Review for Final

Dec 2010

1. Find the domain of \( f(x) = \frac{5}{x-1} \).

2. Find \((f \circ g)(x)\) where \( f(x) = 5x + 2 \), \( g(x) = 3x - 4 \).

3. Find the domain of \((f \circ g)(x)\) where \( f(x) = \frac{5}{x+4} \), \( g(x) = \frac{1}{x} \).

4. Find the inverse function for \( f(x) = \frac{2}{x} \).

5. Find the inverse function for \( x^3 - 1 \).

6. Find the vertical asymptote, if any, of the graph of the rational function \( f(x) = \frac{x+3}{x(x-3)} \).

7. Find the horizontal asymptote, of any, of the graph of the rational \( g(x) = \frac{15x^3}{5x^2+1} \).

8. Solve the polynomial inequality and then graph the solution set on a real number line for \( 3x^2 - 5x \leq 0 \).

9. Solve the polynomial inequality and graph the solution on the real number line for \( \frac{3x+5}{6-2x} \geq 0 \).
10. Use the properties of logarithms to expand as much as possible \( \log_8\left(\frac{64}{\sqrt[3]{x+1}}\right) \).

11. Use the properties of logarithms to expand as much as possible \( \log_2\left(\frac{xy^4}{16}\right)^{1/5} \).

12. Use properties of logarithmic expression to condense as much as possible \( \frac{1}{3}(\log_4 x - \log_4 y) \).

13. Solve the exponential equation and give your answers up to two decimal places where necessary. \( e^{4x} - 3e^{2x} - 18 = 0 \).

14. Solve the exponential equation and give your answers up to two decimal places where necessary. \( 4e^{7x} = 10273 \).

15. Solve the logarithmic equation and give your answers up to two decimal places where necessary. Be sure to reject any solution not in the domain of the original logarithmic equation. \( \log(x + 5) + \log_6 x = 2 \).

16. Solve the logarithmic equation and give your answers up to two decimal places where necessary. Be sure to reject any solution not in the domain of the original logarithmic equation. \( \log_2(x - 1) - \log_2(x + 3) = \log_2(\frac{1}{x}) \).

17. An artifact originally had 16 grams of carbon-14 present. The decay model \( A = 16e^{-0.000121t} \) describes the amount of carbon-14 present after \( t \) years. Use this model find how many grams of carbon-14 will be present in 11430 years?

18. Use the exponential decay model \( A = A_0e^{-0.000121t} \). Skeletons were found at a construction site in San Francisco in 1989. The skeletons contained 88% of the expected carbon-14 found in a living person. In 1989, how old were the skeletons?
19. A bird species in danger of extinction has a population that is decreasing exponentially \( A = A_0e^{kt} \). Five years ago the population was 1400 and today only 1000 of the birds are alive. Once the population drops below 100, the situation will be irreversible. When will this happen?

20. convert the angle \(-270^\circ\) to radians.

21. Convert the angle \(\frac{\pi}{5}\) to degrees.

22. Find a positive angle less than 360\(^\circ\) or 2\(\pi\) that is coterminal with \(\frac{17\pi}{5}\).

23. The Washington Monument is 555 feet high. If you stand one quarter of a mile, or 1320 feet, from the base of the monument and look to the top, find the angle of elevation to the nearest degree.

24. A road is inclined at a angle of 5\(^\circ\). After driving 5000 feet along this road, fund the driver’s increase in altitude. Round to the nearest foot.

25. Find the exact value of each of the remaining trigonometric functions of \(\theta\), give \(\cos \theta = \frac{-4}{\sqrt{5}}\), \(\theta\) is in quadrant III.

26. Find the reference angle for \(-150^\circ\).

27. A point \((-4, 3)\) on the terminal side of an angle \(\theta\) is given. Find the exact value of each of the six trigonometric functions of \(\theta\).

28. Determine the amplitude and period of the function and then graph one period for \(y = 3\sin(\pi x + 2)\).

29. Determine the amplitude and period of the function and then graph one period for \(y = \cos 2x\).
30. Use a right triangle to write each expression as an algebraic expression. Assume that \(x\) is positive and the given inverse function is defined in \(x\) for \(\cos(\sin^{-1} \frac{1}{2})\).

31. The tallest television transmitting tower in the world is in North Dakota. From a point on level ground 5280 feet (one mile) from the base of the tower, the angle of elevation is 21.3°. Approximate the height of the tower to the nearest foot.

32. A boat leaves the entrance to a harbor and travels 150 miles on a bearing of N53°E. How many miles north and how many miles east from the harbor has the boat traveled?

33. Verify \(\tan x + \cot x = \sec x \csc x\).

34. Verify \(\frac{\sin x}{\tan x} + \frac{\cos x}{\sec x} = 1\).

35. Use the sum and difference formulas to find \(\sin(45° - 30°)\).

36. Verify \(\sin(x + y) + \sin(x - y) = 2\sin x \cos y\).

37. Verify \(\sin^2 x + \cos 2x = \cos^2 x\).

38. Find all solutions for \(\cos x = -\frac{1}{2}\).

39. Find all solutions in the interval \([0, 2\pi)\) for \(\sin 2x = \frac{\sqrt{3}}{2}\).

40. A plane leaves airport A and travels 580 miles to airport B on a bearing of N34°E. The plane later leaves airport B and travels to airport C 400 miles away in a bearing S74°E. Find the distance from airport A
to airport C to the nearest tenth of a mile.

41. Solve the triangle with \( a = 5, b = 7, c = 10 \).

42. Convert the polar equation to a rectangular equation \( r \sin \theta = 3 \).

43. Find the unit vector that has the same direction as \( \mathbf{v} = 6\mathbf{i} \).

44. Let \( \mathbf{u} = 2\mathbf{i} - 5\mathbf{j} \) and \( \mathbf{v} = -3\mathbf{i} + 7\mathbf{j} \), find \( \mathbf{u} - \mathbf{v} \).

45. A wine company needs to blend a California wine with 5% alcohol content and a French wine with a 9% alcohol content to obtain 200 gallons of wine with a 7% alcohol content. How many gallons of each kind of wine be used?

46. Solve the system

\[
\begin{align*}
x + y + 2z &= 11 \\
x + y + 3z &= 14 \\
x + 2y - z &= 5
\end{align*}
\]

47. A certain brand of razor blades comes in packages of 6, 12, and 24 blades, costing $2, $3 and $4 per package respectively. A store sold 12 packages containing a total of 162 razor blades and took in $35. How many packages of each type were sold?

48. Write the partial fraction decomposition of \( \frac{4x^2-7x-3}{x^3-x} \).

49. Write the partial fraction decomposition of \( \frac{3x^2+49}{x(x+7)^2} \).
50. The difference between the squares of two numbers is 3. Twice the square of the first number increased by the square of the second number is 9. Find the numbers.

51. \( A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 9 \\ 0 & 7 \end{bmatrix}, \) Find \( 3A + 2B. \)

52. \( A = \begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -2 \\ -1 & 6 \end{bmatrix}, \) Find \( BA. \)

**Answers**

1. \( (−∞, 0) \cup (0, 4) \cup (4, ∞). \)

2. \( (f \circ g)(x) = 15x − 18. \)

3. \( \frac{5x}{1+4x}. \)

4. \( \frac{2}{x}. \)

5. \( (x + 1)^{1/3}. \)

6. \( x = 0, x = 3. \)

7. no horizontal asymptote.

8. \( [0, 5/3]. \)

9. \( [-5/3, 3]. \)
10. \( 2 - \frac{1}{2} \log_8(x + 1). \)

11. \( \frac{1}{5} \log_2 x + \frac{2}{5} \log_2 y - \frac{4}{5}. \)

12. \( \log_4\left(\frac{x}{y}\right)^{1/3}. \)

13. 0.90

14. 1.12

15. \{4\}

16. \{3\}.

17. 4 grams

18. 1056 years.

19. About 39 years after the year the population was 1400.

20. \( -\frac{3\pi}{2} \) radian.

21. 20°.

22. \( \frac{7\pi}{5} \).

23. 23°.
24. 436 feet.

25. \(\sin \theta = -\frac{4}{5}\), \(\tan \theta = \frac{4}{3}\).

26. 30°.

27. \(\sin \theta = \frac{3}{5}\), \(\cos \theta = -\frac{4}{5}\).

28. 3, 2; \(-\frac{2}{\pi}\).

29. 1, \(\pi\).

30. \(\frac{\sqrt{x^2 - 1}}{x}\).

31. 2059 feet

32. 90 mi north and 120 mi east.

33. –

34. –

35. \(\frac{\sqrt{5} - \sqrt{2}}{4}\).

36. –.

37. –.
38. \( x = \frac{2\pi}{3} + 2n\pi \) or \( x = \frac{4\pi}{3} + 2n\pi \).

39. \( \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3} \).

40. About 799.9 mi.

41. \( C = 112^\circ, A = 28^\circ, B = 40^\circ \).

42. \( y = 3 \).

43. i.

44. \( 5i - 12j \).

45. California 100 gallon, French 100 gallon.

46. \( \{(2, 3, 3)\} \).

47. 5 packages of 6, 3 packages of 12 and 4 packages of 24.

48. \( \frac{3}{x} + \frac{4}{x+1} - \frac{3}{x-1} \).

49. \( \frac{1}{x} + \frac{2}{x+7} - \frac{28}{(x+7)x} \).

50. 2 and 1, 2 and -1, -2 and 1 or -2 and -1.

51. \( \begin{bmatrix} 22 & 21 \\ 9 & 20 \end{bmatrix} \).

52. \( \begin{bmatrix} -7 & 3 \\ 29 & 15 \end{bmatrix} \).