DEPARTMENT OF MATHEMATICS COLLOQUIUM

University of West Georgia

2:00 PM, FRIDAY, APRIL 21, 2006, BOYD 301

Speaker: **Prof. Michael V. Klibanov**
University of North Carolina at Charlotte ,

Title: **Carleman Estimates and Their Applications to Inverse and Control Problems**

Abstract.

Carleman estimates were discovered in a monumental work of Torsio Carleman in 1939, which was only 18 pages long and included only two references to the classical works of Holmgren and Hadamard. His goal was to extend the classical Holmgren uniqueness theorem for the case of non-analytic coefficients in the differential operator. This work (which was his best one, in my opinion) was forgotten for almost twenty years until Calderon has "re-discovered" it in 1958 and extended the result of Carleman to more general equations. Since then, hundreds or maybe even thousands publications are dedicated to this idea.

The traditional way of using Carleman estimates has been for proofs of uniqueness results for ill-posed Cauchy problems. Most popular examples are: (1) Cauchy problem for the elliptic equation when the Dirichlet and Neumann boundary data are given at a piece of the boundary only, (2) non-standard Cauchy problems for parabolic and hyperbolic equations when the Dirichlet and Neumann boundary data are given at a piece of the lateral boundary, but initial conditions at $t = 0$ are unknown. In 1973 S.P. Shishatskii has extended that idea to the proofs of conditional Holder stability results for these problems, which are ill-posed in the classical Hadamard sense.

The main contribution of the speaker is that he has adapted the idea of Carleman for proofs of global uniqueness and stability results for coefficient inverse problems (1981), see the original paper and many others in the pdf format at
http://www.math.uncc.edu/people/research/mklibanv.php3
The fundamental difference between an ill-posed Cauchy problem and a coefficient inverse problem is that the latter contains two unknowns in one equation: the solution of a PDE and the unknown coefficient, whereas the former contains only one unknown (solution of that PDE). Before this work only the so-called "local" uniqueness results were known for coefficient inverse problems. Since then many works are devoted to the idea of the presenter. Currently, this is the only method, which enables one to prove global uniqueness and stability results for NON-OVERDETERMINED coefficient inverse problems in dimension $n > 1$.

It was discovered later (1991, 93) by the speaker (with co-authors) that Carleman estimates can also be applied to obtain exact observability results in the control theory. These results lead (almost immediately) to the exact controllability results, one of the goals of this theory. Finally, and quite surprisingly, the idea was modified by him and Timonov (2003) for numerical studies of coefficient inverse problems. Namely, for the development of globally convergent numerical methods, unlike currently known locally convergent ones (e.g., linearization, Newton-like methods, least squares), etc. The major problem with those locally convergent methods is that they suffer from the problem of local minima of cost functionals. This problem, in turns, leads to unreliable solutions. So, the introduction of Carleman Weight Functions (which are involved in Carleman estimates) in the numerical scheme enables one overcome the problem of local minima.

If time allows, I will also present uniqueness results for the problem of recovering the function from the modulus of its Fourier transform.

REFERENCE. M.V. Klibanov and A. Timonov, *Carleman Estimates for Coefficient Inverse Problems and Numerical Applications*, VSP, Utrecht, 2004, see www.vsppub.com

All are welcome.