then improved by Conder in 1993 to $\frac{1}{3}$. Here we improve the upper bound to

$$\sigma_6 \le 0.3941.$$

11:40-12:00 Room 4

Paul T. Ottaway, Dalhousie University

A Partial Order Token Sliding Game

Given a partial order P and a token placed on an element x designated as the starting point, Left and Right take turns sliding the token to other elements of P. If the token is currently at x, Left may only slide the token to elements y such that $y < x$ while Right may only slide the token to elements z such that $z > x$. The only proviso is that the token may never occupy the same element twice during the game. We describe a strategy for playing this game and show that all such games have values which are numbers. We also give a construction for all dyadic rationals.

11:40-12:00 Room 5

Gretchen L. Matthews, Clemson University; Rhett S. Robinson, University of North Carolina - Chapel Hill

A Variant of the Frobenius Problem and Generalized Suzuki Semigroups

Given positive integers $a_1, \ldots, a_k$, the Frobenius problem asks for the largest integer not representable as a linear combination of $a_1, \ldots, a_k$ with nonnegative integral coefficients. A related question is to determine the set of numbers $B(S)$ that are representable as differences of elements of the semigroup $S := \left\{ \sum_{i=1}^{k} c_i a_i : c_i \in \mathbb{N}_0 \right\}$. It is natural to iterate this construction to obtain a chain of semigroups $B_i(S)$. This chain has applications in commutative algebra, algebraic geometry, and coding theory. In this talk, we consider this construction for generalized Suzuki semigroups.

12:10-12:30 **Dennis Eichhorn,** California State University, East Bay
Room 5

The Odd Values of $p(n)$

The parity of $p(n)$, the ordinary partition function, has been studied for over a century, yet it still remains something of a mystery. For example, although empirical evidence (the first several million values) seems to indicate that $P_o(N) = \#\{n < N : p(n) \text{ is odd}\} \sim N/2$, no one has even been able to show that $P_o(N) \geq \sqrt{N}$ for N sufficiently large. Until recently, the best known lower bound for $P_o(N)$ was proven using properties of ℓ-adic Galois representations and the theory of modular forms. In this talk, we give a better lower bound using elementary generating function techniques coupled with results from classical analytic number theory. Restriction to arithmetic progressions and an outline for further improvements will also be discussed.

12:40-1:00 Room 4

Thomas Stoll, University of Vienna

On Families of Nonlinear Recurrences Related to Digits

Consider the sequence of positive integers $(u_n)_{n\geq 1}$ defined by $u_1 = 1$ and

$$u_{n+1} = \left\lfloor \sqrt{2}\,(u_n + 1/2) \right\rfloor, \qquad n \geq 1.$$

In 1970, R. L. Graham and H. O. Pollak discovered the unexpected fact that $u_{2n+1} - 2u_{2n-1}$ is just the n-th digit in the binary expansion of $\sqrt{2} = (1.011010100\ldots)_2$. This result has also been cited in P. Erdős/R. L. Graham *"Old and New Problems and Results in Combinatorial Number Theory"* (Genève, 1980). Therein, the authors suspected that similar results would also hold "for $\sqrt{m}$ and other algebraic numbers", but they concluded that "we have no idea what they are".

Fix $w \in \mathbb{R}_{>0}$. We first give two infinite families of similar nonlinear recurrences such that $u_{2n+1} - 2u_{2n-1}$ indicates the n-th binary digit of w. Furthermore, for all integral $g \geq 2$, we establish a recurrence such that $u_{2n+1} - gu_{2n-1}$ denotes the n-th digit of w in the g-ary digital expansion. For instance, let $v_1 = 1$ and

$$v_{n+1} = \begin{cases} \lfloor a(v_n + 1/2) \rfloor, & \text{if } n \text{ is odd;} \\ \lfloor b(v_n + 1/2) \rfloor, & \text{if } n \text{ is even,} \end{cases}$$

where $a = (9 - 3\sqrt{2})/14$ and $b = 6 + 2\sqrt{2}$. Then $v_{2n+1} - 3v_{2n-1}$ is the n-th digit in the ternary expansion of $\sqrt{2} = (1.102011221\ldots)_3$.

12:40-1:00 Room 5

Aaron Siegel, Mathematical Sciences Research Institute; Thane Plambeck

Miserable Monoids: How to Lose When You Must

We present solutions for numerous previously-unsolved misère octal games. The solutions were obtained using new software, *MisereSolver*, that exploits a theoretical framework recently introduced by Plambeck. We also introduce several new insights into the general structure of such games.

Normal-play impartial games—those in which the player who makes the last move wins—have been well-understood for more than fifty years. Every such game G is equivalent to a Nim-heap N, in the sense that for every impartial game X, $G + X$ and $N + X$ have the same outcomes. In the 1970s, Conway introduced a parallel theory of misère-play games, where the player who makes the last move *loses*. Conway adopted the same definition of equivalence: $G = H$ iff, for every misère game X, $G+X$ and $H+X$ have the same outcomes. In contrast to the normal-play theory, the set of misère equivalence classes is vast, loosely structured, and difficult to categorize. As a result, the misère theory is far more complicated than the classical normal-play theory.

Conway, Sibert, Allemang and others noticed a way to manage these difficulties. If the goal is to determine the winning strategy for a particular game Z, then we can weaken our equivalence by considering only those positions that actually arise in the play of Z. Formally, write $G \equiv H$ iff $G + X$ and $H + X$ have the same outcomes, for every game X that occurs in Z. Conway noticed that even when Z has infinitely many distinct positions, the set of equivalence classes is often finite. Conway, Sibert and Allemang exploited this reduction to solve misère Kayles and a few other games; but such analyses are still very difficult, and until recently little further progress was made.

Then in 2004, Plambeck noticed that this reduction can be made systematic. The equivalence classes modulo Z form a commutative monoid $\mathcal{Q}$ under addition, the *misère quotient* of Z. A winning strategy for Z can easily be recovered from $\mathcal{Q}$; and—most important—the correctness of $\mathcal{Q}$ can be verified algorithmically. By exploiting this new framework, *MisereSolver* can reproduce the Kayles solution in just a few seconds, and can solve many octal games whose strategies are substantially more complicated.

Our presentation will include a detailed construction of $\mathcal{Q}$, several illustrative examples, and a handful of results that describe the structure of $\mathcal{Q}$ in general. Despite recent advances, the number of open questions remains vast, and we will conclude by summarizing some of the most interesting ones.

1:10-1:30 Room 4

Brian Hopkins, Saint Peter's College

Counting Bulgarian Gardens of Eden

A lower bound is provided for the number of partitions of n with Dyson rank -2 or less.
The motivation for this investigation comes from "Bulgarian solitaire," popularized by Martin Gardner in 1983. A move in the game can be considered as a shift operation on partitions of n, inducing a finite dynamical system. The cycles of these systems are understood; a next step is studying the partitions with no preimage. Following the terminology of cellular automata, these are called Garden of Eden partitions. They are characterized by having rank -2 or less.
The number of Garden of Eden partitions of n is at least the number of conjugate pairs of partitions of $n-1$. This is established with an explicit injection most easily described with the Frobenius notation. The number of conjugate pairs can be expressed in terms of the partition function. The map is bijective up to $n = 18$.
An upper bound for the number of Garden of Eden partitions is conjectured.

1:10-1:30 Room 5

Nayandeep Deka Baruah, Tezpur University

Nonic Analogues of the Rogers-Ramanujan Functions with Applications to Partitions

We define the nonic Rogers-Ramanujan type functions

$$D(q) := \sum_{n=0}^{\infty} \frac{(q;q)_{3n} q^{3n^2}}{(q^3;q^3)_n (q^3;q^3)_{2n+1}} = \frac{(q^5;q^9)_\infty (q^4;q^9)_\infty (q^9;q^9)_\infty}{(q^3;q^3)_\infty},$$

$$E(q) := \sum_{n=0}^{\infty} \frac{(q;q)_{3n} (1-q^{3n+2})}{(q^3;q^3)_n (q^3;q^3)_{2n+1}} = \frac{(q^7;q^9)_\infty (q^2;q^9)_\infty (q^9;q^9)_\infty}{(q^3;q^3)_\infty},$$

$$F(q) := \sum_{n=0}^{\infty} \frac{(q;q)_{3n+1} q^{3n(3n+1)}}{(q^3;q^3)_n (q^3;q^3)_{2n+1}} = \frac{(q^8;q^9)_\infty (q;q^9)_\infty (q^9;q^9)_\infty}{(q^3;q^3)_\infty},$$

where the later equalities are due to L. J. Slater (Further identities of the Rogers-Ramanujan type, *Proc. London Math. Soc.(2)* **54**(1952), 147-167). We establish several modular identities involving $D(q)$, $E(q)$, and $F(q)$, like,

$$D(q^2)E(q) - qE(q^2)F(q) + qF(q^2)D(q) = 1$$

and

$$F(q)D(q^5) + qD(q)E(q^5) - q^3E(q)F(q^5) = 1,$$

which are analogous to the famous forty identities for the Rogers-Ramanujan functions. Furthermore, by the notion of colored partitions, we find several partition theoretic results from some of our identities.