DEPARTMENT OF MATHEMATICS COLLOQUIUM
UNIVERSITY OF WEST GEORGIA

4:00 PM, MONDAY, NOVEMBER 24, 2008, BOYD 304

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Title: **ADAPTIVE SAMPLING RECOVERY BASED ON QUASI-INTERPOLANT WAVELET REPRESENTATIONS**

We investigate a problem of approximate adaptive sampling recovery of functions on the interval $[0,1]$ expressing the adaptive choice of n sampled values of a function to be recovered, and of n terms from a given family of functions Φ. More precisely, for each function f on $[0,1]$, we choose a sequence $\xi = \{\xi^s\}_{s=1}^n$ of n points in $[0,1]$, a sequence $a = \{a_s\}_{s=1}^n$ of n functions defined on $\mathbb{R}^n$ and a sequence $\Phi_n = \{\varphi_{k_s}\}_{s=1}^n$ of n functions from a given family Φ. By this choice we define a (non-linear) sampling recovery method so that f is approximately recovered from the n sampled values $f(\xi^1), f(\xi^2), ..., f(\xi^n)$, by the n-term linear combination

$$S(f) = \ S(\xi, \Phi_n, a, f) := \ \sum_{s=1}^n a_s(f(\xi^1), ..., f(\xi^n))\varphi_{k_s}.$$

In searching an optimal sampling method, we study the quantity

$$\nu_n(f, \Phi)_q := \inf_{\Phi_n, \xi, a} \|f - S(\xi, \Phi_n, a, f)\|_q,$$

where the infimum is taken over all sequences $\xi = \{\xi^s\}_{s=1}^n$ of n points, $a = \{a_s\}_{s=1}^n$ of n functions defined on $\mathbb{R}^n$, and $\Phi_n = \{\varphi_{k_s}\}_{s=1}^n$ of n functions from Φ.

Let $U^\alpha_{p,\theta}$ be the unit ball in the Besov space $B^\alpha_{p,\theta}$, and $\mathbf{M}$ the set of the B-spline wavelets

$$M_{k,s}(x) := \ M(2^k x - s),$$

which do not vanish identically on $[0,1]$, where M is the symmetric B-spline of even order $2r > \alpha$ with knots at the points $-r, ..., -1, 0, 1, ..., r$. For $1 \le p, q \le \infty$, $0 < \theta \le \infty$ and $\alpha > 1/p$, we proved the following asymptotic order

$$\nu_n(U^\alpha_{p,\theta}, \mathbf{M})_q \ := \ \sup_{f \in U^\alpha_{p,\theta}} \nu_n(f, \mathbf{M})_q \ \asymp \ n^{-\alpha}.$$

An asymptotically optimal adaptive sampling recovery method for $\nu_n(U^\alpha_{p,\theta}, \mathbf{M})_q$ is constructed based on a quasi-interpolant wavelet representation of functions in terms of the B-splines $M_{k,s}$.

For $1 \le p < q \le \infty$, the asymptotic order of this asymptotically optimal sampling adaptive recovery method is better than the asymptotic order of any non-adaptive sampling recovery method of the form $R(H, \xi, f) := H(f(\xi^1), ..., f(\xi^n))$ with a fixed mapping $H : \mathbb{R}^n \to \mathbf{C}([0,1])$ and n fixed points $\xi = \{\xi^s\}_{s=1}^n$.