

INTEGERS CONFERENCE 2007

University of West Georgia
Carrollton, Georgia

Abstracts

1. **Fisseha Abebe**, Clark Atlanta University

Distance Metric for Protein Partitions

The composition of proteins in terms of amino-acid sequences within three protein theoretical families: serine, leucine, and valine, with respective sizes of 8, 7, and 5 amino acids, have been studied using combinatorial methods. All possible partitions are examined, and a metric is defined to measure the distance between any random partition and the natural partition of the genetic code. Distribution of distance measures has shown patterns of departures from the natural partition.

Joint work with William Seffens, Clark Atlanta University.

2. **George E Andrews**, Pennsylvania State University

Multipartition Identities and the Tri-Pentagonal Number Theorem

In the Journal of Combinatorial Theory, 91(2000), 464-475, Bailey chains were extended to multiple series, and new Pentagonal Number Theorems were deduced. The triple series involving pentagonal numbers (= Tri-Pentagonal Number Theorem) was most intriguing. In the first half of this talk, we shall both interpret the related triple q-series as the generating function for certain tri-partitions, and we shall show that the triple pentagonal number side of the identity can be reduced to a linear combination of three infinite products. In the last half, we shall then discuss the further possibilities of multipartition identities and congruences. In these latter considerations, modular forms, mock theta functions, false theta functions and total interlopers make mysterious appearances.

3. **Mohamed El Bachraoui**, United Arab Emirates University

Mobius Inversion Formulas for Functions of Several Variables

We extend the Mobius inversion formula for functions of one single variable to functions of several variables. As applications, we count for $n \geq m$ the number of relatively prime subsets of sets of the form $\{m, m+1, \dots, n\}$ and we give a formula for phi functions for such sets. Our results generalize the work of M. Nathanson on relatively prime sets and phi functions for sets $\{1, 2, \dots, n\}$.

4. **Bill Banks**, University of Missouri-Colombia

Nicolas Numbers and the Robin Criterion

In 1984, G. Robin proved that the Riemann hypothesis is true if and only if the *Robin inequality* $\sigma(n) < e^\gamma n \log \log n$ holds for every integer $n > 5040$, where $\sigma(n)$ is the sum of divisors function, and γ is the Euler-Mascheroni constant. We exhibit a broad class of subsets of the natural numbers such that the Robin inequality holds for all but finitely many $n \in$. As a special case, we determine the finitely many numbers of the form $n = a^2 + b^2$ that do not satisfy the Robin inequality. In fact, we prove our assertions with the *Nicolas inequality* $n/\varphi(n) < e^\gamma \log \log n$; since $\sigma(n)/n < n/\varphi(n)$ for $n > 1$ our results for the Robin inequality follow at once.

Joint work with Derrick Hart, Pieter Moree and Wesley Nevans.

5. **Jonathan Bayless**, Dartmouth College

On the Unit Group Function

Carmichael's conjecture that there is not a unique preimage for any value of Euler's φ -function is well-known and widely believed. Since the unit group modulo n has order $\varphi(n)$, one might be tempted to conjecture that there is never a unique preimage for the function U , which maps an integer n to the isomorphism class of the unit group modulo n . However, this is false (consider $n = 24$). We give a lower bound on the number of counterexamples up to x , and we also show most numbers n are not counterexamples.

6. **Vitaly Bergelson**, The Ohio State University

New Extensions of the Polynomial Szemerédi Theorem

The Polynomial Szemerédi theorem (joint result with A. Leibman) states that if $p_i, i = 1, 2, \dots, k$ are polynomials with integer coefficients which satisfy $p_i(0) = 0$, then any set A in \mathbb{N} which has positive upper density contains "many" polynomial configurations of the form $\{a, a + p_1(n), a + p_2(n), \dots, a + p_k(n)\}$. (The classical Szemerédi theorem corresponds to the case where $p_i(n) = in, i = 1, 2, \dots, k$).

We will discuss two new extensions of the Polynomial Szemerédi Theorem.

One of these extensions (joint work with A. Leibman and E. Lesigne) establishes necessary and sufficient conditions for a set of polynomials to satisfy the Polynomial Szemerédi Theorem.

Another extension (joint work with R. McCutcheon) deals with the "upgrade" of the Polynomial Szemerédi Theorem to the so called generalized polynomials, namely functions which are obtained from regular polynomials via iterated use of the floor function.

7. **Matthew Boylan**, University of South Carolina

Odd Coefficients of Weakly Holomorphic Modular Forms

Let $f(z)$ be a half-integral weight modular form with integer coefficients $a(n)$ whose poles (if it has any) are supported at the cusps. Fix a prime ℓ . In this talk, we estimate the number of $a(n)$'s not divisible by ℓ and give applications to the study of the ordinary partition function, $p(n)$, and other functions of arithmetic interest whose generating functions are of this type.

Joint work with Scott Ahlgren, University of Illinois at Urbana-Champaign.

8. **Rod Canfield**, University of Georgia

Locally Restricted Compositions

A *composition* of an integer n is a k -tuple of positive integers whose sum is n . The number of these is 2^{n-1} . In 1976 Carlitz considered compositions in which no two adjacent parts are equal, obtaining a closed form for their generating function. Via this generating function, it can be shown that the number of these *Carlitz compositions* is asymptotically Ar^{-n} , where the constants $A = 0.456\dots$ and $r = 0.571\dots$ are roots of transcendental equations. We consider a generalization called *locally restricted compositions*. There are infinitely many locally restricted classes. Two examples: the 2-rowed compositions, which are those whose parts can be arranged as a $2 \times k$ array in which no two horizontally or vertically adjacent parts are equal; and the Carlitz_2 compositions, those in which no part equals a neighbor within distance 2. For each such class we can express the generating function in terms of an infinite matrix. An eigenvalue of this matrix can be used to determine constants A and r such that the total number of compositions in the class is asymptotically Ar^{-n} .

Joint work with Ed Bender, UCSD.

9. **Tsz Ho Chan**, University of Memphis

Almost Squares and Integers Almost Divisible by Another Integer

An almost square is an integer n that can be factored as $n = ab$ where $a, b \approx \sqrt{n}$. In this talk, we discuss short intervals containing an almost square. It leads to a question of finding short interval $[x, x + L]$ that contains an integer divisible by some integer in $[x^c, 2x^c]$ with $0 < c < 1$.

10. **Roberto B. Corcino**, Clayton State University/Mindanao State University

Some Theorems of the q -Analogue of the Generalized Stirling Numbers and the Combinatorics of 0-1 Tableaux

A q -analogue of the unified generalization of Stirling numbers was defined by R.B. Corcino, L.C. Hsu, and E.L. Tan in a joint paper where some basic properties and explicit formulas were derived. In this paper, we establish two types of recurrence relations - the vertical

and horizontal recurrence relations, and obtain certain rational generating function for the q -analogue. This generating function plays a very important role in deriving certain form of explicit formula which is used in giving combinatorial interpretations of the q -analogue in the context of 0-1 tableau. Moreover, using the combinatorics of 0-1 tableaux, we obtain certain formula which is a kind of generalization of Carlitz identity.

11. **Carlos Pereira dos Santos**, ISEC - Portugal

Nimber Constructions

In this work we analyse some nimber positions in Chess, Konane and Amazons. We introduce the concept of Nim-dimension, develop techniques directed to the construction of higher nimbers, unknown so far in those games, and exhibit them.

Joint work with Jorge Nuno Silva, UL - Portugal.

12. **Dennis Eichhorn**, University of California at Irvine

The Distribution of the Points (x, x^{-1}) modulo n .

If we plot the points $(x, x^{-1}) \pmod{n}$ scaled to fit inside the unit square, as n tends to infinity, the points are uniformly distributed. However, when examining these point sets for individual values of n , structure emerges. In this talk, we explore the distribution of these points, and in particular we treat their propensity to fall into lines with slope ± 1 . This talk is joint work with Mizan Khan, Alan Stein, and Christian Yankov.

Joint work with Mizan Khan, Eastern Connecticut State University; Alan Stein, University of Connecticut; and Christian Yankov, Eastern Connecticut State University.

13. **Carrie E. Finch**, Washington & Lee University

Irreducibility of a Class of Polynomials

In 1929, Schur proved that polynomials of the form

$$a_m \frac{x^m}{m!} + a_{m-1} \frac{x^{m-1}}{(m-1)!} + \cdots + a_2 \frac{x^2}{2} + a_1 x + a_0 \quad (1)$$

are irreducible over the rationals, where a_0, a_1, \dots, a_m are arbitrary integers and $|a_0| = |a_m| = 1$. More recently, Filaseta and Allen showed that polynomials of the form

$$a_m \frac{x^{2m}}{3 \cdot 5 \cdots (2m-1)} + a_{m-1} \frac{x^{2m-2}}{3 \cdot 5 \cdots (2m-3)} + \cdots + a_2 \frac{x^4}{3 \cdot 5} + a_1 \frac{x^2}{3} + a_0 \quad (2)$$

are irreducible when $0 < |a_m| < 2m - 1$. One of the common features of the polynomials in (1) and (2) is that the denominators are products of integers in arithmetic progression. In this paper, we discuss the irreducibility of polynomials of the form

$$a_m \frac{x^m}{a \cdot (a+2) \cdots (a+2(m-1))} + a_{m-1} \frac{x^{m-1}}{a \cdot (a+2) \cdots 2(m-2)} + \cdots + a_2 \frac{x^2}{a(a+2)} + a_1 \frac{x}{a} + a_0.$$

14. **Tim Flowers**, Clemson University

Asymptotics of Generalized Euler Polynomials

We present preliminary results on the asymptotic behavior of certain generalized Bernoulli and Euler polynomials. We follow an idea of Strodt and a method presented in some recent work of Borwein, Calkin, and Manna.

15. **Shanzhen Gao**, Florida Atlantic University

Heron Sequences

We study sequences whose consecutive terms determine Heron triangles. With a few exceptions, we show that given two positive integers u, v , there is an infinite sequence every three consecutive terms of which determine a Heron triangle. We also construct arbitrarily long sequences every three consecutive terms of which are the sides of a Heron triangle.

Joint work with Paul Yiu, Department of Mathematical Sciences, Florida Atlantic University and K.R.S.Sastry, DoddaKalsandra Post, Bangalore, 560062, India.

16. **David Garth**, Truman State University

Self-Generating Sets and Numeration Systems

We will explore connections between self-generating sets and numeration systems. Let F be a collection of affine functions of the form $2^kx + b$, where $0 \leq b < 2^k$. We define a self-generating set S as follows. Let $0 \in S$, and if $x \in S$ and $f \in F$, then $f(x) \in S$. Kimberling noticed that for $F = \{2x, 4x + 1\}$ the resulting S is the set of natural numbers whose binary expansions contain no consecutive ones. These expansions correspond to the greedy expansions of the natural numbers with respect to the Fibonacci sequence. In this talk we consider the question of which numeration systems of the natural numbers contain digit expansions that can be realized as the base two expansions of the integers in a self-generated set. We give necessary and sufficient conditions for a numeration system, defined as a collection of words over $\{0, 1\}$, to have a base sequence. This will be used to generate several examples of based numeration systems that are self-generated. We also show that the base sequence of any self-generated numeration system must satisfy a linear recurrence relation. Finally, we show that any based numeration system with digits in $\{0, 1\}$ is a fractal basis. This also generalizes a result of Kimberling.

17. **Dan Goldston**, San Jose State University

Larger Than Average Gaps Between Primes

I will discuss the problem of producing larger than average gaps between consecutive primes using the method of Goldston, Pintz, and Yildirim, and the rather disappointing results that are obtained. If, following Gallagher, one assumes the Hardy-Littlewood conjectures then it is possible to do much better. In principle this conditional approach should actually produce gaps close to the size of Cramer's conjecture, but this will require a new type of singular series average which may be exceedingly difficult to prove.

18. **Georges Grekos**, Universite Saint-Etienne, France

Representation Functions, Sidon Sets and Bases

We give some new properties of the “representation function”, counting the number of representations of each $n \in \mathbf{N} = \{0, 1, 2, \dots\}$ as sum of 2 elements from an infinite subset A of the set \mathbf{N} of natural numbers. We deduce that the number of such representations by an asymptotic 2-basis A of \mathbf{N} cannot be bounded by 2, i.e. A cannot be a Sidon set. We then show that a Sidon set cannot be a 3-basis of \mathbf{N} , [it is an open problem whether a Sidon set can be an *asymptotic* 3-basis of \mathbf{N} or not] and we describe an algorithm for producing, if it exists, a Sidon set which is an asymptotic 3-basis of \mathbf{N} , with a specific starting point of representation.

Joint work with Labib Haddad, 120 rue de Charonne, 75011 Paris, France; Charles Helou, Penn State University, PA 19063, USA; and Jukka Pihko, University of Helsinki, 00014 Finland.

19. **Udi Hadad**, Weizmann Institute of Science, Rehovot, Israel

Complexity of a Sequences Membership Question

The question whether an m -tuple $(x_1, \dots, x_m) \in_{\geq 0}^m$ is in (a^1, \dots, a^m) , where the a^i are given integer sequences, can sometimes be decided *efficiently* (in polynomial time). More often, the answer is unknown, the best known algorithms being exponential. We present a polynomial approximate algorithm for deciding this question for some sequences a_i . Specifically, we consider the complementary sequences $a_n = \{a_i, b_i : 0 \leq i < n\}$, $b_n = a_n + \lfloor n/k \rfloor$ (sequences A102528/9 in Sloane's encyclopedia for $k = 2$). Using Fekete's Lemma we show that the polynomially computable sequences $s_n = \lfloor n\alpha \rfloor$, $t_n = \lfloor n\beta \rfloor$ where $\alpha = (\sqrt{17} + 3)/4$, $\beta = \alpha + 1/2$ are very good approximations, namely, $s_{n-1} \leq a_n \leq s_n$, $t_{n-1} \leq b_n \leq t_n$ for all $n \geq 1$ ($k = 2$). We conjecture that the percentage of n for which $a_n = s_n - 1$ is about 73%, $a_n = s_n$, 19%, $a_n = s_n - 2$, 8%. Similar results for b_n, t_n . Analogous results for every fixed $k > 1$. Existence of a limiting distribution, with one of the percentages dominant, would lead to first probabilistic algorithms for determining the winning positions of certain impartial games, where the (a^1, \dots, a^m) are the second player winning positions.

Joint work with Aviezri S. Fraenkel, Weizmann Institute of Science, Rehovot, Israel.

20. **Peter Hegarty**, Chalmers University of Technology and Göteborg University

When Almost All Sets are Difference Dominated

We investigate the relationship between the sizes of the sum and difference sets attached to a subset of $\{0, 1, \dots, N\}$, chosen randomly according to a binomial model with parameter $p(N) = \Omega(N^{-1})$. We show that the random subset is almost surely difference dominated, as $N \rightarrow \infty$, for any choice of $p(N)$ tending to zero, thus confirming a conjecture of Martin and O’Byrant. The proofs use recent strong concentration results.

Furthermore, we exhibit a threshold phenomenon regarding the ratio of the size of the difference- to the sumset. If $p(N) = o(N^{-1/2})$ then almost all sums and differences are almost surely distinct, and in particular the difference set is almost surely about twice as large as the sumset. If $N^{-1/2} = o(p(N))$ then both the sum and difference sets almost surely have size $(2N + 1) - O(p(N)^{-2})$, and so the ratio in question is almost surely very close to one. If $p(N) = c \cdot N^{-1/2}$ then as c increases from zero to infinity (i.e.: as the threshold is crossed), the same ratio almost surely decreases continuously from two to one, according to an explicitly given function of c .

We also extend our results to the comparison of the generalised difference sets attached to an arbitrary pair of binary linear forms. For certain pairs of forms f and g , we show that there in fact exists a sharp threshold at $c_{f,g} \cdot N^{-1/2}$, for some computable constant $c_{f,g}$, such that one form almost surely dominates below the threshold and the other almost surely above it.

The heart of our approach involves using different tools to obtain strong concentration of the sizes of the sum and difference sets about their mean values, for every value of the parameter $p(N)$.

Joint work with Steven J. Miller, Brown University.

21. Neil Hindman, Howard University

Largeness of the Set of Finite Products in a Semigroup

We investigate when the set of finite products of distinct terms of a sequence $\langle x_n \rangle_{n=1}^{\infty}$ in a semigroup (S, \cdot) is large in any of several standard notions of largeness. These include *piecewise syndetic*, *central*, *syndetic*, *central**, and *IP**. In the case of a “nice” sequence in $(S, \cdot) = (\mathbb{N}, +)$ one has that $FS(\langle x_n \rangle_{n=1}^{\infty})$ has any or all of the first three properties if and only if $\{x_{n+1} - \sum_{t=1}^n x_t : n \in \mathbb{N}\}$ is bounded from above.

Joint work with Chase Adams, III, Howard University.

22. M.H.Hooshmand, University Science of Malaysia

b-Parts of Real Numbers

Consider $[a]$ the largest integer not exceeding a and put $(a) = a - [a]$ (the decimal part of a). Let b be a nonzero constant real number. For every real number a , we set

$$[a]_b = b \left[\frac{a}{b} \right] \quad , \quad (a)_b = b \left(\frac{a}{b} \right),$$

and call the notation $[a]_b$ "b-integer part of a " and $(a)_b$ "b-decimal part of a ". In this talk we give number theoretic explanations of b -parts of real numbers and generalize the division algorithm (from integers to all real numbers), by using b -parts. Finally b -additive operation (that is b -decimal part of the additive operation) and b -bounded groups will be introduced and some of their properties will be presented.

23. **Brian Hopkins**, Saint Peter's College

Between Conjugation and Bulgarian Solitaire

The operation of partition conjugation can be thought of as shifting all columns of the partition's Ferrers diagram to rows. This gives a directed graph on all partitions of n consisting of 2-cycles (conjugate pairs) and singletons (self-conjugate partitions). Shifting a single column of the Ferrers diagram to the first row is the operation defining Bulgarian Solitaire, which has been studied for some 25 years. Under this operation, the directed graph of n can have as few as one component. In this talk, we consider Bulgarian solitaire and conjugation as extreme cases of a general column-to-row operation. What happens when 2 columns are shifted to rows? when $n - 1$ columns are shifted to rows? We present preliminary results on the number of components, cycle partitions, and Garden of Eden partitions on the system of partitions of n when shifting k columns to rows.

24. **Eugen Ionascu**, Columbus State University

Counting Equilateral Triangles in $\{0, 1, \dots, n\}^3$

We are investigating a characterization of all equilateral triangles in Z^3 . Based on this we calculate the number of such triangles in $\{0, 1, 2, \dots, n\}^3$ for $n < 1105$.

25. **Susil Kumar Jena**, KIIT University, Bhubaneswar-751024, Orissa, India

A New Technique Called the Method of Infinite Ascent

Unlike the four known techniques -the Direct Method, the Method of Induction, the Method of Contradiction and the Method of Infinite Descent- this new technique called the 'Method of Infinite Ascent' will prove helpful in analysing and proving some problems in Diophantine Arithmetic. There are some conjectures in diophantine equations which are presumed to have no solution in integers. With the known techniques what best we can do for them is to prove that they can have finite number of integral solutions, if they exist. However, to go further in proving the impossibility of an integral solution we will need this new technique. Even for some proved results relating to the said diophantine equations this new method will be easier to apply to reach at the same conclusion. In this short paper, we will apply this idea on proving the celebrated Fermat's Last Theorem for powers 3 and 4 and open up a new direction of attack to the problem without the complex mathematics of elliptic functions and modular forms. In the VII Joint Meeting of the American Mathematical Society and the

Mexican Society of Mathematics held in Zacatecas, Mexico during May 23-26, 2007 we have presented a paper with the title: The Method of Infinite Ascent applied on the Diophantine Equation $A^6 + nB^3 = C^2$ in which we have seen the utility of this novel method.

26. **Veselin Jungic**, Simon Fraser University

On Arithmetic Progressions With Odd Differences

We are interested in the following problem inspired by the Brown-Graham-Landman conjecture on large sets. For given $k, n \in \mathbb{N}$, how big can $A \subset \{1, 2, \dots, n\}$ be so that A intersects each $i \pmod{k}$ class and is free of k -term odd difference arithmetic progressions?

Joint work with J. Fox and R. Radoičić.

27. **Srinivasa Raghava Kanduru**, Annamalai University

Some New Type Of Partition Function Identities (Eulerian Forms)

The object of this paper is to study some new 'Eulerian Forms' related to partition functions. Their series expansion is obtained. A few coefficients are outlined. Also some very interesting false - theta type series are derived related to Ramanujan series, which he discussed in his 'Lost Note Book'.

Key words & Phrases: Partition function, Eulerian Forms, False-theta Series.

28. **Jeong-Hyun Kang**, University of West Georgia

Distance Graph – p -adic approach

For a given subset D of the positive integers, the integer distance graph $G(\mathbb{Z}, D)$ is defined with the set of integers as vertex set, and two vertices are adjacent if the (Euclidean) distance between them belongs to D . It has been an active research problem to characterize the chromatic number according to a given distance set D . The integer distance graphs were first systematically studied by Eggleton–Erdős–Skilton in 1985, and have been investigated in many ways. In this talk, we approach the problem under the p -adic norm. The chromatic numbers of some distance sets will be determined under p -adic distance. We discuss how the p -adic results can be connected to and complement some of the results in Euclidean norm.

Joint work with Hiren Maharaj, Clemson University.

29. **Louis W. Kolitsch**, The University of Tennessee at Martin

A Note on the Parity of the Ordinary Partition Function

It is well known that the values of the ordinary partition function are even and odd infinitely often. In this talk the parity of $p(n)$, the number of ordinary partitions of n , will be described in terms of the parity of other partition functions.

30. **Takao Komatsu**, Hirosaki University

A Combinatorial Proof of the Continued Fraction Expansion of e

We give a combinatorial proof of the continued fraction expansion of e in terms of integrals. Let p_n/q_n be the n -th convergent of the continued fraction expansion of $e^{1/s} = [1; (2k-1)s, 1, 1]_{k=0}^{\infty}$ ($s \geq 2$). It is known that $1/(q_{n+1} + q_n) < |p_n - eq_n| < 1/q_n$. Osler expressed this error, $p_n - eq_n$, in terms of integrals. Our method is based upon the concept of leaping convergents. As some applications, similar results can be obtained concerning $e^{2/s}$, $se^{1/(ls)}$, $e^{1/(ls)}/s$, $e^{1/(3s+1)}/3$, $(e^{1/(3s)} + 1)/3$ etc.

31. **Bryna Kra**, Northwestern University

Ergodic Theory, Additive Combinatorics and Nilmanifolds

Much recent work in ergodic theory has been motivated by interactions with combinatorics and with number theory. A striking example is Szemerdi's Theorem, which states that a set of integers with positive upper density contains arbitrarily long arithmetic progressions. Soon after Szemerdi's proof, Furstenberg gave a new proof using ergodic theory. This opened new questions in ergodic theory, and developments in ergodic theory, in turn, have lead to breakthroughs in additive combinatorics. It turns out that algebraic constraints (nilsystems) play a key role in understanding these phenomenon, both in additive combinatorics and in ergodic theory. I will give an overview of the role of nilsystems in the recent developments, explaining the beginnings of a theory of higher order Fourier analysis that is a tool for addressing open problems in the area.

32. **Urban Larsson**, Göteborg University

2-Pile Nim With a Restricted Number of Move-size Imitations

We study a variation of the combinatorial game of 2-pile Nim. Move as in 2-pile Nim but with the following constraint:

Provided the previous player has just removed say $x > 0$ tokens from the pile with a less number of tokens, the next player may remove x tokens from the pile with more tokens. But for each move, in "a strict sequence of previous player - next player moves", such an *imitation* takes place, the value of an *imitation counter* is increased by one unit. As this counter reaches a predetermined natural number, then by the rules of this game, if the previous player once again removes a positive number of tokens from the pile with less tokens, the next player may not remove this same number of tokens from the pile with more tokens.

We show that the winning positions of this game in a sense resembles closely the winning positions of the game of Wythoff Nim - more precisely a version of Wythoff Nim with a Muller twist. If time permits, we will mention possible generalisations to the above.

33. **David Leach**, University of West Georgia

Zumkeller Numbers and Half-Zumkeller Numbers

A perfect number n can be thought of as a number whose divisors can be partitioned into two sets such that (1) the sets have equal sums and (2) one set contains only n itself. We define numbers that satisfy condition (1) to be *Zumkeller numbers*. If the proper divisors of n can be partitioned into equal-summed sets, n is said to be *half-Zumkeller*. In this talk we examine Zumkeller numbers and half-Zumkeller numbers, and explore the relationships between them.

Joint work with Matt Walsh, IPFW; Sally Clark and Mark Liatti, Huntingdon College; Janet Dalzell; and John Holliday, Georgia College and State University.

34. **Joon Yop Lee**, POSTECH, Korea

Involutions on Theta Function Identities

Combinatorial proofs of the following theta function identities will be given

$$\begin{aligned} \sum_{n=0}^{\infty} (-1)^n q^{n^2} &= \sum_{n=0}^{\infty} \frac{q^{2n}}{(q)_{2n}} \prod_{k=1}^{\infty} (1 - q^k), \\ \sum_{n=0}^{\infty} (-1)^n q^{(n+1)^2} &= \sum_{n=0}^{\infty} \frac{q^{2n+1}}{(q)_{2n+1}} \prod_{k=1}^{\infty} (1 - q^k), \\ \sum_{n=0}^{\infty} (-1)^n q^{n^2} &= \sum_{n=0}^{\infty} \frac{q^{n(2n+1)}}{(q)_{2n}} \prod_{k=1}^{\infty} (1 - q^{2k+1}), \\ \sum_{n=0}^{\infty} (-1)^n q^{(n+1)^2} &= \sum_{n=0}^{\infty} \frac{q^{(n+1)(2n+1)}}{(q)_{2n+1}} \prod_{k=1}^{\infty} (1 - q^{2k+1}), \end{aligned}$$

using partition bijections. Using these partition bijections, generalizations of these will be given. And a derived identity will be considered in a similar context.

35. **Xian-Jin Li**, Brigham Young University

On the Convergence of Euler's Product for Hecke Zeta Functions

It is well-known that Euler's product formula for the Riemann zeta function $\zeta(s)$ is still valid for $\Re(s) = 1$ and $s \neq 1$. In this paper, we extend this result to zeta functions of number fields

k . In particular, we show that the Dedekind zeta function $\zeta_k(s)$ and Hecke's zeta functions $\zeta(s, \chi)$ can be written as Euler's product on the line $\Re(s) = 1$ except at the point $s = 1$. This result played an important role in a recent result of the author, which is a generalization of Tate's adelic Poisson summation formula.

36. **Florian Luca**, National Autonomous University of Mexico

On the Distribution of Totients

For a positive integer n , write $\phi(n)/n = a/b$ with coprime integers $a = a(n)$ and $b = b(n)$, where $\phi(n)$ is the Euler function of n . For a fixed integer a , we consider the number of integers b for which the above relation holds for some n . We also fix b and count the corresponding a 's. We discuss the greatest common divisor of n and $\phi(n)$ and apply our results to the relation $\phi(n) \mid f(n)$, where f is a polynomial.

Joint work with P. Erdős and C. Pomerance.

37. **Neil Lyall**, University of Georgia

Polynomial Configurations in Sumsets

We will discuss some quantitative results concerning the existence of certain polynomial patterns in the sum/difference set of an arbitrary subset of the integers of positive (upper) density.

Joint work with Akos Magyar, University of Georgia.

38. **Gretchen Matthews**, Clemson University

Frobenius Numbers of Generalized Fibonacci Semigroups

Given relatively prime positive integers, the numerical semigroup generated by a_1, \dots, a_n is $S := \{\sum_{i=1}^n c_i a_i : c_i \in \mathbb{Z}^+ \cup \{0\}\}$. The largest integer not in S is called the Frobenius number of S . Recently, J. M. Marín, J. L. Ramírez Alfonsín, and M. P. Revuelta determined the Frobenius number of a Fibonacci semigroup, that is, a numerical semigroup generated by a certain set of Fibonacci numbers. In this talk, we find the Frobenius numbers of generalized Fibonacci semigroups obtaining the result of Marín et. al. as a special case. In addition, we determine the pseudo-Frobenius numbers of such semigroups.

39. **Neil McKay**, Dalhousie University

All-small Games and Uptimal Notation

The games \uparrow^n and more generally G^n have been studied by Conway, Berlekamp, Moews and others. As stated in *Lessons in Play*, Conway and Ryba suggest a definition of G^x for dyadic rational values of x . We suggest a slightly different definition, one that is consistent with the present definition for integer powers. In particular, we will discuss the case $G = \uparrow$. A game which is the sum of powers of \uparrow is said to be an *uptimal*. We claim that all values of the game ALL-SMALL PUSH are uptimals and conjecture their values. I will also give explicit positions with values of the form \uparrow^x . This is work from my Masters thesis, supervised by R.J. Nowakowski.

40. **Kellen Myers**, Colgate University

Results on Standard and Off-Diagonal Rado Numbers

We will discuss an off-diagonal generalization of one of Rado's results on the regularity of linear equations, including several relevant off-diagonal Rado numbers and bounds. Additionally, we will present new results for particular standard Rado numbers of certain linear equations. Joint work with Aaron Robertson, Colgate University.

41. **Brenden Nagle**, University of South Florida

On an Extremal Hypergraph Problem Concerning Forbidden Families

For an integer $k \geq 2$, let $\mathbf{F}^{(k)}$ be a fixed (but possibly infinite) collection of k -graphs. A k -graph $\mathcal{G}^{(k)}$ is said to be $\mathbf{F}^{(k)}$ -free if it contains no sub-hypergraph isomorphic to an element of $\mathbf{F}^{(k)}$. Let $\text{Forb}(n, \mathbf{F}^{(k)})$ denote the collection of $\mathbf{F}^{(k)}$ -free k -graphs $\mathcal{G}^{(k)}$ on vertex set $\{1, \dots, n\}$. In this talk, we shall present the asymptotic $\log_2 |\text{Forb}(n, \mathbf{F}^{(k)})| = \text{ex}(n, \mathbf{F}^{(k)}) + o(n^k)$, where $\text{ex}(n, \mathbf{F}^{(k)})$ is the well-known Turán number of the family $\mathbf{F}^{(k)}$. We shall present an analogous result for forbidding the elements of $\mathbf{F}^{(k)}$ as induced sub-hypergraphs.

The results presented here generalize several earlier known for graphs ($k = 2$), which we briefly review in this talk. The new results we present rely on recent hypergraph regularity techniques. Some of this work is joint with Vojtěch Rödl and Mathias Schacht and some with my former Masters student Ryan Dotson.

42. **Mel Nathanson**, Lehman College (CUNY) and the Institute for Advanced Study

Representation Functions of Bases With Respect to Linear Forms

Let $F(x_1, \dots, x_m) = u_1x_1 + \dots + u_mx_m$ be a linear form with nonzero, relatively prime integer coefficients u_1, \dots, u_m . For any set A of integers, let $F(A) = \{F(a_1, \dots, a_m) : a_i \in A \text{ for } i = 1, \dots, m\}$. The *representation function* associated with the form F is

$$R_{A,F}(n) = \text{card}(\{(a_1, \dots, a_m) \in A^m : F(a_1, \dots, a_m) = n\}).$$

The set A is a *emphbasis* with respect to F for almost all integers if the set $\mathbf{Z} \setminus F(A)$ has asymptotic density zero. Equivalently, the representation function of a basis for almost all

integers is a function $f : \mathbf{Z} \rightarrow \mathbf{N}_0 \cup \{\infty\}$ such that $f^{-1}(0)$ has density zero. Given such a function, the inverse problem for bases is to construct a set A whose representation function is f . In this paper the inverse problem is solved for binary linear forms.

43. **Jaroslav Nešetřil**, Charles University

44. **Richard Nowakowski**, Dalhousie University

Rolled Games

There are several games (e.g. Hackenbush Strings, Maze, and Roll-the-Cricket-Pitch) that have a similar abstract structure: given a position G , $G^{LL} \subset G^L$, that is, the left options of the left options are left options of the original position. The canonical forms can be arbitrarily long but their *reduced canonical forms* are either numbers of switches. We illustrate the proofs with reference to Roll-the-Cricket-pitch.

Joint work with P. Ottaway, Dalhousie University.

45. **Kevin O’Bryant**, CUNY, College of Staten Island

Sizes of Skewed Sumsets

Is there a set A for which the sumset $\{a + b : a, b \in A\}$ has more elements than the skewed sumset $\{2a + b : a, b \in A\}$? We answer this question and many others like it in joint work with Nathanson, Orosz, Ruzsa, and Silva.

46. **Ken Ono**, University of Wisconsin Madison

Freeman Dyson’s Challenge for the Future: The Story of Ramanujan’s Mock Theta Functions

The legend of Ramanujan is one of the most romantic stories in the modern history of mathematics. It is the story of an untrained mathematician, from south India, who brilliantly discovers tantalizing examples of phenomena well before their time. Indeed, the legacy of Ramanujan’s work (as a whole) is well documented and includes direct connections to some of the deepest results in modern number theory such as the proof of the Weil Conjectures, and the proof of Fermat’s Last Theorem. However, one final problem remained. In his last letter to Hardy (written on his death bed), Ramanujan gave examples of 17 functions he referred to as “mock theta functions”. Without a definition and without good clues, number theorists were unable to make any real sense out of these peculiar functions. Nevertheless,

these examples make important appearances in many disparate areas of mathematics, a fact which inspired Freeman Dyson to proclaim: "Mock theta-functions give us tantalizing hints of a grand synthesis still to be discovered. Somehow it should be possible to build them into a coherent group-theoretical structure... This remains a challenge for the future. My dream is that I will live to see the day when our young physicists, struggling to bring the predictions of superstring theory into correspondence with the facts of nature, will be led to enlarge their analytic machinery to include not only theta-functions but mock theta-functions." –Freeman Dyson, 1987

In this lecture I will outline the history of Ramanujan's final enigma, and I will present the solution.

47. **Ram Krishna Pandey**, Indian Institute of Technology Delhi

On the Density of Integral Sets with Missing Difference

I will speak on the following problem posed by Motzkin. Let S be a set of nonnegative integers, and let $S(x)$ denote the number of elements $n \in S$ such that $n \leq x, x \in \mathbb{R}$. The upper and lower densities of S , denoted by $\bar{\delta}(S)$ and $\underline{\delta}(S)$ respectively, are defined as follows:

$$\bar{\delta}(S) := \limsup_{x \rightarrow \infty} \frac{S(x)}{x}, \quad \underline{\delta}(S) := \liminf_{x \rightarrow \infty} \frac{S(x)}{x}.$$

If $\bar{\delta}(S) = \underline{\delta}(S)$, we denote the common value by $\delta(S)$, and say that S has density $\delta(S)$. Given a set of positive integers M , S is said to an M -set if $a \in S, b \in S \Rightarrow a - b \notin M$. Motzkin's problem is to determine

$$\mu(M) := \sup_S \bar{\delta}(S)$$

where S varies over the class of all M -sets. Cantor and Gordon in 1973 settled the problem when $|M| \leq 2$. Motzkin's problem is still unsolved in general when $|M| \geq 3$. Haralambis, Rabinowitz and Prolux, Eggleton, Erdős, and Skilton, Kemnitz and Kolberg, Chang, Liu and Zhu, Gupta and Tripathi, Liu and Zhu, worked out the answer in some special cases, including some infinite families. In this talk we discuss the problem for the family $M = \{a, b, c\}$ where $c = na$ or nb . Our results generalizes some known results by Haralambis in 1977.

Joint work with Amitabha Tripathi, Indian Institute of Technology Delhi.

48. **Paul Pollack**, Dartmouth College

Arithmetic Properties of Polynomial Specializations Over Finite Fields

Several classical problems in number theory could be resolved if we understood the frequency with which polynomials simultaneously assume prime values. Schinzel's Hypothesis H and its quantitative refinements by Hardy-Littlewood and Bateman-Horn present us with a plausible heuristic understanding of this phenomenon, but their conjectures have proved difficult to attack. We discuss some recent results towards an analogue of Schinzel's Hypothesis H in the setting of polynomials over finite fields, together with some applications of these results. Fundamental to this work is a link between the cycle types of random permutations and

the factorization types of certain polynomial specializations. This last result implies that for certain ranges of the parameters, one can understand not only simultaneous prime values, but many other statistics (e.g., smoothness) of simultaneous polynomial specializations.

49. **Carl Pomerance**, Dartmouth College

Pseudopowers

An x -pseudopower to base g is a positive integer which is not a power of g yet is so modulo p for all primes $p \leq x$. We improve an upper bound for the least such number due to E. Bach, R. Lukes, J. Shallit, and H. C. Williams. The method is based on a combination of some bounds of exponential sums with new results about the average behavior of the multiplicative order of g modulo prime numbers.

Joint work with Sergei Konyagin and Igor Shparlinski.

50. **Aaron Robertson**, Colgate University

Van der Waerden Numbers and Related Functions

Let $w(k, \ell)$ be the least $n \in \mathbb{Z}^+$ such that any 2-coloring of $\{1, 2, \dots, n\}$ admits either a k -term arithmetic progression of the first color, or an ℓ -term arithmetic progression of the second color. We will present new bounds on various instances of $w(k, \ell)$ as well as on several functions related to these van der Waerden numbers.

Joint work with Tom Brown, Simon Fraser University and Bruce Landman, University of West Georgia.

51. **Frank Ruskey**, University of Victoria, Canada

Solutions of Certain Meta-Fibonacci-like Recurrence Relations

This talk is concerned with the enigmatic behavior of “nested” recurrence relations of the form

$$A(n) = A(n - s - A(n - a_1) - \dots - A(n - a_p)) + A(n - s - t - A(n - b_1) - \dots - A(n - b_q))$$

where $0 \leq s, 0 \leq t, a_1 \leq \dots \leq a_p$, and $b_1 \leq \dots \leq b_q$. Examples include Hofstadter’s famous Q sequence ($p = q = 1, s = t = 0, a_1 = 1, b_1 = 2$) and the Conolly meta-Fibonacci recurrence $A(n) = A(n - A(n - 1)) + A(n - 1 - A(n - 2))$. In general, the solutions are quite sensitive to the initial conditions. Sometimes the solutions correspond to counting certain combinatorial objects and nice explicit expressions, generating functions, and asymptotics can be obtained. In other cases the true behavior does not become apparent until n is “large.” And in many cases the behavior appears to be somewhat chaotic, with predictable trends, but hard to describe exact values.

Joint work with Brad Jackson, San Jose State University and Steve Tanny, University of Toronto.

52. **Vermont Rutherford**, Florida Atlantic University

Enumeration of Grid-based Seating Arrangements

53. **Carla Savage**, North Carolina State University

Integer Analogs of Lecture Hall Theorems

We consider integer analogs of generating function identities that arise in the enumeration of integer solutions $(\lambda_1, \dots, \lambda_n)$ to the inequalities $\lambda_n/s_n \geq \lambda_{n-1}/s_{n-1} \geq \dots \geq \lambda_1/s_1 \geq 0$, for integer sequences $\{s_i\}$. In doing so, we (i) pose an integer analog of an open question of Bousquet-Mélou and Eriksson about polynomial sequences $\{s_i\}$ and (ii) show how ℓ -nomial coefficients arise naturally in this enumeration problem.

54. **Wolfgang A. Schmid**, University of Graz, Austria

Sets of Lengths in Monoids of Zero-sum Sequences over Finite Abelian Groups

Let $(G, +)$ be a finite abelian group. A sequence $g_1 \dots g_n$ of elements of G is called a zero-sum sequence over G if $g_1 + \dots + g_n = 0$. We identify sequences that just differ by the ordering of the elements; thus, the set of all zero-sum sequences over G , with concatenation as operation, is a commutative monoid. Each zero-sum sequence can be factorized into minimal zero-sum sequences, i.e., zero-sum sequences that do not have a proper subsequence that is a zero-sum sequence. In general a zero-sum sequence has several distinct factorizations and indeed even the lengths of the factorizations (i.e., the number of minimal zero-sum sequences in the factorization) may differ. The set of lengths of a zero-sum sequence is the set of all lengths of factorizations of the sequence. The investigation of factorizations of zero-sum sequences was initiated by W. Narkiewicz and is motivated by its close connection to arithmetical questions in rings of algebraic integers (where the finite abelian group is the class group). Though, for each abelian group with at least 3 elements there are zero-sum sequences with arbitrarily large sets of lengths, it is known that there are only finitely many types of sets of lengths. More precisely, by a theorem of A. Geroldinger (1988) and a subsequent refinement by G. Freiman and A. Geroldinger (2000), it is known that for each finite abelian group there is a constant M such that each set of lengths L of a zero-sum sequence over G is an almost arithmetical multiprogression (AAMP) with bound M , i.e., $y + D + \{0, d, \dots, \ell d\} \subset L \subset y + D + \{-Md, \dots, (\ell + M)d\}$ for some $1 \leq d \leq M$ a set $\{0, d\} \subset D \subset \{0, \dots, d\}$ and integers y, ℓ . In this talk, we present a sort of converse to this result. Namely, for each M we construct a finite abelian group G such that each AAMP with bound M is up to a shift the set of lengths of some zero-sum sequence over G .

55. **James Sellers**, Penn State University

Observations on the Parity of the Total Number of Parts in Odd-Part Partitions

In recent years, numerous functions which count the number of parts of various types of partitions have been studied. In this talk, we consider the function $pt_o(n)$ which counts the total number of parts in all oddpart partitions of n (or what Chen and Ji recently called the number of rooted partitions of n into odd parts). In particular, we prove a number of results on the parity of $pt_o(n)$, including infinitely many Ramanujanlike congruences satisfied by the function.

56. **Andrew Sills**, Georgia Southern University

Identities of the Ramanujan-Slater Type

I will present a number of q -series identities similar in spirit to those found in Ramanujan's lost notebook and Lucy Slater's list of Rogers-Ramanujan type identities. Time permitting, I will include some combinatorial consequences and implications for Lie algebras. Some of the results to be presented are the result of joint work with James McLaughlin.

Joint work with James McLaughlin, West Chester University.

57. **Jorge Nuno Silva**, University of California Berkeley

Nimber Constructions

We propose the concept of Nim-dimension of a combinatorial game. We analyse three nimber construction methods (symmetry, decollage, collage). We present some examples: *2 in Chess, *4 in Konane, *4 in Amazons.

This is joint work with Carlos Pereira dos Santos, ISEC - Portugal.

58. **Matthew Smith**, University of Georgia

On Solution-free Sets for Simultaneous Additive Equations

I will use a combination of the Hardy-Littlewood circle method and the methods developed by Gowers in his recent proof of Szemerédi's Theorem on long arithmetic progressions to obtain quantitative estimates for the upper density of a set containing no solutions to a translation and dilation invariant system of diagonal polynomials of degrees $1, 2, \dots, k$.

59. **Pante Stanica**, Naval Postgraduate School

Nagy Graphs and Homogeneous Bent Boolean Functions

Homogeneous Boolean functions that have the largest distance from the set of affine functions are currently sought after (theoretically and computationally). We present some recent results that connect the mentioned search with a subgraph of the Johnson graph, called Nagy graph, in the cubic case and propose an avenue for research on this problem, which further connects Graph Theory and cryptographic Boolean functions.

60. **Fraser Stewart**, University of Dundee

Misère Hackenbush is NP-hard

Hackenbush is a game played on a graph with coloured edges, where players take it in turns to remove edges of their own colour. Under normal play rules the last player to move is the winner and this game has been used extensively to study the theory of normal play games, it is for this reason that we wish to study this game under misère games, where the last player to move is the loser, to try and help understand the general structure of misère games. However for normal play games it has been shown that even for Red-Blue Hackenbush (all edges are coloured either red or blue) under normal play, determining the winner of a given position is NP-hard. I will extend this result to misère play, by proving that determining the winner of a given position of Red-Blue-Green Hackenbush, under misère play, is also NP-hard. I will also demonstrate that for some restricted variants the winner can be determined in polynomial time and give a classification for which positions a given player can win.

61. **Holly Swisher**, Oregon State University

An Investigation of k -component Multipartitions

Multipartitions are generalizations of partitions that have significance in the representation of simple Lie algebras, but are also arithmetically interesting in their own right. Andrews has recently found an infinite family of congruences for the multipartition function $P_k(n)$ and has connected one of the bipartition congruences to the famed crank statistic. Here we further explore this subject, focusing on the combinatorics of generalizing the crank to k -component multipartitions, and its relationship to congruences. We also investigate a geometric generalization of Ferrers diagrams, and how this leads to an interesting class of multipartitions and identities.

Joint work with Joanna Furno and Patrick Waters.

62. **Laszlo Szalay**, University of West Hungary, Sopron, Hungary

Fibonacci Diophantine Triplets

A set $\{a_1, a_2, \dots, a_s\}$ of positive integers is called Diophantine s -tuple if the product of any two of them increased by 1 is a perfect square. The first Diophantine quadruple, the set $\{1, 3, 8, 120\}$ was found by Fermat. It seems strongly true that there does not exist Diophantine quintuples, but the proof is still beyond reach. The above inspired us to investigate the following question. What would happen if one replaces the perfect squares by Fibonacci numbers (or generally, by the terms of a given binary recurrence) in the problem of Diophantos? We say that a set $\{a_1, a_2, \dots, a_s\}$ of integers form Fibonacci diophantine s -tuple if $a_i a_j + 1$ is a Fibonacci number for all $1 \leq i < j \leq s$. We showed that there is no Fibonacci diophantine triplet.

Joint work with Florian Luca, UNAM, Morelia, Mexico.

63. **Frank Thorne**, University of Wisconsin-Madison

Maier Matrices Beyond \mathbb{Z}

The Maier matrix method has been used to prove many interesting results concerning irregularities in the distribution of primes and related arithmetic sequences. I will present generalizations of the Maier matrix method to $\mathbb{F}_q[t]$ and to certain number fields, and describe my work obtaining analogues of results of Maier, Shiu, and Granville and Soundararajan.

64. **Vira Vasilyeva**, National Technical University of Ukraine "KPI"

On Existence of Monochromatic Short Arithmetic Progressions in 2-coloring of Integers

Let Z be the set of integers and let $A = \{a_1, a_2, \dots, a_n, \dots\}$ be a subset of integers. Denote by $A + A$ such set: $A + A = \{a_i + a_j, a_i, a_j \in A\}$. Does there exist monochromatic infinite set $A + A$ in every 2-coloring of Z ? We can write a set $A + A$ in such way:

$$A + A = \{2a_1, a_1 + a_2, 2a_2, \dots, 2a_1, a_1 + a_n, 2a_n, \dots, \\ 2a_2, a_2 + a_3, 2a_3, \dots, 2a_2, a_2 + a_n, 2a_n, \dots, 2a_n, a_n + a_{n+1}, 2a_{n+1}, \dots\}$$

So $A + A$ is the set of 3-term arithmetic progressions connecting in special way. Next theorem states existence of such progressions but question about $A + A$ is still open.

Theorem 1. In every 2-coloring of Z there exists infinite set of monochromatic numbers $\{2a_1, 2a_2, \dots, 2a_n, \dots\}$ such that for some integers $b_{i_j}, c_{i_j}, i, j \in N$ the set of linked 3-term arithmetic progressions $\{2a_i, a_i + b_{i_1}, 2b_{i_1}, b_{i_1} + b_{i_2}, 2b_{i_2}, b_{i_2} + b_{i_3}, 2b_{i_3}, \dots, 2b_{i_n}, \dots\}$ and the set of 3-term arithmetic progressions with common first term $\{2a_i, a_i + c_{i_1}, 2c_{i_1}, a_i + c_{i_2}, 2c_{i_2}, \dots, a_i + c_{i_n}, 2c_{i_n}, \dots\}$ are monochromatic. In this theorem it's impossible to increase neither number of colors nor number of terms arithmetic progressions. We can write the set $A + A$ in another way:

$$A + A = \{2a_1, a_1 + a_2, a_1 + a_3, \dots, a_1 + a_n, \dots, \\ 2a_2, a_2 + a_3, \dots, a_2 + a_n, \dots, 2a_{n-1}, a_{n-1} + a_n, \dots, 2a_n, \dots\}$$

So this set is a collection of infinite sequences and each one is a shift of another one.

Theorem 2. In any 2-coloring of Z there exist two infinite monochromatic sequences $\{2a_1, a_1 + a_2, a_1 + a_3, \dots, a_1 + a_n, \dots\}$ and $\{2a_2, a_2 + a_3, \dots, a_2 + a_n, \dots\}$

65. **Kevin Ventullo**, Illinois Institute of Technology

An Upper Bound on the Degree of Accessibility of Translations of the Primes

If D is a set of positive integers, a k -term sequence of positive integers is called a D -diffsequence if all differences between consecutive members of the sequence belong to D . A set D is called r -accessible if every r -coloring of the integers admits arbitrarily long monochromatic D -diffsequences. The *degree of accessibility* of a set D , denoted $doa(D)$, is the largest value of r such that D is r -accessible. It is known that $doa(P) \leq 2$, where P is the set of prime numbers, and that $doa(P + c) \geq 2$, where c is a fixed positive odd integer. In the talk, I will present a recently proven result about $doa(P + c)$, namely that $doa(P + c) \leq q$, where q is the smallest prime factor of c .

66. **Van Vu**, Rutgers University

Structure Theorems Concerning Sumsets and Applications

Let A be a set of elements of an abelian group (we will focus on Z and Z_p) and let $S(A)$ be the collection of all partial sums of A . The following questions are among the most popular in combinatorial number theory:

- (1) Does $S(A)$ contains some nice structure (a long AP, say) ?
- (2) Does $S(A)$ contains 0 ?
- (3) Does $S(A)$ contains the whole group ?
- (4) (At most) how many partial sums can represent the same element in $S(A)$?

I will discuss the so-called "Structural" method to these problems and survey some recent results obtained by this method (mostly based on joint work with Szemerédi, Tao, and my student Nguyen) and also some applications. Here are a few samples:

Theorem A: If A is an infinite sequence of positive integers with density at least $1000n^{-1/2}$, then $S(A)$ contains an infinite AP.

Theorem B: Let A be a subset of Z_p . The main reason for $S(A)$ not containing 0 is that its elements are small (thus not adding up to p).

Theorem C: If there are many partial sums representing the same element in $S(A)$, then A looks like an arithmetic progression.

67. **Jeffrey Paul Wheeler**, University of Memphis

The Erdős-Heilbronn Problem for Finite Groups

The Erdős-Heilbronn Conjecture, proven in 1994 by Dias da Silva and Hamidoune, states that for any two subsets A and B of $\mathbb{Z}/p\mathbb{Z}$ we have $|A\dot{+}B| \geq \min\{p, |A| + |B| - 3\}$, where $A + B := \{a + b \pmod{p} : a \in A \text{ and } b \in B \text{ and } a \neq b\}$. We generalize this result from $\mathbb{Z}/p\mathbb{Z}$ to arbitrary finite (including non-abelian) groups.

Joint work with Paul Balister, University of Memphis.

68. **Kevin Williamson**, The Catholic University of America

The Ramsey Properties of the Fibonacci Numbers

In *Ramsey Theory on the Integers*, Bruce Landman and Aaron Robertson introduce the notion of accessibility. Let S be a set of positive integers. A k -term S -diffsequence is a sequence of k positive integers $x_1 < x_2 < \dots < x_k$ such that $|x_{i+1} - x_i| \in S$ for $i = 1, 2, \dots, k - 1$. S is said to be r -accessible if every r -coloring of the set of positive integers yields arbitrarily long monochromatic S -diffsequences. If S is r -accessible for all positive integers r , then S is said to be *accessible*. The first half of this talk will be devoted to various lemmas useful in determining the accessibility of a set. The second half will be devoted to progress regarding the accessibility of the Fibonacci numbers.

Joint work with Derek Anderson, University of Pennsylvania; Megan Craven, St. Peter's College; Bruce Landman, University of West Georgia; and Aaron Shapiro, The Catholic University of America.

69. **Tomohiro Yamada**, Dep. of Math., Kyoto Univ.

On an equation involving the function $\sigma(N)$

We shall discuss some results on the equation $\sigma(\sigma(N)) = A\sigma(N) + BN$.