Exercise Set 5.1

Q6
a) Yes, $3 \in \{1, 2, 33\}$.  b) No, $1 \not\in \{1\}$.

Q10  a) Yes, $\mathbb{Z}^+ \subseteq \mathbb{Q}$.

b) No, as $-\sqrt{2} \in \mathbb{R}^-$, but $-\sqrt{2} \not\in \mathbb{Q}$.

c) No, $\frac{1}{2} \in \mathbb{Q}$ but $\frac{1}{2} \not\in \mathbb{Z}$.

d) No, $0 \in \mathbb{Z}$, but $0 \not\in (\mathbb{Z}^- \cup \mathbb{Z}^+)$.

Q15

(a) $A \cap B$

(b) $B \cup C$
Chapter 5.2

1) Prove that \( A \subseteq B \) if and only if \( A \cap B = A \)

Let \( p \) correspond to \( x \in A \) and let \( q \) correspond to \( x \in B \). Then the above statement is equivalent to \( (p \to q) \iff ((p \land q) \iff p) \)

The truth table for this statement form is:

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( (p \to q) )</th>
<th>( (p \land q) )</th>
<th>( (p \iff q) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T )</td>
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<td>( T )</td>
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<td>( T )</td>
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<tr>
<td>( T )</td>
<td>( F )</td>
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<td>( F )</td>
<td>( T )</td>
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<tr>
<td>( F )</td>
<td>( F )</td>
<td>( T )</td>
<td>( F )</td>
<td>( T )</td>
</tr>
</tbody>
</table>

Since the statement form is a tautology the set law is true for all sets \( A \) and \( B \).
Exercise Set 5.3

Q40 First note that $A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3)\}$. So

$$\mathcal{P}(A \times B) = \{\emptyset, \{(1, 2)\}, \{(1, 3)\}, \{(2, 2)\}, \{(2, 3)\}, \{(1, 2), (1, 3)\},$$
$$\{(1, 2), (2, 2)\}, \{(1, 2), (2, 3)\}, \{(1, 3), (2, 2)\}, \{(1, 3), (2, 3)\},$$
$$\{(2, 2), (2, 3)\}, \{(1, 2), (1, 3), (2, 2)\}, \{(1, 2), (1, 3), (2, 3)\},$$
$$\{(1, 2), (2, 2), (2, 3)\}, \{(1, 3), (2, 2), (2, 3)\},$$
$$\{(1, 2), (1, 3), (2, 2), (2, 3)\}\}.$$

Q42 a) $\mathcal{P}(\emptyset) = \{\emptyset\}$.

b) $\mathcal{P}(\mathcal{P}(\emptyset)) = \{\emptyset, \{\emptyset\}\}$, so $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset))) = \{\emptyset, \{\emptyset, \{\emptyset\}\}\}$.

Q45 No, as $S_a = \{\{a\}, \{a, b\}, \{a, c\}\}$, $S_b = \{\{b\}, \{a, b\}, \{b, c\}\}$ and $S_a \cap S_b = \{\{a, b\}\} \neq \emptyset$.

Exercise Set 11.1

Q4

Q9 i) $e_1, e_2, e_7$ ii) $v_1, v_2$ iii) $e_2, e_7$ iv) $e_1, e_3$ v) $e_4, e_5$ vi) $v_4$

vii) $\deg(v_3) = 2$ viii) total degree is 14.

Q21

Q23 Let $v$ represent the number of vertices and since the sum of the degrees of the vertices must equal twice the number of edges, we have $3v = 18$. So $v = 6$. Now the corresponding graph is:
Q46 Label the vertices according to the following scheme.

<table>
<thead>
<tr>
<th>Label</th>
<th>Committee</th>
</tr>
</thead>
<tbody>
<tr>
<td>UE</td>
<td>Undergraduate Education</td>
</tr>
<tr>
<td>GE</td>
<td>Graduate Education</td>
</tr>
<tr>
<td>C</td>
<td>Colloquium</td>
</tr>
<tr>
<td>L</td>
<td>Library</td>
</tr>
<tr>
<td>H</td>
<td>Hiring</td>
</tr>
<tr>
<td>P</td>
<td>Personnel</td>
</tr>
</tbody>
</table>

The committee structure corresponds to the graph on the left which can be also represented by the tripartite graph on the right.

Now it is clear that committee GE and C should meet at some time, as should UE and P, and L and H.