Exercise Set 10.1

Q5  a) Yes as $3 \mid (10 - 1)$. Yes as $3 \mid (1 - 10)$. Yes as $3 \mid (2 - 2)$. No as $3 \nmid (8 - 1)$.
b) Five possible examples are $3T0$, $6T0$, $-6T0$, $0T0$, $-99T0$.
c) Five possible examples are $1T1$, $-5T1$, $4T1$, $100T1$, $10T1$.
d) Five possible examples are $2T2$, $5T2$, $-10T2$, $-100T2$, $-4T2$.

Q10  a) No as $\{a\} \cap \{c\} = \emptyset$.
b) Yes as $\{a, b\} \cap \{b, c\} = \{b\} \neq \emptyset$.
c) Yes as $\{a, b\} \cap \{a, b, c\} = \{a, b\} \neq \emptyset$.

Q26

![Graph Image]

Exercise Set 10.2

Q17 $\forall m, n \in \mathbb{Z} \quad m0n \Leftrightarrow m - n$ is odd.

Not reflexive since $m - m = 0$ which is even $\forall m \in \mathbb{Z}$.

Yes, symmetric. To prove this, assume $m0n$ that is $m - n = 2p + 1$ for some $p$. It follows that $n - m = -2p - 1 = 2(-p - 1) + 1$, which is odd. So $n0m$.

Not transitive as $22 - 1$ is odd and $1 - 22$ is odd, but $22 - 22$ is even.

Exercise Set 10.3

Q21 b) $\forall m, n \in \mathbb{Z}, mDn \Leftrightarrow m^2 \equiv n^2(mod 3)$.

We shall prove this is an equivalence relation.

First note $\forall m, \ m^2 - m^2 = 0 = 0 \times 3$. Hence $3 \mid (m^2 - m^2)$. Thus $m^2 \equiv m^2(mod 3)$ and the relation is reflexive.

Next we shall prove $\forall m, n \in \mathbb{Z}$ if $m^2 \equiv n^2(mod 3)$ then $n^2 \equiv m^2(mod 3)$. If $m^2 \equiv n^2(mod 3)$ then $3 \mid (m^2 - n^2)$ or $m^2 - n^2 = 3k$ where $k \in \mathbb{Z}$. But this implies
that $n^2 - m^2 = 3l$ where $l = -k \in \mathbb{Z}$. Thus $3|(n^2 - m^2)$ and so $n^2 \equiv m^2 \pmod{3}$. So the relation is symmetric.

Next we must prove the relation is transitive. That is, we must prove $\forall m, n, p \in \mathbb{Z}$ if $m^2 \equiv n^2 \pmod{3}$ and $n^2 \equiv p^2 \pmod{3}$ then $m^2 \equiv p^2 \pmod{3}$. So assume that $m^2 \equiv n^2 \pmod{3}$ and $n^2 \equiv p^2 \pmod{3}$. Implying that $m^2 - n^2 = k3$ and $n^2 - p^2 = l3$ where $k, l \in \mathbb{Z}$. By adding these equations we get $m^2 - n^2 + n^2 - p^2 = k3 + l3 = (k + l)3$, where $k + l \in \mathbb{Z}$. Hence $m^2 \equiv p^2 \pmod{3}$ and the relation is transitive.

Since the relation is reflexive, symmetric and transitive, it is an equivalence relation.

Q16 Equivalence classes are:

- $[0] = \{4p | p \in \mathbb{Z}\}$;
- $[1] = \{4p + 1 | p \in \mathbb{Z}\}$;
- $[2] = \{4p + 2 | p \in \mathbb{Z}\}$; and
- $[3] = \{4p + 3 | p \in \mathbb{Z}\}$.

Q21 Equivalence classes are:

- $[0] = \{q | q = 3p, \text{ for some } p \in \mathbb{Z}\}$
- $[1] = \{q | q = 3p + 1 \text{ or } q = 3p + 2, \text{ for some } p \in \mathbb{Z}\}$.

**Exercise Set 10.5**

Q1  

- c) From the diagram below we can see that this relation is antisymmetric.

![Diagram](image)

- d) This relation is not antisymmetric, as $(1, 2), (2, 1) \in R_4$ but $1 \neq 2$.

Q9 The relation $R$ is defined by $x, y \in \mathbb{R}$, $xRy \iff x^2 \leq y^2$.

Since $x^2 \leq x^2$, $xRx$ and $R$ is reflexive.

Assume $xRy$ and $yRz$, then $x^2 \leq y^2$ and $y^2 \leq z^2$, so $x^2 \leq z^2$ and $R$ is transitive.

Assume $x, y \in \mathbb{R}$ and $xRy$ and $yRx$. Then $x^2 \leq y^2$ and $y^2 \leq x^2$. Thus $x^2 = y^2$. But $x$ is not necessarily equal to $y$. For example $x = -2$ and $y = 2$. Consequently $R$ is not antisymmetric.

Thus $R$ is not a partial order.