

APPLIED MATHEMATICS SEMINAR
Department of Mathematics
University of West Georgia
4:40 PM, MONDAY, OCTOBER 16, 2006, BOYD 304

Speaker: Dr. Vu Kim Tuan, UWG

Title: **UNCERTAINTY PRINCIPLES FOR THE HANKEL TRANSFORM**

Abstract

The Hankel transform of order $\nu > -1/2$ is defined as follows

$$f_\nu(x) := \int_0^\infty \sqrt{xy} J_\nu(xy) f(y) dy,$$

where $J_\nu(z)$ is the Bessel function of the first kind. If $f \in L_2(\mathbb{R}_+)$, then $f_\nu \in L_2(\mathbb{R}_+)$, and the Parseval formula $\|f\|_{L_2(\mathbb{R}_+)} = \|f_\nu\|_{L_2(\mathbb{R}_+)}$ holds. The following inequality for the Hankel transform

$$\|xf(x)\|_{L_2(\mathbb{R}_+)} \cdot \|xf_\nu(x)\|_{L_2(\mathbb{R}_+)} \geq (\nu + 1) \|f\|_{L_2(\mathbb{R}_+)}^2$$

has been established by Rösler (1999). It is an analogue of the classical Heisenberg-Weyl uncertainty principle for the Fourier transform. Roughly speaking, the uncertainty principle for the Fourier transform says that a function and its Fourier transform cannot both be sharply localized. That is, it is impossible for a nonzero function and its Fourier transform to be simultaneously small. "Smallness" had taken different interpretations in different contexts. Hardy, Cowling and Price, and Beurling showed such impossibility when "smallness" is interpreted as sharp pointwise or integrable decay. Benedicks, Slepian, Pollak and Landau, Donoho and Stark gave qualitative uncertainty principles for the Fourier transforms.

The purpose of this talk is to obtain uncertainty principles similar to Hardy's, Beurling's, Cowling-Price's, Gelfand-Shilov's, and Donoho-Stark's principles for the Hankel transform and generalized Fourier transforms arising from eigenfunctions expansions of Sturm-Liouville problems.

All are welcome.