Exercise Set 7.3

Q3  b) \( G \) is not one-to-one as \( G(a) = G(d) = y \) but \( a \neq d \).
    \( G \) is not onto as there exists no element \( m \) of \( X \) such that \( G(m) = z \).

Q5  b) One possible example is \( g = \{(1, 1), (2, 2), (3, 1)\} \).
    c) One possible example is \( h = \{(1, 1), (2, 1), (3, 1)\} \).
    d) One possible example is \( k = \{(1, 2), (2, 3), (3, 1)\} \).

Q14 \( f(x) = \frac{2x + 1}{x}, x \neq 0 \). Assume

\[
\begin{align*}
f(x_1) &= f(x_2) \\
\Rightarrow \quad \frac{2x_1 + 1}{x_1} &= \frac{2x_2 + 1}{x_2} \\
\Rightarrow \quad (2x_1 + 1)x_2 &= (2x_2 + 1)x_1 \\
\Rightarrow \quad 2x_1x_2 + x_2 &= 2x_2x_1 + x_1 \\
\Rightarrow \quad x_2 &= x_1
\end{align*}
\]

Hence \( f \) is one-to-one.

Q44 Note that if \( y = \frac{2x + 1}{x} \) then \( y = 2 + \frac{1}{x} \) or \( x = \frac{1}{y - 2} \) provided \( y \neq 2 \). So for all \( y \in \mathbb{R} \setminus \{2\} \) there exists an \( x \in \mathbb{R} \) such that \( f(x) = y \), namely \( x = \frac{1}{y - 2} \). Hence \( f(x) \) is onto.

Since \( f(x) \) is one-to-one and onto, \( f(x) \) is a one-to-one correspondence. The inverse function is

\[
f^{-1} : \mathbb{R} \setminus \{2\} \Rightarrow \mathbb{R} \setminus \{0\} \\
f^{-1}(y) = \frac{1}{y - 2} .
\]
Exercise Set 8.1

Q6 \( t_k = t_{k-1} + 2t_{k-2} \), \( t_0 = -1 \), \( t_1 = 1 \). So \( t_2 = 1 + 2(-1) = -1 \) and \( t_3 = -1 + 2(1) = 1 \).

Q10 The sequence \( b_0, b_1, b_2 \ldots \), is defined by \( b_n = 5^n \), so \( 5b_{k-1} = 5.5^{k-1} = 5^k = b_k \). Hence \( b_k = 5b_{k-1} \) for all \( k \geq 1 \).

Exercise Set 8.3

Q12 \( e_k = 0e_{k-1} + 9e_{k-2} \) for all \( k \geq 2 \) with \( e_0 = 0, e_1 = 2 \). Therefore the characteristic equation is \( t^2 - 9 = 0 \). So \( (t - 3)(t + 3) = 0 \) and \( t = -3 \) or \( t = 3 \). Hence \( e_n = C(3)^n + D(-3)^n \). Since \( e_0 = 0 \) and \( e_0 = C(3)^0 + D(-3)^0 \), we have

\[ 0 = C + D. \]

Since \( e_1 = 2 \) and \( e_1 = C(3)^1 + D(-3)^1 \), we have

\[ 2 = 3C - 3D. \]

Substituting \( C = -D \) into \( 2 = 3C - 3D \) we get \( 2 = 3C + 3C \) or \( C = \frac{1}{3} \). This in turn implies that \( D = -\frac{1}{3} \). Thus \( e_n = \frac{1}{3}(3)^n - \frac{1}{3}(-3)^n = (3)^{n-1} + (-3)^{n-1} = 3^{n-1}(1 + (-1)^{n-1}) \). So summarising

\[ e_n = 3^{n-1}(1 + (-1)^{n-1}). \]