

# INTEGERS CONFERENCE 2011

University of West Georgia, Carrollton, Georgia

## Abstracts

1. **Sarah Anderson**, Clemson University

### Distance Graphs on $\mathbb{Z}^n$ with $l_1$ Norm

Let  $G = (\mathbb{Z}^n, D)$  be a graph with vertex set  $\mathbb{Z}^n$  such that  $x, y \in \mathbb{Z}^n$  are adjacent if and only if  $\|x - y\|_{l_1} \in D$ , where  $D = \{d \mid d < d_1 \text{ or } d_2 < d \leq n\}$  for  $1 \leq d_1 \leq d_2 \leq n$ . In this talk, we will find lower and upper bounds for  $\chi(G)$ . However, in order to find lower bounds for  $\chi(G)$ , we will first need to examine coding theory, and define a  $(n, [d_1, d_2])$ -code to be the set  $\mathcal{C} \subset \{0, 1\}^n$  such that for all  $x, y \in \mathcal{C}$  with  $x \neq y$ ,  $d_1 \leq d(x, y) \leq d_2$  for  $1 \leq d_1 \leq d_2 \leq n$ , where  $d$  denotes Hamming distance. We will show  $\chi(G) \geq \frac{2^n}{A(n, [d_1, d_2])}$ , where  $A(n, [d_1, d_2])$  denotes the maximum size of a  $(n, [d_1, d_2])$ -code. In addition, we will use codes in spherical caps to find an upper bound for  $A(n, [d_1, d_2])$ .

This is joint work with Jeong-Hyun Kang and Hiren Maharaj.

2. **William Banks**, University of Missouri

### Carmichael Numbers Composed of Piatetski-Shapiro Primes

We show that for any real number  $c$  in the range  $1 < c < 147/145$  there are infinitely many Carmichael numbers composed solely of primes of the form  $p = [n^c]$ , where  $n$  is a natural number and  $[x]$  denotes the floor function. This result has been obtained during a recent joint investigation with R. Baker, J. Brudern, I. Shparlinski and A. Weingartner.

3. **Arie Bialostocki**, University of Idaho

### On Non Linear Generalizations of the EGZ Theorem

The Erdős-Ginzburg-Ziv Theorem states that every sequence of residues modulo  $n$  of length  $2n - 1$  contains a zero-sum subsequence of length  $n$ . Thus it is associated with the function  $f(x_1, x_2, \dots, x_n) = \sum_{i=1}^n x_i$ . We will discuss results and conjectures related to the dissertation of Luong Trahn and address the question which non linear functions induce Erdős-Ginzburg-Ziv type theorems in view of weaker and stronger versions of the EGZ theorem. In particular we will consider the weighted version due to David Gryniewicz and various old and new related questions.

4. **Sadek Bouroubi**, University of Science and Technology Houari Boumediene

### On Quadrilaterals Inscribed in a Regular $n$ -gon and Partitions of an Integer

Let  $\Delta(n)$  denote the number of essentially different possible triangles inscribed in a regular  $n$ -gon. In the 1938's, Norman Anning asked the question concerning the value of  $\Delta(n)$ . Using several arguments, some authors showed that  $\Delta(n) = \{n^2/12\}$ , where  $\{x\}$  is the nearest integer to  $x$ . Here we take back the same question, but concerning the number of non-isometric ordered quadrilaterals, inscribed in a regular  $n$ -gon, denoted  $\square(n)$ , then, we give it's formula. On the other hand we give a connecting formula between  $\Delta(n)$  and  $\square(n)$ , which will be considered as a combinatorial interpretation of a certain identity on the integer partitions.

5. **Jacob Bowen**, San Francisco State University

**Plane Partitions and Hamming Distances**

We study  $2 \times n$  plane partitions using Stanley's theory of  $P$ -partitions. We develop an identity on sums of Hamming distances using generating functions. These generating functions can be calculated easily using a theorem due to Stanley, or can be computed manually by summing on the elements of the underlying poset. Computing the generating functions the former way gives a simple formula involving  $q$ -binomial coefficients; using the latter method yields a sum of powers of Hamming distances. We generalize the  $q$ -binomial formula to extend a theorem on plane partition chains due to Andrews, Paule, and Riese.

6. **Kyle Burke**, Wittenberg University

**Matchmaker: Nimber Patterns and Conjectures**

Matchmaker is an impartial game based off of the Stable Marriage Problem. A position of the game is two universally-ranked sets of  $n$  candidates, and a subset of a bipartite matching between them. Each turn a player updates the matching by either adding an edge between two unmatched candidates or pairing up two already-matched candidates that prefer each other over their current mate. In this case, the new connection is added, the old connections are removed and also the two newly-unpaired candidates are automatically matched together. There is one final state in the ruleset: when all candidates are paired with the other candidate of the same rank. In this talk, we will present the game then discuss a number of conjectures regarding the nimber values for different Matchmaker positions. Validation of these conjectures could lead to an efficient algorithm for evaluating the game.

7. **Scott Chapman**, Sam Houston State University

**Sets of Length and Delta Sets of Numerical Monoids**

Let  $S$  be a numerical monoid with  $n_1, n_2, \dots, n_k$  a minimal generating set (which we denote by  $S = \langle n_1, n_2, \dots, n_k \rangle$ ). If  $x \in S$ , then we call a representation  $x = x_1 n_1 + \dots + x_k n_k$  an *irreducible factorization* of  $x$  in  $S$  and  $x_1 + \dots + x_k$  its *length*. The *set of lengths* of  $x \neq 0$  is then defined as

$$\mathcal{L}(x) = \{n \mid \exists x_1, \dots, x_k \text{ such that } x = x_1 n_1 + \dots + x_k n_k \text{ and } n = x_1 + \dots + x_k\},$$

and the set of lengths of  $S$  as

$$\mathcal{L}(S) = \{\mathcal{L}(x) \mid 0 \neq x \in S\}.$$

If we write  $\mathcal{L}(x) = \{n_1, n_2, \dots, n_k\}$  with  $n_1 < n_2 < \dots < n_k$ , then the Delta set of  $x$  is defined as

$$\Delta(x) = \{n_i - n_{i-1} \mid 2 \leq i \leq k\}$$

and the Delta set of  $S$  as

$$\bigcup_{0 \neq x \in S} \Delta(x).$$

Five papers have appeared in the literature over the last 5 years which study the structure of the sets  $\mathcal{L}(x)$ ,  $\mathcal{L}(S)$ ,  $\Delta(x)$  and  $\Delta(S)$ . We will review the main results of these papers and in particular discuss the following results.

- (a) If  $S = \langle n, n+k, n+2k, \dots, n+tk \rangle$  is generated by an arithmetic sequence, then  $\Delta(S) = \{k\}$ .
- (b) Sets of lengths do not characterize numerical monoids, i.e., if  $\mathcal{L}(S) = \mathcal{L}(S')$  then  $S = S'$  may not hold.
- (c) If  $S = \{s_1, s_2, s_3, \dots\}$  with  $s_i < s_{i+1}$  for all  $i$ , then the sequence  $\Delta(s_1), \Delta(s_2), \Delta(s_3), \dots$  is eventually periodic.

## 8. Jonathan Chappelon, University Montpellier 2

### Modular Schur Numbers

A set of integers is called *l-sum-free modulo m* if it contains no elements  $x_1, \dots, x_l, y$  satisfying  $x_1 + \dots + x_l \equiv y \pmod{m}$ . It is called *weakly l-sum-free modulo m* if it contains no pairwise distinct elements  $x_1, \dots, x_l, y$  satisfying  $x_1 + \dots + x_l \equiv y \pmod{m}$ . For  $k \geq 1$  subsets and  $l \geq 1$  summands, the *modular Schur number*  $S_m(k, l)$  is the largest integer  $n$  for which the set  $\{1, 2, \dots, n\}$  admits a  $k$ -partition into  $l$ -sum-free modulo  $m$  subsets. The *modular weak Schur number*  $WS_m(k, l)$  is the largest integer  $n$  for which the set  $\{1, 2, \dots, n\}$  admits a  $k$ -partition into weakly  $l$ -sum-free modulo  $m$  subsets. In this talk, we present the exact values of the modular Schur numbers  $S_m(k, l)$  and the modular weak Schur numbers  $WS_m(k, l)$ , for  $k \geq 1$ ,  $l \geq 1$  and for the moduli  $m \leq 3$ .

This is joint work with M.P. Revuelta and M.I. Sanz.

## 9. Fang Chen, Oxford College of Emory University

### Long Minimal Zero-sum Sequences in the Group $C_2 \oplus C_{2k}$

A sequence in an additively written abelian group  $G$  is called a *minimal zero-sum sequence* if its sum is the zero element of  $G$  and none of its proper subsequences has sum zero. The structure of the longest minimal zero-sum sequences in the group  $C_2 \oplus C_{2k}$  is known; they have length  $2k + 1$ . In this talk we characterize the minimal zero-sum sequences in  $C_2 \oplus C_{2k}$  ( $k \geq 3$ ) with lengths at least  $2\lfloor k/2 \rfloor + 4$ . In particular the characterization covers sequences

whose lengths are just a bit greater than half of the maximum possible one. We state the main result for length at least  $k + 4$ . Let  $\alpha$  be a minimal zero-sum sequence of length  $\ell \geq k + 4$  in the group  $C_2 \oplus C_{2k}$  where  $k \geq 3$ . There exist a term  $a$  of  $\alpha$  with order  $2k$ , a basis  $\{e, a\}$  of  $C_2 \oplus C_{2k}$  containing  $a$ , with  $\text{ord}(e) = 2$ , and a representation  $\alpha = \prod_{j=1}^{\ell} (y_j e + z_j a)$  of  $\alpha$  with  $y_j \in \{0, 1\}$ ,  $z_j \in \mathbb{Z}$  so that:

- (i)  $0 < z_j < k$  if  $y_j = 0$  ( $1 \leq j \leq \ell$ ) and  $0 < z_i + z_j < k$  if  $y_i = y_j = 1$  ( $1 \leq i < j \leq \ell$ );
- (ii)  $\sum_{j=1}^{\ell} z_j = 2k$ .

The conclusion is slightly weaker in the remaining case  $\ell = k + 3$ , with  $k$  odd.

The characterization cannot be extended in the same form to shorter sequences. The argument is based on structural results about minimal zero-sum sequences in cyclic groups.

This is joint work with Svetoslav Savchev.

10. **Éva Czabarka**, University of South Carolina

### Phylogenetic Trees and Stirling Numbers

P.L. Erdős and L.A. Székely provided a bijection between rooted semi-labeled trees and set partitions, and hence Stirling numbers of the second kind. This, with the asymptotic normality of the Stirling numbers of the second kind (Harper) translates into the asymptotic normality of rooted leaf-labeled trees with a fixed number of vertices and a variable number of internal vertices. Phylogenetic trees are rooted leaf-labeled trees where the only internal vertex that can have degree 2 is the root. We apply Harper's method and the Erdős-Székely bijection to obtain the asymptotic normality of phylogenetic trees in several sense.

This is joint work with P.L. Erdős, V. Johnson, A. Kupczok and L.A. Székely.

11. **Marcia Edson**, Murray State University

### On Properties of Representations in Certain Linear Numeration Systems

Given  $a \geq b$ , let  $G_0 = 1$ ,  $G_1 = a + 1$ , and  $G_{n+2} = aG_{n+1} + bG_n$  for  $n \geq 0$ . For each choice of  $a$  and  $b$ , we have a linear recurrence that defines a numeration system. Every positive integer  $n$  may be written as the sum of the  $G_n$ , with alphabet  $A = \{0, 1, \dots, a\}$ , in one or more different ways. Let  $R_{(a,b)}(n)$  be the function that counts the number of distinct representations of an integer as a sum of the  $G_n$ . We extend results of J. Berstel, P. Kocábová, Z. Masáková, and E. Pelantová, and M. Edson and L. Q. Zamboni and give two distinct methods for calculating  $R_{(a,b)}(n)$ . One formula involves products of  $2 \times 2$  matrices and the other sums of binomial coefficients modulo 2. For the main result, we consider the limiting measure  $\mu_\beta$  of a convergent infinite convolution of measures (Bernoulli convolutions), where  $\beta$  is the dominating root of the characteristic equation of the recurrence above. We study the Garsia entropy of these measures and calculate explicitly the limiting entropy associated with  $\mu_\beta$ . This result extends those of J. Alexander and D. Zagier, and P. J. Grabner, P. Kirschenhofer, and R. F. Tichy. We then see that all these results can be generalized further to confluent numeration systems.

12. **Geremias Polanco Encarnacion**, University of Illinois at Urbana Champaign

### **Beatty Ratios and Sturmian Sequences**

Let  $\alpha$  and  $\beta$  be irrational numbers such that  $\frac{1}{\alpha} + \frac{1}{\beta} = 1$ , and let  $A := \{\lfloor \alpha n \rfloor\}_{n=1}^{\infty}$  and  $B := \{\lfloor \beta n \rfloor\}_{n=1}^{\infty}$ . Then, it follows that every positive integer  $n$  will appear once in  $A$  or  $B$ , and no integer will appear in both. In the literature,  $A$  and  $B$  are called beatty sequences, which are special case of sturmian sequences. These sequences play a role in various fields of physics, mathematics, biology and computer science. In particular in number theory, they are connected with prime numbers, semigroups, characters, divisor functions, other arithmetical functions, etc... In this talk we study differences of ratios of beatty sequences of the form  $\frac{a_{n+1}}{a_n} - \frac{b_{n+1}}{b_n}$ . We show that sturmian sequences can be defined in terms of this ratios. We also show that the series involving this and similar type of ratios is convergent. Furthermore, we find an identity for such series that bears a superficial resemblance to (a discrete version of) Frullani's Integral.

13. **Jessica Enright**, University of Alberta

### **Impartial Set Representation Games**

We describe a class of impartial combinatorial games on graphs using set representations. In these games, the players antagonistically build a set representation of a graph. We give hardness results for determining the winner of a position of these types of games in general, and give polynomial-time algorithms to solve special cases of these games on trees.

14. **Anastassia Etropolski**, Emory University

### **Powers of the eta-Function and Hecke Operators**

Half-integer weight Hecke operators and their distinct properties play a major role in the theory surrounding partition numbers and Dedekind's eta-function. Generalizing a recent paper of Ono, here we obtain closed formulas for the Hecke images of all negative powers of the eta-function. These formulas are generated through the use of Faber polynomials. In addition, congruences for a large class of powers of Ramanujan's Delta-function are obtained in a corollary. We further exhibit a fast calculation for many large values of vector partition functions.

This is joint work with Sarah Pitman.

15. **Carrie Finch**, Washington & Lee University

### **Nonlinear Sierpiński and Riesel Numbers**

In 1960, Sierpiński proved that there exist infinitely many odd positive integers  $k$  such that  $k \cdot 2^n + 1$  is composite for all positive integers  $n$ . Such values of  $k$  are known as *Sierpiński numbers*. Extending the ideas of Sierpiński to a nonlinear situation, Chen showed that there exist infinitely many positive integers  $k$  such that  $k^r \cdot 2^n + d$  is composite for all positive integers  $n$ , where  $d \in \{-1, 1\}$ , provided that  $r$  is a positive integer with  $r \not\equiv 0, 4, 6, 8 \pmod{12}$ . Filaseta, Finch and Kozek improved Chen's result by completely lifting the restrictions on  $r$  when  $d = 1$ , and they asked if a similar result exists if  $k^r$  is replaced by  $f(k)$ , where  $f(x)$  is an arbitrary nonconstant polynomial in  $\mathbb{Z}[x]$ . In this talk, we address this question when  $f(x) = ax^r + bx + c \in \mathbb{Z}[x]$ . In particular, we show, for various values of  $a, b, c, d$  and  $r$ , that there exist infinitely many positive integers  $k$  such that  $f(k) \cdot 2^n + d$  is composite for all integers  $n \geq 1$ . When  $d = 1$  or  $-1$ , we refer to such values of  $k$  as *nonlinear Sierpiński* or *nonlinear Riesel* numbers, respectively.

16. **Alex Fink**, North Carolina State University

### Lattice Games and Computation

Lattice games were introduced by Ezra Miller and Alan Guo as a framework subsuming many of our favourite examples of impartial combinatorial games, and intended to be more amenable to analysis of both commutative-algebraic and efficient algorithmic flavours. However, it turns out that lattice games can have arbitrarily bad behaviour: the outcome classes of their positions can be made to perform arbitrary computations. I will present some of the theory of lattice games, including the construction achieving this computational universality, and suggest where a plausible hope for a tractable theory might still lie.

17. **Aviezri S. Fraenkel**, Weizmann Institute of Science

### Rational Take-away Games

WYTHOFF is played on a pair of nonnegative integers,  $(M, N)$ . A move consists of either (i) subtracting any positive integer from precisely *one* of  $M$  or  $N$  such that the result remains nonnegative, or (ii) subtracting the *same* positive integer from both  $M$  and  $N$  such that the results remain nonnegative. The first player unable to move loses. Can we extend these results to the case where  $(M, N)$  are *rational* numbers? Using the Calkin-Wilf tree, we show how to play RATWYT, and any *take-away* game, when  $(M, N)$  are rational numbers, where suitable other rational numbers are subtracted from them according to the game rules.

18. **Shanzhen Gao**, Florida Atlantic University

### Some Enumerative Problems in Walks and Lattice Paths

A well-known long standing problem in combinatorics and statistical mechanics is to find the generating function for self-avoiding walks (SAW) on a two-dimensional lattice, enumerated by perimeter. A SAW is a sequence of moves on a square lattice which does not visit the

same point more than once. It has been considered by more than one hundred researchers in the past one hundred years, including George Polya, Tony Guttmann, Laszlo Lovasz, Donald Knuth, Richard Stanley, Doron Zeilberger, Mireille Bousquet-Mélou, Thomas Prellberg, Neal Madras, Gordon Slade, Agnes Dittel, E.J. Janse van Rensburg, Harry Kesten, Stuart G. Whittington, Lincoln Chayes, Iwan Jensen, Arthur T. Benjamin, and many others. More than three hundred papers and a few volumes of books were published in this area. A SAW is interesting for simulations because its properties cannot be calculated analytically. Calculating the number of self-avoiding walks is a common computational problem. We will discuss some problems and integer sequences arising from some special SAWs, walks with several step vectors and lattice paths.

19. **Alfred Geroldinger**, Karl-Franzens Universität

**On Products of Two Irreducible Elements**

Let  $H$  be a Krull monoid with finite class group  $G$  (e.g., the ring of integers of an algebraic number field). The Davenport constant  $D(G)$  of  $G$  is the maximal length of a minimal zero-sum sequence over  $G$ . A straightforward observation shows that a product of two irreducible elements (atoms) of  $H$  can be written as a product of  $D(G)$  irreducible elements at most. We study this extremal case and consider the set  $\mathcal{V}_{\{2, D(G)\}}(H)$  of all possible  $l \in \mathbb{N}$  such that there is an equation

$$u_1 u_2 = v_1 \cdot \dots \cdot v_l = w_1 \cdot \dots \cdot w_{D(G)},$$

where all  $u_1, u_2, v_i, w_j$  are irreducible elements. This is the same as studying the set of all  $l \in \mathbb{N}$  for which there exist a minimal zero-sum sequence  $U$  over  $G$  of length  $|U| = D(G)$  and minimal zero-sum sequences  $V_1, \dots, V_l$  over  $G$  such that

$$U(-U) = V_1 \cdot \dots \cdot V_l.$$

It turns out that in many cases this set  $\mathcal{V}_{\{2, D(G)\}}(H)$  is characteristic for the group  $G$ . The main result is based on the recent characterization of all minimal zero-sum sequences of length  $D(G)$  over groups of rank two.

This is joint work with Paul Baginski, David J. Grynkiewicz and Andreas Philipp.

20. **Darren Glass**, Gettysburg College

**Communal Partitions of Integers**

There is a well-known formula due to Andrews that counts the number of incongruent triangles with integer sides and a fixed perimeter, which is equivalent to counting the number of triples such that no single entry is more than half the sum of all three. In this talk, we consider the analogous question counting the number of  $k$ -tuples of nonnegative integers none of which is more than  $1/(k-1)$  of the sum of all  $k$  integers. We give an explicit function for the generating function which counts these  $k$ -tuples in the case where they are ordered, unordered, or partially ordered. Finally, we discuss the application to algebraic geometry which motivated this question.

21. **Gary Greaves**, Royal Holloway University of London

### Algebraic Integers and Combinatorial Objects

One gains a better understanding of certain sets of algebraic integers by associating them to combinatorial objects. In a series of recent papers, McKee and Smyth have used various kinds of graphs and matrices to study Pisot and Salem numbers. The smallest *known* Salem number  $\tau_0$  is the larger real zero of the polynomial

$$L(z) = z^{10} + z^9 - z^7 - z^6 - z^5 - z^4 - z^3 + z + 1.$$

Lehmer's conjecture states that the Mahler measure  $\tau_0$  of  $L$  is the smallest Mahler measure of any monic integer polynomial. I will describe the general method of McKee and Smyth used to study Pisot and Salem numbers, and focus on how it has been applied to obtain results supporting Lehmer's conjecture.

22. **David Gryniewicz**, Karl-Franzens-Universität Graz

### Freiman Homomorphisms

Freiman homomorphisms are the natural maps preserving the structure of a sumset

$$A + B = \{a + b : a \in A, b \in B\},$$

where  $A$  and  $B$  are subsets of an abelian group. Despite the importance that homomorphisms play in other areas of mathematics, there has been little effort until recently to study them in the context of sumsets, with attention almost exclusively limited to the case when  $A = B$ . In this talk, we briefly go over the basic definitions and properties of Freiman homomorphisms done in the setting of an arbitrary sumset  $A + B$ , including the universal ambient group, and then explain how the theory can be developed to the point where the following result follows as a consequence of a fundamental bound on the torsion subgroup of the universal ambient group. In short, the theorem shows that every finite, torsion-free sumset has an isomorphic copy with its summands contained as large subsets of a  $d$ -dimensional parallelogram.

**Theorem.** Let  $G$  be a torsion-free abelian group and let  $A, B \subseteq G$  be finite and nonempty subsets. Let  $d = \dim^+(A + B)$  be the Freiman dimension of  $A + B$ . Then there exists a Freiman isomorphic sumset  $A' + B' \subseteq \mathbb{Z}^d \subseteq \mathbb{R}^d$  with  $0 \in A' + B'$  and a  $d$ -dimensional parallelogram  $P \subseteq \mathbb{R}^d$  such that

$$\langle A' + B' \rangle = \mathbb{Z}^d, \quad A' \cup B' \subseteq P \quad \text{and} \quad \text{Vol}(P) \leq 2^{|A \cup B| - 1 - d},$$

where  $\text{Vol}(P)$  denotes the volume of  $P$ .

It is conjectured by Konyagin and Lev that the above theorem holds with the stronger conclusion of  $P$  being a rectangle having sides parallel to the standard basis vectors in  $\mathbb{Z}^d$ . Combining the above theorem with classical results from the Geometry of Numbers, one can obtain that  $P$  is a rectangle having sides parallel to the standard basis vectors in  $\mathbb{Z}^d$ , but at the expense of weakening the bound on the volume of  $P$  by a multiple of an (explicit) constant depending only on  $d$ .

23. **Neil Hindman**, Howard University

### **Images of $C$ Sets Under Nonhomogeneous Spectra**

Let  $\alpha > 0$  and  $0 < \gamma < 1$ . Define  $g_{\alpha,\gamma} : \mathbb{N} \rightarrow \mathbb{N}$  by  $g_{\alpha,\gamma}(n) = \lfloor \alpha n + \gamma \rfloor$ . The set  $\{g_{\alpha,\gamma}(n) : n \in \mathbb{N}\}$  is called the *nonhomogeneous spectrum of  $\alpha$  and  $\gamma$* . By extension, we refer to the maps  $g_{\alpha,\gamma}$  as spectra. In 1996 V. Bergelson, B. Kra, and I showed that if  $A$  is an *IP* set, a central set, an *IP\** set, or a central\* set, then  $g_{\alpha,\gamma}[A]$  is the corresponding object. We extend this result to include several other notions of largeness, including  $C$  sets,  $J$  sets, strongly central sets, and piecewise syndetic sets. Of these,  $C$  sets are particularly interesting because they are the sets which satisfy the conclusion of the Central Sets Theorem (so have many of the strong combinatorial properties of central sets) but have a much simpler elementary description than do central sets.

This is joint work with John Johnson.

24. **Brian Hopkins**, Saint Peter's College

### **Spotted Tilings and $n$ -color Compositions**

Spotted tilings are a new combinatorial interpretation of  $n$ -color compositions. Previous and new enumeration results are proven using this tool, an interpretation of the Fibonacci numbers, and a case of Terquem's problem. The spotted tilings also allow for MacMahon's zig-zag graphs to be applied to  $n$ -color compositions, addressing a question of Agarwal in the 2000 paper where these compositions were introduced.

25. **Eugen J. Ionascu**, Columbus State University

### **Ehrhart Polynomials Associated with Cubes and Equilateral Triangles in 3-dimensional Space, which have Integer Coordinates**

The Ehrhart's polynomial,  $L(P, t)$ , associated with a lattice polytope  $P$  is given by the number of lattice points contained in a dilation of the polytope, i.e.  $tP$  with  $t \in \mathbb{N}$ . The degree of the polynomial is given by the dimension of the polytope. We consider this object associated with cubes and equilateral triangles in  $\mathbb{Z}^3$ .

26. **Marie Jameson**, Emory University

### **Congruences for Broken $k$ -diamond Partitions**

In 2007, Paule and Andrews constructed a class of directed graphs called broken  $k$ -diamonds, and used them to define  $\Delta_k(n)$  to be the number of broken  $k$ -diamond partitions of  $n$ . These objects are of interest because the generating function of  $\Delta_k(n)$  is essentially a modular form. The study of these generating functions led to two conjectures of Paule and Radu, the proofs of which we discuss here.

27. **Veselin Jungić**, Simon Fraser University

### **Chaotic Orderings of the Rationals and Reals**

In 1980 Erdős and Graham asked whether or not every linear ordering of the reals must have a monotonic  $k$ -term arithmetic progression for every  $k$ . The question was recently answered Hayri Ardal, Tom Brown, and the speaker.

28. **William J. Keith**, University of Lisbon

### **Ramanujan Congruence Analogues for the Han/Nekrasov-Okounkov Hooklength Formula**

Congruences for powers of the partition function are a subject of much study, beginning with those for the partition function itself, such as  $p(5k+4) \equiv 0 \pmod{5}$ . In this talk we will consider the polynomials  $p_n(b)$  that arise in the expansion of the formula for  $\prod (1 - q^k)^{b-1} = \sum \frac{q^n}{n!} p_n(b)$ . These polynomials exhibit analogous behaviors in some arithmetic progressions. For example, in  $p_{5k+4}(b)$ , the coefficients equally populate the nonzero residue classes mod 5. We will prove this as well as many other symmetries for these polynomials.

29. **Zachary A. Kent**, Emory University

### **Witt Rings and Matroids**

The study of Witt rings of formally real fields in the algebraic theory of quadratic forms has led to a particularly good understanding of the finitely generated torsion free Witt rings. In this paper, we work primarily with a somewhat more general class of rings which can be completely characterized by (binary) matroids. The different types of standard constructions and invariants coming from algebra and from combinatorics lead to previously unstudied problems for both areas; in particular, there are new invariants for Witt rings and new constructions for matroids with many open questions.

30. **Kağan Kursungöz**, Sabanci University

### **Variations on a Result of Bressoud**

The Rogers-Ramanujan-Gordon identities state that the number of partitions of  $n$  such that  $f_i + f_{i+1} < k$  and  $f_1 < a$  are the same as the number of partitions of  $n$  such that  $f_{j(2k+1)} = f_{j(2k+1)+a} = f_{(j+1)(2k+1)-a} = 0$ . Here,  $f_i$  denotes the number of occurrences (the frequency) of  $i$  in a given partition. The analytical counterpart is called the Andrews-Gordon Identities. In 1981, Bressoud found a generalization of the Andrews-Gordon Identities, and he gave an interpretation. We will give an alternative interpretation using the Gordon-marking of a partition and *clusters*. We will discuss the obstacles in further generalizing these results in the same fashion.

31. **Urban Larsson**, Chalmers University of Technology & University of Gothenburg

### **A Take-away Game Emulating the Rule 110 Cellular Automaton**

We study 2-player combinatorial games of take-away whose winning strategies emulate the rule 110 cellular automaton. Given initial conditions consisting of a central data pattern and repetitive left and right patterns, this cellular automaton was proved undecidable around year 2000 by Matthew Cook. As a consequence we show that certain questions are undecidable for our games.

32. **Andrew Ledoan**, University of Tennessee Chattanooga

### **On the Differences Between Consecutive Prime Numbers**

In 1976 Gallagher showed that the prime numbers are distributed in short intervals according to a Poisson distribution around their average spacing, subject to the truth of a modified uniform version of the Hardy-Littlewood prime  $k$ -tuple conjecture. In this talk, I will present an extension of Gallagher's theorem. More precisely, I will prove an asymptotic formula for the number of differences between consecutive prime numbers of a specified length, subject to the above prime  $k$ -tuple conjecture.

This is joint work with Professor Daniel A. Goldston.

33. **Joon Yop Lee**, POSTECH, Korea

### **Poly-Bernoulli Numbers and Lonesum Matrices**

A lonesum matrix is a matrix that can be uniquely reconstructed from its row and column sums. Kaneko defined the poly-Bernoulli numbers  $B_m^{(n)}$  by a generating function, and Brewbaker computed the number of binary lonesum  $m \times n$  matrices and showed that this number coincides with the poly-Bernoulli number  $B_m^{(-n)}$ . We compute the number of  $q$ -ary lonesum  $m \times n$  matrices, and then provide generalized Kaneko's formulas by using the generating function for the number of  $q$ -ary lonesum  $m \times n$  matrices. In addition, we define two types of  $q$ -ary lonesum matrices that are composed of strong and weak lonesum matrices, and suggest further researches on lonesum matrices.

This is joint work with Hyun Kwang Kim and Denis S. Krotov.

34. **Robert Lemke Oliver**, Emory University

### **Pretentiously Detecting Power Cancellation**

Granville and Soundararajan have recently introduced the notion of pretentiousness in the study of multiplicative functions taking values in the complex unit disk, with the idea that two functions that are "close" to each other in some specific sense should have the same, or

at the very least similar, behavior. We investigate whether this applies to detecting power cancellation in the partial sums of such functions. We find that it does not, and we introduce a modified notion of distance which does permit us to do so.

35. **Linyuan Lu**, University of South Carolina

### Monochromatic 4-term Arithmetic Progressions in 2-colorings of $\mathbb{Z}_n$

This talk is motivated by a recent result of Wolf on the minimum number of monochromatic 4-term arithmetic progressions (4-APs, for short) in  $\mathbb{Z}_p$ , where  $p$  is a prime number. Wolf proved that there is a 2-coloring of  $\mathbb{Z}_p$  with 0.000386% fewer monochromatic 4-APs than random 2-colorings; the proof is probabilistic and non-constructive. In this talk, we present an explicit and simple construction of a 2-coloring with 9.3% fewer monochromatic 4-APs than random 2-colorings. This problem leads us to consider the minimum number of monochromatic 4-APs in  $\mathbb{Z}_n$  for general  $n$ . We obtain both lower bound and upper bound on the minimum number of monochromatic 4-APs in  $\mathbb{Z}_n$ . Wolf proved that any 2-coloring of  $\mathbb{Z}_p$  has at least  $(1/16 + o(1))p^2$  monochromatic 4-APs. We improve this lower bound into  $(7/96 + o(1))p^2$ .

Our results on  $\mathbb{Z}_n$  naturally apply to the similar problem on  $[n]$ . In 2008, Parillo, Robertson, and Saracino constructed a 2-coloring of  $[n]$  with 14.6% fewer monochromatic 3-APs than random 2-colorings. In 2010, Butler, Costello, and Graham extended their methods and used an extensive computer search to construct a 2-coloring of  $[n]$  with 17.35% fewer monochromatic 4-APs (and 26.8% fewer monochromatic 5-APs) than random 2-colorings. Our construction gives a 2-coloring of  $[n]$  with 33.33% fewer monochromatic 4-APs (and 57.89% fewer monochromatic 5-APs) than random 2-colorings.

This is joint work with Xing Peng.

36. **Florian Luca**, Universidad Nacional Autonoma de Mexico

### Balancing with Powers of Fibonacci Numbers

Let  $(F_n)_{n \geq 0}$  be the Fibonacci sequence. In my talk, I will show that the only solution of the Diophantine equation

$$F_1^k + F_2^k + \cdots + F_{n-1}^k = F_{n+1}^\ell + \cdots + F_{n+r}^\ell$$

in positive integers  $(k, \ell, n, r)$  is  $(8, 2, 4, 3)$ . This confirms a conjecture of Behera, Liptai, Panda and Szalay. This is joint work with A. Dujella (Zagreb) and S. Díaz Alvarado (Toluca).

37. **Neil Lyall**, University of Georgia

### Polynomial Patterns in Subsets of the Integers I

It is a striking and elegant fact (proved independently by Furstenberg and Sarkozy) that any subset of the integers of positive upper density necessarily contains two distinct elements whose difference is given by a perfect square. We will present a new proof of this result and time permitting also discuss a number of variations, extensions and generalizations.

38. **Akihiro Matsuura**, Tokyo Denki University

### **The Tower of Hanoi with Weighted Cost**

We consider a generalization of the classical three-peg Tower of Hanoi problem, where each undirected edge between pegs has a positive weight and the problem is to transfer all the disks from one peg to another with the minimum sum of weights, instead of the minimum number of moves. We present an optimal algorithm for this problem and we also show the exact value of the minimum sum of weights which varies according to the number of disks, the relation of the weights, and the pair of pegs.

This is joint work with Shizuka Nishimaki.

39. **Neil A. McKay**, Dalhousie University

### **Wythoff Partizan Subtraction**

Wythoff Partizan Subtraction is a heap game where one player may remove a number of tokens if it is a member of the lower Wythoff sequence and the other may remove a number of tokens if it is a member of the (complementary) upper Wythoff sequence. We observe that the reduced canonical forms of positions are a restricted set of numbers and switches. The sequences of heap sizes with the same reduced canonical forms are related to the Wythoff sequences and the infinite Fibonacci word. We also note that outcome classes of a single heap are not periodic.

This is joint work with Urban Larsson, Richard J. Nowakowski, and Angela A. Siegel.

40. **Rebecca Milley**, Dalhousie University

### **The Misère Monoid of Single-handed Consecutive Move Ban Games**

A combinatorial game  $G$  is a *consecutive move ban* (CMB) game if there are no left options from any left option  $G^L$  and no right options from any right option  $G^R$ , and if every follower of  $G$  is also a CMB game. The structure of these games makes them interesting subjects for misère play, and a detailed analysis of misère CMB strategy was given by Paul Ottaway in 2009. From his analysis we can attempt to determine the associated *misère monoid* – a construction developed by Plambeck and Siegel (2008) to encode some of the otherwise elusive algebraic structure of misère games. This talk will present the monoid for a subset of CMB games we call *single-handed* (games  $G$  with either  $G^L = \emptyset$  or  $G^R = \emptyset$ ) and discuss our progress on the monoid for general CMB games.

This is joint work with Richard Nowakowski and Meghan Allen.

41. **Mojtaba Moniri**, Western Illinois University

### **Spectra and Binary Complexity of Certain Reals $\arctan(\alpha)/\pi$**

Plouffe (98) computed some reals  $\arctan(\alpha)/\pi$  in binary. Recently,  $\arctan$  drew interest of logicians Tent-Ziegler and Skordev-Weiermann-Georgiev:  $\beta = \arctan(\alpha)/\pi$  is  $\mathcal{M}^2$ -computable if  $\alpha$  is. These were via Cauchy approximations, one may consider the spectrum (or Beatty sequence)  $(\lfloor n\beta \rfloor)_{n \in \mathbb{N}}$ , etc. There is a hierarchy of primitive recursive reals represented in various ways (Chen-Su-Zheng 07). There are more subtleties restricting to levels like  $\mathcal{M}^2$  or Grzegorzcyk's  $\mathcal{E}^2$  missing exponentiation.

We study Beatty and binary complexity of certain reals  $\beta = \beta(\alpha) = \arctan(\alpha)/\pi$  via two algorithms. Let  $\beta = \beta(\frac{1}{2})$ ,  $(a_n, b_n)$  defined by  $a_1 = 2$ ,  $b_1 = 5$ ,  $a_{n+1} = 2a_n + b_n - 6$ , and  $b_{n+1} = 2b_n - a_n - 4$ , and  $(c_n, d_n)$  defined by  $c_1 = 1$ ,  $d_1 = 2$ ,  $c_{n+1} = 2c_n d_n$ , and  $d_{n+1} = d_n^2 - c_n^2$  (the latter double sequence originates from Plouffe's construction). For all  $n$ ,  $\lfloor 2^n \beta \rfloor$  is odd if and only if  $\#\{m \leq 2^n \mid (b_m - 5)(b_{m+1} - 5) < 0\}$  is odd if and only if  $c_n d_n < 0$ . All components grow exponentially; is it true that  $(\lfloor n\beta \rfloor)_{n \in \mathbb{N}} \in \mathcal{E}^2$ ?

This is joint work with Iraj Kalantari.

42. **Rishi Nath**, City University of New York

### **Simultaneous Core Partitions**

$p$ -core partitions are partitions which contain no hook of a fixed length  $p$ . They arise in various areas including the block theory of symmetric groups,  $k$ -Schur functions, Shi arrangements and Coxeter groups. Here we survey new research into simultaneous core partitions, that is, those containing no hooks for two distinct integers  $p$  and  $q$ . The work of J. Olsson, M. Fayers, and others will be discussed.

43. **Mel Nathanson**, CUNY

### **Geometric Group Theory and Arithmetic Diameter**

This talk describes a series of problems in additive number theory that arise from imposing various arithmetically defined metrics on the integers. A simple example: Compute the minimum number of terms needed to represent an integer as sums and differences of powers of 2 and powers of 3.

44. **Heinrich Niederhausen**, Florida Atlantic University

## Motzkin Numbers and Chebyshev Polynomials

Christian Krattenthaler wrote in his 2001 paper *Permutations with Restricted Patterns and Dyck paths* “Whenever you encounter generating functions which can be expressed in terms of continued fractions or Chebyshev polynomials, then expect that Dyck or Motzkin paths are at the heart of your problem, and will help to solve it.” We would like to look for more connections between (generalized) Motzkin numbers and Chebyshev polynomials. We enumerate paths from  $(0, 0)$  to  $(n, j)$  that do not go below the  $x$ -axis, and take steps from  $\{\nearrow, \searrow, \xrightarrow{w}\}$ . The horizontal steps of lengths  $w$  get weight  $\omega$ . If  $j = 0$ , the generating function of this weighted enumeration  $W_{n;w,\omega}$  equals  $W(t; w, \omega) = \sum_{n \geq 0} W_{n;w,\omega} t^n = 1 + \omega t^w W(t; w, \omega) + t^2 W(t; w, \omega)^2 = (z - \sqrt{z^2 - 1})/t$  if  $z = (1 - \omega t^w)/2t$ . The *Chebyshev polynomials of the second kind*  $U_n(x)$  have the generating function  $U(t; x) = \sum_{n=0}^{\infty} U_n(x) t^n = 1/(1 - 2xt + t^2)$  and satisfy the recursion  $U_n(x) = 2xU_{n-1}(x) - U_{n-2}(x)$ , with  $U_0(x) = 1$ . Here is the first connection between Motzkin numbers ( $w = 1$ ) and Chebyshev polynomials: The inverse Motzkin numbers, obtained from the compositional inverse  $I(t; 1, \omega)$  of  $tW(t; 1, \omega)$  equals  $I(t; 1, \omega) = t/(1 + \omega t + t^2) = tU(t; -\omega/2)$ . Furthermore, the Chebyshev polynomials play a role when we expand  $\mathcal{W}(t, j)$ , the generating function of the the number of paths  $W(n, j)$  that end at  $(n, j)$ , staying above the  $x$ -axis (we drop  $w$  and  $\omega$  from the notation),  $\mathcal{W}(x, t) = \sum_{j \geq 0} \mathcal{W}(t, j) x^j = (z - x - \sqrt{z^2 - 1})U(x; z)/(2t)$ . Denote the general Motzkin numbers in a band, i.e., the number of paths staying between the  $x$ -axis and the height  $y = k$  (not touching  $y = k$ ), by  $W^{(k)}(n, j)$ . Then  $\mathcal{W}^{(k)}(t, x) = \sum_{j \geq 0} W^{(k)}(t, j) x^j = \left(\frac{1}{U_k(z)} - x\right)U(x; z)/t$ .

45. **Richard J. Nowakowski**, Dalhousie University

### The Game of Partizan Euclid

When using the Euclidean Algorithm to find the gcd of two numbers  $(p, q)$ , we may write  $p = a \times q + r$  and continue with the pair  $(q, r)$ , or write  $p = (a + 1) \times q - (q - r)$  and continue with  $(q, q - r)$ . We turn this into a partizan game, starting with  $(p, q)$  and allowing Left to move to  $(q, (p \bmod q))$  and Right to  $(q, q - (p \bmod q))$  provided  $p$  is not a multiple of  $q$  in which case the game is over. Playing a sum seems difficult (who wins  $(34, 21) + (21, 13) + (13, 8)$ ?) but analyzing one game shows the interesting structure of this game. We show that any game tree can be represented as a directed path with an extra leaf at every vertex (based on the most inefficient way of finding the gcd); this allows determining the outcome class to become a word problem and word reductions show that every game is equivalent to one of 12.

This is joint work with Neil McKay.

46. **Ken Ono**, Emory University

### Adding & Counting

One easily sees that

$$4 = 3 + 1 = 2 + 2 = 2 + 1 + 1 = 1 + 1 + 1 + 1,$$

and so we say that there are 5 partitions of 4. The stuff of partitions seems like mere child's play. The speaker will explain how the simple task of adding and counting has fascinated many of the world's leading mathematicians: Euler, Ramanujan, Hardy, Rademacher, to name just a few. As is typical in number theory, many of the most fundamental (and simple to state) questions have remained unsolved. In 2010, the speaker, with the support of the American Institute for Mathematics and the National Science Foundation, assembled a group of researchers to attack some of these problems. He will describe their findings.

47. **Carl Pomerance**, Dartmouth College

### Dense Product-free Sets

A subset  $S$  of a ring  $R$  is *product free* if  $ab \neq c$  whenever  $a, b, c \in S$ . How large can  $S$  be when  $R = \mathbb{Z}/n\mathbb{Z}$  or  $R = \mathbb{Z}$ ? It is easy to come up with examples of product-free sets of integers with asymptotic density  $1/2$ , and in  $\mathbb{Z}/n\mathbb{Z}$  with almost  $n/2$  elements. For example, if  $p$  is an odd prime, the quadratic nonresidues for  $p$  form a product-free subset of  $\mathbb{Z}/p\mathbb{Z}$  of size  $(p-1)/2$ . So, “ $1/2$ ” seems to be a natural guess, and in a recent paper with A. Schinzel we proved this for  $\mathbb{Z}/n\mathbb{Z}$  for a very large proportion of numbers  $n$  and for all  $n \leq 9 \times 10^8$ . However, in new work with P. Kurlberg and J. Lagarias, we show that for each  $\epsilon > 0$ , there are numbers  $n$  with product-free subsets of  $\mathbb{Z}/n\mathbb{Z}$  of size  $> (1-\epsilon)n$ . The smallest example we were able to find that beats  $n/2$  has  $n \approx 10^{1.61 \times 10^8}$ . If  $n_\epsilon$  is the least  $n$  that beats  $(1-\epsilon)n$ , we show that  $n_\epsilon$  tends to infinity about doubly exponentially in  $1/\epsilon^{17}$ .

48. **Alex Rice**, University of Georgia

### Polynomial Patterns in Subsets of the Integers II

We will discuss generalizations of the Sárközy-Furstenberg Theorem which states that any subset of the natural numbers of positive upper density necessarily contains two distinct elements which differ by a perfect square. In particular, we will replace the squares with  $f(\mathbb{N})$  and  $g(\mathcal{P})$ , where  $f$  and  $g$  are in certain classes of integer polynomials. We will survey what is known and state new results, including a new record bound for quadratic polynomials.

49. **Neville Robbins**, San Francisco State University

### On Divisor Partitions

If  $n$  is a natural number, then a divisor partition of  $n$  is a partition such that all parts are divisors of  $n$ . We obtain formulas for the number of divisor partitions of  $n$  when  $n$  has few divisors.

50. **Aaron Robertson**, Colgate University

## On Van der Waerden Numbers

We will present some new asymptotic lower bounds for certain van der Waerden numbers.

51. **Larry Rolén**, Emory University

### Benford's Law for Coefficients of Modular Forms and Partition Functions

It has long been observed that many naturally occurring statistics and arithmetic functions have surprising properties. For example, if we examine the first digits of a sequence in base 10, instead of the a priori estimate that each digit should appear equally often we find that the first digit is a 1 about 6 times as often as it is a 9. Although this is a well-known heuristic, it has only been proven for a relatively small class of arithmetic functions. Using recent results of Ken Ono and Kathrin Bringmann on coefficients of harmonic Maass forms as well as classical theory of uniform distribution, we prove that the coefficients of an infinite class of modular forms satisfy the Benford distribution. This allows us to generate large classes of sequences which were previously unknown to be Benford. In particular, we will show this for the partition function  $p(n)$  as well as numerous classes of natural partition functions.

52. **Esengül Saltürk**, Yildiz Technical University Department of Mathematics

### Generalized Gaussian Numbers

While studying the number of linear codes over integers modulo  $p^m$  ( $p$  prime,  $m$  positive integer), we obtained a direct formula. Due to the similarity to Gaussian numbers, we name these numbers as Generalized Gaussian Numbers. Hence we study some properties of these numbers and also obtain new number sequences. This research is supported by Yildiz Technical University Research Support Unit.

This is joint work with İrfan Şiap.

53. **Carlos Pereira dos Santos**, High Institute of Education and Science

### A Universal Combinatorial Ruleset

Berlekamp asked the question “What is the habitat of \*2?”. This talk is about an even more general question: “What is the habitat of the short Conway’s group?”. It is presented a proof that all the short combinatorial games are positions of a particular well-known ruleset: KONANE is the first known *universal ruleset*.

This is joint work with Alda Carvalho.

54. **Carla Savage**, North Carolina State University

## Generalized Lecture Hall Partitions and Eulerian Polynomials

Lecture hall partitions, introduced by Bousquet-Melou and Eriksson in 1997, are nonnegative integer sequences  $(x_1, x_2, \dots, x_n)$  satisfying  $x_i/i \leq x_{i+1}/(i+1)$  for  $1 \leq i < n$ . They are known for their mysteriously simple generating function and their relationship to Euler's odd-distinct partition theorem. In this talk, we generalize to  $s$ -lecture hall partitions, satisfying  $x_i/s_i \leq x_{i+1}/s_{i+1}$  for  $1 \leq i < n$ , where  $s$  is an arbitrary positive integer sequence. We introduce  $s$ -inversion sequences and show that the enumeration of  $s$ -lecture hall partitions gives rise to new families of Eulerian polynomials, defined by ascent statistics on  $s$ -inversion sequences. These results unify, generalize, and extend several familiar results about Eulerian polynomials and lecture hall partitions. This includes joint work with Michael Schuster, Gopal Viswanathan, and Thomas Pensyl.

55. **James Sellers**, Pennsylvania State University

## Enumeration of Line-Hamiltonian Multigraphic Degree Sequences

In this talk, we will consider an enumeration problem which is very closely related to recent graph-theoretic results of Lai and Liang. In particular, we will discuss their characterization of integer partitions which can be realized as the degree sequence of a multigraph  $G$  with the property that the line graph of  $G$  is Hamiltonian, and we will provide a closed-form formula for the number of such degree sequences. We will then connect this to the similar result, obtained in 2008 by Rodseth, Sellers, and Tverberg, for connected multigraphs.

56. **Mark Shattuck**, University of Tennessee Knoxville

## Restricted Partitions and Motzkin Left Factors

If  $n \geq 1$ , then let  $L_n$  enumerate the lattice paths from the origin to the line  $x = n - 1$  using  $(1, 1)$ ,  $(1, -1)$ , and  $(1, 0)$  steps that never dip below the  $x$ -axis (termed *Motzkin left factors*). The sequence  $L_n$  counts, among other things, certain restricted subsets of permutations and Catalan paths. Here, we provide new combinatorial interpretations for these numbers in terms of finite set partitions. In particular, we identify four classes of the partitions of size  $n$ , all of which have cardinality  $L_n$  and each avoiding two classical patterns of length four. A two-variable polynomial generalization of  $L_n$  may be obtained by considering a pair of statistics on the partition class in one of the cases. In a similar manner, seemingly new combinatorial interpretations for the Catalan number  $C_n$  may be given in terms of the cardinality of certain subsets of the partitions of size  $n$  which avoid a pattern of length four and another of length five.

57. **Pante Stanica**, Naval Postgraduate School

## Generalized Bent Functions and nega-Hadamard Transform

In this talk, we define generalized bent functions on  $\mathbb{Z}_2^n$  with values in  $\mathbb{Z}_q$ , where  $q \geq 2$  is any positive integer, via the nega-Hadamard transform. We characterize this class of functions symmetric with respect to two variables, provide an analogue of Maiorana–McFarland type bent functions in the generalized set up. A class of bent functions called generalized spreads type is introduced and it is demonstrated that recently introduced Dillon type and Maiorana–McFarland type can be described as generalized spreads type functions. Thus, unification of two different types of generalized bent functions is achieved.

58. **László Székely**, University of South Carolina

### ***M*-part Sperner Families, Transversals and Mixed Orthogonal Arrays**

In the 1960’s, Katona and Kleitman discovered independently that the conclusion of the Sperner theorem about the size of Sperner families on an  $n$ -element set (i.e. a family of sets, such that none of them includes another), also applies to 2-part Sperner families, but not to more-part Sperner families.

An  $M$ -part  $L_1, \dots, L_M$ -Sperner family is a family  $\mathcal{F}$  of subsets of an  $n$ -element set partitioned into classes  $X_1, X_2, \dots, X_M$ , such that for any chain  $A_1 \subset A_2 \subset \dots \subset A_{L_i+1}$  from  $\mathcal{F}$ ,  $A_{L_i+1} \setminus A_1$  is not a subset of  $X_i$ .

Although the problem of finding the maximum size of  $M$ -part  $L_1, \dots, L_M$  Sperner families has been reduced to a “problem of numbers” from a “problem of sets” through the concept of homogeneity, and asymptotic results have been obtained, there was not even a conjecture for an optimal structure for  $M > 2$ . We provide now such a conjecture if the size of the underlying set is  $n = M(2^t - 1)$ , and  $L_i = 1$ .

For  $M = 2$ , Péter Erdős and Katona classified all maximum size 2-part Sperner families. Their tools lead us to the concept of full transversals. Full transversals have not been easy to construct and did not play the role that we expected for  $M > 3$ , though Füredi, Griggs, Odlyzko and Shearer constructed full transversals for  $M = 2$  for any  $X_1, X_2, L_1, L_2$ . However, we found a way to construct full transversals based on rounding for any  $M$ .

In an other direction, we generalized the LYM inequality to  $M$ -part  $L_1, \dots, L_M$  Sperner families into a set of  $M$  inequalities. (For a homogeneous family, if one inequality holds with equality, we face a full transversal.) We showed that if all inequalities hold with equality, then we face a mixed orthogonal arrays with strength  $M - 1$ .

In turn, our construction of full transversals provide an infinite number of mixed orthogonal arrays with strength  $M - 1$ . Furthermore, we define a generalization of  $M$ -part Sperner families, the  $d$ -dimensional  $M$ -part Sperner family, which are related to mixed orthogonal arrays with strength  $M - 1$ . (Mixed orthogonal arrays are basic tools for experimental design and are closely connected to many other chapters of design theory.)

This is a joint work with (with various subsets of) Harout Aydinian, Éva Czabarka, Konrad Engel, and Peter Erdős.

59. **Yuval Tanny**, Weizmann Institute of Science

### **A Class of Wythoff-like Games**

We present a class of two-player Wythoff game variations that depends on a given function  $f(k)$ . In this class a move consists of either removing a positive number of tokens from precisely one of two given piles, or to remove  $k$  tokens from one pile and  $\ell$  from the other, subject to the constraint  $0 < k \leq \ell < f(k)$ . We analyze three classes of integer-valued functions  $f(k)$ : constant, superadditive, polynomial of degree  $> 1$  with non-negative coefficients. The nature of the winning positions in the games is essentially unique for each class. The linear class  $f(k) = sk + t$ ,  $s, t \geq 1$  integers, was analyzed previously by A. Fraenkel (1998). We extend this result to the three other classes of functions.

This is joint work with Aviezri S. Fraenkel.

60. **Lola A. Thompson**, Dartmouth College

### Polynomials with Divisors of Every Degree

Which polynomials with integer coefficients have a divisor of every degree? Certainly, any polynomial that splits completely into linear factors satisfies this criterion. However, there are other choices of polynomials that are not as obvious. In this talk, we consider polynomials of the form  $x^n - 1$  and determine when members of this family have a divisor of every degree in a given polynomial ring.

61. **Frank Thorne**, Stanford University

### Four Perspectives on a Curious Secondary Term

In 1971, Davenport and Heilbronn proved asymptotic formulas for the number of cubic fields of bounded discriminant, and for the amount of 3-torsion in the class groups of quadratic fields of bounded discriminant. Numerical data turned out to be a poor match for the theoretical results, and Datskovsky-Wright and Roberts conjectured the existence of secondary terms. This problem lay dormant for awhile, but the past two years have seen four new and very different approaches to these secondary terms, including two independent proofs of the conjecture. I will say something about all of these approaches, developed by Bhargava-Shankar-Tsimerman, Hough, Zhao, and Taniguchi and myself.

62. **Enrique Treviño**, Swarthmore College

### The Smoothed Pólya–Vinogradov Inequality

Let  $\chi$  be a primitive Dirichlet character to the modulus  $q$ . Let  $S_\chi(M, N) = \max_{M < n \leq M+N} \left| \sum_{M < n \leq M+N} \chi(n) \right|$ . The Pólya–Vinogradov inequality states that  $S_\chi(M, N) \ll \sqrt{q} \log q$ . The smoothed Pólya–Vinogradov inequality, recently introduced by Levin, Pomerance and Soundararajan, is a numerically useful version of the Pólya–Vinogradov inequality, where you manage to lose the  $\log q$  factor by smoothing out the sum. The smoothed Pólya–Vinogradov inequality has been used to settle a conjecture of Brizolis, namely that for every prime  $p > 3$ , there is a

primitive root  $g$  and an integer  $x \in [1, p - 1]$  such that  $g^x \equiv x \pmod{p}$ . It has also been used to improve the best known numerically explicit upper bound on the last inert prime in a real quadratic field. In this talk we will present several theorems concerning the smoothed Pólya–Vinogradov inequality.

63. **Joseph Vandehey**, University of Illinois

### On Multiplicative Functions with Bounded Partial Sums

Consider a multiplicative function  $f(n)$  taking values on the unit circle. Is it possible that the partial sums of this function are bounded? We show that if we weaken the notion of multiplicativity so that  $f(pn) = f(p)f(n)$  for all primes  $p$  in some finite set  $P$ , then the answer is yes. We also discuss a result of Bronstein, which shows that functions modified from characters at a finite number of places have unbounded partial sums.

64. **Julia Wolf**, Ecole Polytechnique

### Finite Field Models in Additive Number Theory

The term “finite field philosophy” was coined by Ben Green in a seminal article from 2005, and its implications have since been actively pursued by many other researchers in additive combinatorics. The basic idea is that a high-dimensional vector space over a small fixed finite field often provides a “good model” for a given number theoretic problem in the integers or the cyclic group modulo  $N$ . By a “good model” we mean that an analogous question can be asked, whose solution is many times considerably simplified by the availability of algebraic notions such as “subspace” and “linear independence”, which are non-existent (or much less useful) in the interval 1 up to  $N$ , or  $\mathbb{Z}_N$  with  $N$  a prime. In this talk we present a number of open problems in the finite field setting that have (to our knowledge) not yet been viewed in this light, as well as a survey of some of their more well-known counterparts in the integers or the cyclic group modulo  $N$ .

65. **Carl Yergler**, Davidson College

### On Three Sets with Non-decreasing Diameter

Let  $[a, b]$  denote the integers between  $a$  and  $b$  inclusive, and for a set of integers  $X$ , let  $\text{diam}(X) = \max(X) - \min(X)$ . Moreover  $X <_p Y$  if and only if  $\max(X) < \min(Y)$ . For a positive integer  $m$ , let  $f(m, m, m; 2)$  be the least integer  $N$  such that for every 2-coloring  $\Delta : [1, N] \rightarrow \{0, 1\}$  there exist three subsets  $B_1, B_2, B_3 \subseteq [1, N]$  such that (a)  $B_i$  for  $i = 1, 2, 3$  is monochromatic, (b)  $|B_i| = m$  for  $i = 1, 2, 3$ , (c)  $B_1 <_p B_2 <_p B_3$  and (d)  $\text{diam}(B_1) \leq \text{diam}(B_2) \leq \text{diam}(B_3)$ . We will motivate the basis for this problem, its relation to the Erdős–Ginzburg–Ziv theorem and give an explanation why  $f(m, m, m; 2) = 8m - 5 + \lfloor \frac{2m-2}{3} \rfloor$  for  $m \geq 5$ .

This is joint work with Daniel Bernstein and David Gryniewicz.