**CHAPTER 23 | Options**

**Need to Absorb, Read, Do**

- Need to Absorb - Definitions and terms related to options. Be able to compute the value of a call or put using a 1-period binomial option pricing model. You must be able to compute the value of assets, calls, puts, and bonds using the put-call parity formula. Know the impact of changes to inputs in the Black-Scholes Options Pricing Model. For Real Options, be able to identify the type of option, who is long, who is short, the underlying asset, the exercise price, and sometimes the premium.

- Need to Read - Read the Chapter and my Real Options Handout

- Need to Do - You should download my handout on real options. Make 100 on the Chapter 23 Quiz. Questions and Problems that you should be able to answer: 1-8, 10-18, and 21-25. Some of the problems take many steps to solve. Note, many of the problems may require extended periods of quiet contemplation. You will probably need my videos in addition to the textbook to understand this material. If my materials and the textbook are not sufficient, the Chicago Board Options Exchange (CBOE) has some basic tutorials on financial options.

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Options

Long Implies the **right** but not the **obligation**…

Short Implies the **obligation** but not the **right**…

*This chapter explores various options and their payoff structure.*

Strategic NPV = Passive NPV + Present value of options arising from the active management of the firm’s investment opportunities

Basic Options

**Long a Call Option**

The right to buy an asset at a specified exercise price on or before the exercise date

**Long a Put Option**

The right to sell an asset at a specified exercise price on or before the exercise date

Option Obligations

<table>
<thead>
<tr>
<th>Buyer</th>
<th>Seller</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call option</td>
<td>Right to buy asset</td>
</tr>
<tr>
<td>Put option</td>
<td>Right to sell asset</td>
</tr>
<tr>
<td></td>
<td>Obligation to sell asset</td>
</tr>
<tr>
<td></td>
<td>Obligation to buy asset</td>
</tr>
</tbody>
</table>

Option Value

**Option Value:** The value of an option at expiration is a function of the stock price and the exercise price.

**Long Call Option**

<table>
<thead>
<tr>
<th>Stock Price at Expiration</th>
<th>Value of Call at Expiration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greater than exercise price</td>
<td>Stock price – exercise price</td>
</tr>
<tr>
<td>Less than exercise price</td>
<td>Zero</td>
</tr>
</tbody>
</table>

**Long Put Option**

<table>
<thead>
<tr>
<th>Stock Price at Expiration</th>
<th>Value of Put at Expiration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greater than exercise price</td>
<td>Zero</td>
</tr>
<tr>
<td>Less than exercise price</td>
<td>Exercise price – stock price</td>
</tr>
</tbody>
</table>
Options: Basic Terminology

• The price at which the exchange is made is called the strike or exercise price.
• Exercising the option involves exchanging cash for the underlying asset.
• When a call option is exercised by the holder, the underlying asset is purchased by paying the exercise price.
• When a put option is exercised by the holder, the underlying asset is sold for the exercise price.

Options: Basic Terminology

• An option is in-the-money if exercising it provides an advantage over buying or selling the underlying asset in the open market.
  • A call option is in-the-money if the open market price of the underlying asset is more than the exercise price.
  • A put option is in-the-money if the open market price of the underlying asset is less than the exercise price.

Options: Basic Terminology

• An option is out-of-the-money if buying or selling the underlying asset in the open market provides an advantage over exercising it.
  • A call option is out-of-the-money if the open market price of the underlying asset is less than the exercise price.
  • A put option is out-of-the-money if the open market price of the underlying asset is more than the exercise price.

Options: Basic Terminology

• The exercise value is the amount of advantage that an in-the-money option provides over buying or selling the asset at the open market price.
  • The exercise value of out-of-the-money options is zero.
Options: Basic Terminology

- Options have a finite life.
  - Upon expiration, the option contract is null and void.
- An American option can be exercised at any time prior to expiration.
- A European option can only be exercised at maturity, not before.

Call Option Value

Call Option Value

<table>
<thead>
<tr>
<th>If the following variables increase, ...</th>
<th>... the value of a call option will</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock price</td>
<td>Increase</td>
</tr>
<tr>
<td>Exercise price</td>
<td>Decrease</td>
</tr>
<tr>
<td>Interest rate</td>
<td>Increase</td>
</tr>
<tr>
<td>Time to expiration</td>
<td>Increase</td>
</tr>
<tr>
<td>Volatility of stock price</td>
<td>Increase</td>
</tr>
</tbody>
</table>

Black-Scholes Option Pricing Model

\[ \text{Option}_C = P_s[N(d_1)] - S[N(d_2)]e^{-rt} \]
**Binomial Model - Portfolio Replication Method**

1. Calculate the option delta =
   
   Spread of possible option prices
   Spread of possible share prices

2. Determine the value of the bank loan that would have the same payoff as the out-of-the-money option.

**Binomial Model - Risk Neutral Method**

1. At each point in time, determine the two possible outcomes for the value of the asset.
2. Recognize that the value of an out-of-the-money option is 0 and an in-the-money option is \((P_0 - X)\) for a call and \((X - P_0)\) for a put.
3. If an investor is risk-neutral the expected return = interest rate.
4. Solve for the probability of a price rise using the following formula:
   
   \[
   \text{Risk-free interest rate} = \left[ \left( \text{Prob. Of rise} \times \% \text{ Increase in price} \right) + \left( 1 - \text{Prob. of rise} \right) \times \% \text{ Decrease in price} \right]
   \]
5. Solve for the expected future value of the option using the following formula:
   
   \[
   \text{Expected FV of option} = \left[ \left( \text{Prob. of rise} \times \text{Option value in case of price increase} \right) + \left( 1 - \text{Prob. of rise} \times \text{Option value in case of price decrease} \right) \right]
   \]
6. Current value of the option = \[
   \frac{\text{Expected FV of the option}}{1 + \text{Interest rate}}
   \]

**Put-Call Parity**

- Conceptually
  
  ➔ The RIGHT to buy a stock together with the ABILITY to buy it. (A call option plus the present value of the exercise price.)

  Should be worth the same as
   
   ➔ The RIGHT to sell a stock together with the ABILITY to sell it. (A put option written on a stock that you own.)

- This concept is know as **Put-Call Parity**

- **Stock + Put = Call + PV of Exercise Price**

**Hidden Options and Options on Real Assets**

**Real Options** - Options embedded in real assets

- Option to Expand
- Option to Abandon

Price-setting option  Timing Option
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Option Value: Example

Option values given an exercise price of $720

<table>
<thead>
<tr>
<th>Stock Price</th>
<th>$600</th>
<th>$650</th>
<th>$720</th>
<th>$780</th>
<th>$840</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call Value</td>
<td>$0</td>
<td>0</td>
<td>0</td>
<td>60</td>
<td>120</td>
</tr>
<tr>
<td>Put Value</td>
<td>$120</td>
<td>60</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

What are the payoff limits for call option buyers? Sellers?

What are the payoff limits for put option buyers? Sellers?

Summary of Long and Short

**If Long**
- Limited Losses
- Potential Unlimited Gains
- Reduces Risk
- Reduces Return
- Pays a Premium upfront
- Usually Loses Money

**If Short**
- Limited Gains
- Potential Unlimited Losses
- Increases Risk
- Increases Return
- Receive a Premium upfront
- Usually Makes Money

Call Option Value

Call option value (buyer) given a $720 exercise price.
Call Option Profit
Profit (buyer): Current Price - Exercise Price - Cost of Call
$20 call option (buyer) given a $720 exercise price:

Share Price
Call option value

$100

Profit = ($840 - $720) - $20 = $100

$20 840

Call Option Value
Call option payoff (seller) given a $720 exercise price.

Share Price
Call option payoff

$-120

720 840

Call Option Profit
Profit (seller): Exercise Price - Current Price - Cost of Call
$20 call option (seller) given a $720 exercise price:

Share Price
Call option payoff

$-100 $-120

Profit = $720 - $840 + $20 = -$100

Call Option: Example
How much must the stock be worth at expiration in order for a call holder to break even if the exercise price is $50 and the call premium was $4?

Call Profit = stock value - exercise price - premium

$0 = stock value - $50 - $4

\[ \therefore Stock\ Value = $54 \]
Put Option Value

Put option value (buyer) given a $720 exercise price:

Put option value

$120

600 720

Share Price

Put Option Profit

Profit (buyer): Exercise Price - Current Price - Cost of Put

$30 put option (buyer) given a $720 exercise price:

Profit = $720 - $600 - $30 = $90

Put Option Value

Put option payoff (seller) given a $720 exercise price.

Put option $ payoff

-120

600 720

Share Price

Put Option Profit

Profit (Seller): Current Price - Exercise Price + Cost of Put

$30 put option (seller) given a $720 exercise price.

Profit = $600 - $720 + $30 = -$90
Put Options: Example

What is your return on exercising a put option which was purchased for $10 with an exercise price of $85? The stock price at expiration is $81.

\[ \text{Proceeds} = \text{exercise price} - \text{stock price} \]
\[ \therefore \text{Proceeds} = \$85 - \$81 = \$4 \]

\[ \text{Profit} = \text{proceeds} - \text{initial investment} \]
\[ \text{Profit} = \$4 - \$10 = -\$6 \]
\[ \therefore \text{Return} = \frac{-\$6}{\$10} = -60\% \]

Option Hedging Strategy

Protective Put:

\[ \begin{align*}
\text{Share Price} & \quad \text{Position Value} \\
\text{Protective Put} & \quad \text{Long Stock} \\
\text{Long Put} & \quad \text{Protective Put}
\end{align*} \]

Call Option Value:

Upper and Lower Limits

\[ \text{Stock Price} \]
\[ \text{Upper Limit} \]
\[ \text{Lower Limit} \]

\[ \text{(Stock price - exercise price)} \text{ or 0 whichever is higher} \]
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Black-Scholes Option Pricing Model

\[ \text{Option}_C = P_d [N(d_1)] - S[N(d_2)]e^{-rt} \]
A Simple (Binomial) Model of Option Valuation (Page 308)

Consider the call option sold by Carla to Alex. The option has one year to maturity, and an exercise price of $65,000. At the time of the sale, the land value was $60,000. Assume that in one year, the land value can be either $67,870 (up by 13.12%) or $48,000 (down by 20%) in one year. If the risk-free rate is 4% per year, and assuming that the land should yield this rate, what is the value of the call option on this land?

Risk Neutral Method
1. At each point in time, determine the two possible outcomes for the value of the asset.
2. Recognize that the value of an out-of-the-money option is 0 and an in-the-money option is \((P_0 - X)\) for a call and \((X - P_0)\) for a put.
3. If an investor is risk-neutral the expected return = interest rate.
4. Solve for the probability of a price rise using the following formula: Risk-free interest rate = \([\text{(Prob. of rise)} \times \% \text{ Increase in price}] + [\text{(1-Prob. of rise)} \times \% \text{ Decrease in price}]\)
5. Solve for the expected future value of the option using the following formula: Expected FV of option = \([\text{(Prob. of rise)} \times \text{Option value in case of price increase}] + [\text{(1-Prob. of rise)} \times \text{Option value in case of price decrease}]\)
6. Current value of the option = \(\frac{\text{Expected FV of the option}}{1 + \text{Interest rate}}\)

If the land value goes up to $67,870, the value of the call option would be $67,870 – $65,000 or $2,870.

If the land value declines to $48,000, the call option would expire out-of-the-money and be worthless.
A Simple (Binomial) Model of Option Valuation

- Let \( p_u \) denote the probability that land will go up in value. Then the probability that the land value will decline is \( 1 - p_u \).
- Since the land is to yield the riskless rate of return of 4%,
  \[ 4\% = p_u \times (13.12\%) + (1 - p_u) \times (-20.0\%) \]
- Solving this for \( p_u \), we get \( p_u = 72.5\% \).
- The probability of a decline in land value is thus 27.5%.

A Simple (Binomial) Model of Option Valuation

- The expected value of the call option at maturity is:
  \[ p_u \times \text{Value of call if land goes up} + (1 - p_u) \times \text{Value of call if land goes down} \]
  \[ = (0.725) \times ($2,870) + (0.275) \times ($0) \]
  \[ = $2,080 \]
- The present value of this is the value of the call option today:
  \[ $2,080/1.04 = $2,000 \]

Arbitrage Option Valuation

- We can get the same result with a completely different insight.
- We can replicate the payoff of the call option with a levered position in the land.
- That levered real estate portfolio can be valued.
- Once we have that number, we can back out the value of the call option.

Portfolio Replication Method

1. Calculate the option delta = \( \frac{\text{Spread of possible option prices}}{\text{Spread of possible share prices}} \)
   Example: A stock which currently sells for $100 with an exercise price of $110. In the next period, it could rise to $125($125-$100 = $15 option value) or it could fall to $80($0 option value). The option delta = \( \frac{15-0}{125-80} = 1/3 \). This says that three calls are replicated by using one share and borrowing, or the value of 3 calls = value of 1 share - bank loan.

2. Determine the value of the bank loan that would have the same payoff as the out-of-the-money option, in our case a future value of $80. If the interest rate is 10%, then the bank loan would equal $72.73. (In one year, you repay principal of $72.73 + $7.27 interest) = $80. Thus, the value of 3 calls = $100-$73.73 = $27.27, and the value of 1 call = $9.09.
1. A call option on this land with exercise price of $65,000 will have the following payoffs.
2. We can replicate the payoffs of the call option. With a levered position in the stock.

\[
\begin{align*}
\text{Land}_{t=0} & \quad \text{Land}_{t=1} & \quad \text{Call}_{t=1} \\
& \quad $67,870 & \quad $2,870 \\
$60,000 & \quad $48,000 & \quad $0
\end{align*}
\]

Borrow the present value of $48,000 today and buy the land.
The net payoff for this levered equity portfolio in one period is either $19,870 or $0.

\[
\begin{align*}
\text{Land}_{t=0} & \quad \text{Land}_{t=1} & \quad \text{deb} & \quad \text{portfolio} & \quad \text{Call}_{t=1} \\
& \quad $67,870 & \quad $48,000= & \quad $19,870 & \quad $2,870 \\
$60,000 & \quad $48,000– & \quad $48,000= & \quad $0 & \quad $0
\end{align*}
\]

- The call option represents $2,870/$1,9870 = 14.44% of the value of the land with borrowing.
- If we find the value of that portfolio, we can find the value of the call option at \( t = 0 \).
- The land at \( t = 0 \) is worth $60,000.
- The present value of $48,000 is $46,153.84.
- The value of the call is thus:

\[
\text{Call}_{t=0} = \frac{2,870}{19,870} \times \left( \frac{60,000 - 48,000}{1.04} \right) = 2,000
\]

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Put-Call Parity

- Conceptually
  - The RIGHT to buy a stock together with the ABILITY to buy it. (A call option plus the present value of the exercise price.)
  - Should be worth the same as
    - The RIGHT to sell a stock together with the ABILITY to sell it. (A put option written on a stock that you own.)
  - This concept is known as Put-Call Parity
  - Stock + Put = Call + PV of Exercise Price

Put-Call Parity (Page 650-651)

- Every situation that can be described in terms of a call option has a parallel description in terms of a put option.
- Compare the case where Alex bought a call from Carla to the case where Carla purchased the put option from Paul.
  - Alex’s profits, as a function of land value, show a pattern similar to Carla’s profits when she bought the put option from Paul.

A Simple (Binomial) Model of Option Valuation (Page 308)

Consider the call option sold by Carla to Alex. The option has one year to maturity, and an exercise price of $65,000. At the time of the sale, the land value was $60,000. Assume that in one year, the land value can be either $67,870 (up by 13.12%) or $48,000 (down by 20%) in one year. If the risk-free rate is 4% per year, and assuming that the land should yield this rate, what is the value of the call option on this land?

A Simple (Binomial) Model of Option Valuation

- The expected value of the call option at maturity is:
  \[
p_u (\text{Value of call if land goes up}) + (1 - p_u) (\text{Value of call if land goes down})
  = (0.725) (2,870) + (0.275) (0)
  = 2,080
\]
- The present value of this is the value of the call option today:
  \[
  2,080/1.04 = 2,000
  \]
Put-Call Parity

- The following two portfolios have the same payoffs at expiry:
  - One European call plus an amount of cash equal to present value of the exercise price.
  - One European put plus one share of stock.
- This means that at time zero, two portfolios should have the same value.
- The following slide shows the payoffs.

Example of Put Call Parity

- We observe an asset is selling at $22, a one-year call with exercise price of $25 is selling $1.50, and the risk free rate of 3%. What is the value of a put with an exercise price of $25 with one year to expiration?
  - \( S + P = C + \text{PV of } X \)
  - \( 22 + P = 1.5 + \frac{25}{(1+4\%)} \)
  - \( P = 1.5 + 24.03 - 22 = 3.54 \)
Valuing an Option
Option value = Exercise value + Time premium

- At maturity, the value of an option is its exercise value, which is either
  - zero (if the option is out-of-the-money) or
  - the positive difference between the value of the underlying asset and the option’s exercise price (if the option is in-the-money).

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Options on Real Assets
- Real Options: Options embedded in real assets
  - Option to Abandon: Price-setting option
  - Option to Expand: Timing Option

Options on Financial Assets
- Executive Stock Options
- Warrants
- Convertible Bonds
- Callable Bonds
“Hidden” Options

- Stakeholder relationships in financial contracting.
- Refunding a home mortgage.
- Tax-timing options.
- Options connected with capital investments.
- Variable cost reduction.

Places to Look for Options

- Publicly traded options
- Insurance
- Real estate options
- Convertible bonds
- Warrants
- Employee stock options
- Call provisions in bonds

Common Stock is an Option

- Common stock as a call option on the firm’s assets
  - The exercise price is the amount due to the debtholders
- Common stock as a put option on the firm’s assets
  - The stockholders can “sell” the assets to the debtholders for an exercise price equal to the amount owed
  - \( S (\text{Assets}) = (PV \text{ of } X - P)(\text{Debt}) + C \text{ (Stock)} \)

Common Stock is an Option

a. Lucas Enterprises recently opted to open a new retail outlet. If the outlet outperforms the expectations, the manager can opt to increase the store’s size. If it underperforms, the manager can opt to close the store.

Type of option __ CALL __ Who is long? Who is short? Underlying Asset? Exercise Price?

Type of option __ PUT __

c. Jack and Jill are house hunting. They find House A situated on a hill. They really like the house but want to continue searching the market for one more week before making their final decision to buy the house. To avoid having someone else purchase House A while they continue their house hunting, they decide to place a $2,500 deposit on House A. This deposit will apply to the purchase price if they buy House A. If they do not buy House A, they will forfeit the $2,500.
### Valuing an Option

- Prior to maturity, the value of the option differs from its exercise value, by its “time premium.”
- The time premium is the value of the “optionality.” The option holder can claim certain outcomes and reject others.
- It depends on
  - the remaining time until expiration of the option
  - the risk of the underlying asset
  - riskless rate of return (the time value of money)

### Option Value and Remaining Time to Expiration

- The longer the time to expiration, the higher the time premium.
- You can never be worse off with an option that has a longer time to expiration.
- More time allows more chance for an option to be in-the-money at expiration.

### Option Value and Risk

- The higher the risk of an asset, the greater the time premium.
- With higher risk, the probability of extremely good outcomes increases.
  - This increases the probability the option will be in-the-money at expiration.
  - Even though the probability of extremely bad outcomes increases, the option will be worthless anyway.

### Option Value and the Time Value of Money

- The higher the riskless rate of return, the lower the present value of a known future amount.
- Exercising the option involves either
  - the payment of the exercise price by the option holder (in the case of a call), or
  - the receipt of the exercise price (in the case of a put).
Option Value and the Time Value of Money

- A call holder will pay the exercise price to receive the asset.
  - With higher riskless returns, the present value of this payment is lower and the call option value is higher.
- A put holder will receive the exercise price to receive the asset.
  - With higher riskless returns, the present value of this payment is lower and the put option value is lower.

The Effects on Option Value

<table>
<thead>
<tr>
<th></th>
<th>Call</th>
<th>Put</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remaining time</td>
<td>↑</td>
<td>↑</td>
</tr>
<tr>
<td>Underlying asset risk</td>
<td>↑</td>
<td>↑</td>
</tr>
<tr>
<td>Riskless rate of return</td>
<td>↑</td>
<td>↓</td>
</tr>
</tbody>
</table>

Generalizations about Options

- The largest time premium occurs when the underlying asset’s value equals the exercise price.
  - At this point, the uncertainty about whether the option will expire in- or out-of-the-money is the greatest.
- The time premium declines as the option becomes more in- or out-of-the-money.
  - At this point, there is little uncertainty about whether the option will end up in- or out-of-the-money.

Generalizations about Options

- An American option is never worth less than a comparable European option.
  - The American option gives you everything that a European option does, plus the added feature of possible exercise prior to maturity.
- An option’s time premium is generally positive.
- An out-of-the-money option can never have a negative value.
Generalizations about Options

- It is generally better to sell the option than exercise it.
- By exercising it prior to maturity, you are giving up the time premium.
- If you sell the option, you get both the exercise value and the time premium.

Generalizations about Options

- The further an option is out-of-the-money, the less it is worth.
- The entire value of the “deep” out-of-the-money option is derived from the time premium.
- The time premium decreases as the options gets further out-of-the-money.

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