Exercise Set 1.4

Q19 \((P \land \sim Q \land R) \lor (P \land \sim Q \land \sim R) \lor (\sim P \land Q \land \sim R)\) (Note that this can be simplified to \((P \land \sim Q) \lor (\sim P \land Q \land \sim R)\).)

Exercise Set 2.1

Q15d \(\forall x, y \in \mathbb{Z}\) if \(x\) and \(y\) are odd then \(x.y\) is odd.

Q16b \(\forall x,\) if \(x\) is a computer science student, then \(x\) needs to take assembly language programming. \(\forall\) computer science students \(x,\) \(x\) needs to take assembly language programming.

Q28d True. If it were false we would need to exhibit a number belonging to \(D\) which is of the form \(y_2\), where \(y\) is replaced by a digit other than 3 or 4. But the only element of \(D\) which is of the form \(y_2\) is 32. So we cannot find a counter example. Thus the statement is true.

Q32 Let \(p\) represent the statement \(n\ is\ prime\), \(o\ represent\ the\ statement\ \(n\ is\ odd\), \(t\ represent\ the\ statement\ \(n = 2\). Then the statement form is \(p \rightarrow (o \lor t)\)

The negation of this statement form is
\[ \sim (p \rightarrow (o \lor t)) \equiv (\sim p \lor (o \lor t)) \equiv p \land \sim (o \lor t) \equiv p \land \sim o \land \sim t \]

Thus the negation is
\[ \exists n \in \mathbb{Z}, \text{such that } n \text{ is prime and } n \text{ is not odd and } n \neq 2. \]

**Exercise Set 2.2**

Q4 There exists a book, which all people have read.

Negation: \( \forall \) books \( b \), \( \exists \) a person \( p \) such that \( p \) has not read \( b \).

Equivalently: Given any book, there exists a person who has not read that book.

Q8 There exists a real number \( x \) such that for all real numbers \( y, x + y = 0 \).

Negation: \( \forall x \in \mathbb{R}, \exists y \in \mathbb{R} \) such that \( x + y \neq 0 \)

Equivalently: Given any real number \( x \) there exists a real number \( y \) such that \( x + y \neq 0 \)

Q44 b) Let \( P(x) \) be the statement that \( x \) is positive and \( Q(x) \) be the statement that \( x \) is negative.

Then \( \exists x \in \mathbb{R}, (P(x) \land Q(x)) \) means there exists a real number which is both positive and negative. This is a false statement (note 0 is neither positive or negative).

But \( (\exists x \in \mathbb{R}, P(x)) \land (\exists x \in \mathbb{R}, Q(x)) \) means there exists a real number which is positive and there exists a real number which is negative. This is a true statement, for example, 2 is positive and -100 is negative. Hence these two statements do not have the same truth values.