CHAPTER 7
Bonds and Their Valuation

- Key features of bonds
- Bond valuation
- Measuring yield
- Assessing risk

Key Features of a Bond

1. Par value: Face amount; paid at maturity. (Usually $1,000 for corporate bonds.)
2. Maturity: Years until bond must be repaid.
3. Issue date: Date when bond was issued.
4. Coupon interest rate: Stated interest rate (generally fixed). Multiply by par to get $ of interest.

5. Yield to maturity: rate of return earned on a bond held until maturity (also called the “promised yield” or expected return).

Effect of a call provision

- Allows issuer to refund the bond issue if rates decline (helps the issuer, but hurts the investor).
- Borrowers are willing to pay more, and lenders require more, for callable bonds.
- Most bonds have a deferred call and a declining call premium.
Other types (features) of bonds

- Convertible bond – may be exchanged for common stock of the firm, at the holder’s option.
- Warrant – long-term option to buy a stated number of shares of common stock at a specified price.
- Putable bond – allows holder to sell the bond back to the company prior to maturity.
- Income bond – pays interest only when interest is earned by the firm.
- Indexed bond – interest rate paid is based upon the rate of inflation.

Financial Asset Values

The “price” of any asset should equal the present value (based on an appropriate discount rate) of its expected cash flows.

\[
PV = \frac{CF_1}{(1+k)^1} + \frac{CF_2}{(1+k)^2} + \ldots + \frac{CF_n}{(1+k)^n}.
\]

\[
0 \quad k \quad 1 \quad 2 \quad \ldots \quad n
\]

| Value | \( CF_1 \) | \( CF_2 \) | \( CF_n \) |
The discount rate \( (k_i) \) is the **opportunity cost of capital** (i.e., the rate that could be earned on alternative investments of equal risk).

\[
k_i = k^* + IP + MRP + DRP + LP
\]

- The expected cash flows of a bond are the interest payments and the par value.

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**What is the value of a 10-year, 10% coupon, $1000 par bond if \( k_d = 10\% \)?**

\[
V_B = \frac{100}{(1 + k_d)} + \ldots + \frac{100}{(1 + k_d)^{10}} + \frac{1,000}{(1 + k_d)^{10}}
\]

\[
= \frac{90.91}{1.1} + \ldots + \frac{38.55}{1.1^{10}} + \frac{385.54}{1.1^{10}}
\]

\[
= 90.91 + \ldots + 38.55 + 385.54
\]

\[
= 1,000.
\]
**Formula**

\[
V_B = \sum_{t=1}^{m\cdot N} \frac{\text{INT}/m}{(1 + k_d/m)^t} + \frac{\text{Par}}{(1 + k_d/m)^{m\cdot N}}
\]

\(V_B\) = value of bond  
\(N\) = number of years  
\(\text{INT}\) = annual coupon payment ($)  
\(\text{Par}\) = par value  
\(k_d\) = annual discount rate  
\(m\) = number of coupon pmts each year

**Calculator:** The bond consists of a 10-year, 10% annuity of $100/year plus a $1,000 lump sum at \(t = 10\):

<table>
<thead>
<tr>
<th>INPUTS</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N) 10</td>
<td>(I/YR) 10</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>INPUTS</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N) 10</td>
<td>(I/YR) 10</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The calculator can calculate the present value of the annuity and maturity value in one step (always use end mode).

**PV annuity** = $614.46

**PV maturity value** = $385.54

**PV bond** = $1,000.00

**Bond Valuation using the calculator:**

- **N** = total number of coupon payments
- **I** = discount rate (expected return; required return; yield-to-maturity)
- **PV** = price of the bond
- **FV** = Par value of bond
- **PMT** = coupon payment ($) each payment period

**USE THE END MODE**
What would happen if expected inflation rose by 3%, causing $k = 13\%$?

<table>
<thead>
<tr>
<th>INPUTS</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>13</td>
</tr>
</tbody>
</table>

When $k_d$ rises, above the coupon rate, the bond’s value falls below par, so it sells at a discount.

What would happen if inflation fell, and $k_d$ declined to 7%?

<table>
<thead>
<tr>
<th>INPUTS</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>7</td>
</tr>
</tbody>
</table>

Price rises above par, and bond sells at a premium, if coupon > $k_d$. 
- If coupon rate < \( k_d \), discount.

- If coupon rate = \( k_d \), par bond.

- If coupon rate > \( k_d \), premium.

**Non-annual Coupon Payments**

All calculator entries represent the same items if coupon payments are made during the year.

The only difference is that the \( N \), \( I \), and \( PMT \) must be adjusted to account for the non-annual payments.
Non-annual Coupon Payments

What’s the value of a 10-year, 12% quoted annual coupon bond with semi-annual coupon payments if \( k_d = 10\% \)?

<table>
<thead>
<tr>
<th>INPUTS</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>2x10</td>
<td>10/2</td>
</tr>
<tr>
<td>N</td>
<td>I/YR</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Find the value of 10-year, 10% coupon, semiannual bond if \( k_d = 13\% \).

<table>
<thead>
<tr>
<th>INPUTS</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>2x10</td>
<td>13/2</td>
</tr>
<tr>
<td>N</td>
<td>I/YR</td>
</tr>
<tr>
<td>20</td>
<td>6.5</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
What is the “yield to maturity” (YTM)?

- YTM is the rate of return earned on a bond held to maturity. Also called the “promised yield.”
- The yield-to-maturity is determined by solving for the discount rate implied by the current selling price of the bond. All items are the same on the calculator, you just solve for I.

What’s the YTM on a 10-year, 9% annual coupon, $1,000 par value bond that sells for $887?

<table>
<thead>
<tr>
<th>INPUTS</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10.91</td>
</tr>
<tr>
<td>N</td>
<td>I/YR</td>
</tr>
<tr>
<td>-887</td>
<td>PV</td>
</tr>
<tr>
<td>90</td>
<td>PMT</td>
</tr>
<tr>
<td>1000</td>
<td>FV</td>
</tr>
</tbody>
</table>
Find YTM if price were $1,134.20.

Sells at a premium. Because coupon = 9% > \( k_d = 7.08\% \), bond’s value > par.

What’s the quoted annual YTM on a 10-year, 9% coupon, semi-annual payment, $1,000 par value bond that sells for $887?

Quoted annual YTM = $5.441 \times 2 = 10.882\%$
How does adding a “call provision” affect a bond?

- Issuer can refund if rates decline. That helps the issuer but hurts the investor.
- Therefore, borrowers are willing to pay more, and lenders require more, on callable bonds.
- Most bonds have a deferred call and a declining call premium.

Yield to call using the calculator:

- \( N \) = ***total number of coupon payments until bonds are callable
- \( I \) = YTC (compute)
- \( PV \) = price of the bond
- \( FV \) = ***Call price of bond
- \( PMT \) = coupon payment ($) each payment period

USE THE END MODE
A 10-year, 10% semiannual coupon, $1,000 par value bond is selling for $1,135.90. It can be called after 4 years at $1,050.

What is the bond’s nominal yield to maturity (YTM)?

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{INPUTS} & 10 \times 2 & -1135.9 & 50 & 1000 \\
\text{OUTPUT} & & & \text{N} & \text{I/YR} & \text{PV} & \text{PMT} & \text{FV} \\
& & & & & & & 4.00 \\
\end{array}
\]

YTM = 4.00 \times 2 = 8.00\% 

What is the bond’s nominal yield to call (YTC)?

(To calculate YTC we replace N with the number of coupon payments prior to call and we replace FV with the price that must be paid to call the bonds.)

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{INPUTS} & 4 \times 2 & -1135.9 & 50 & 1050 \\
\text{OUTPUT} & & & \text{N} & \text{I/YR} & \text{PV} & \text{PMT} & \text{FV} \\
& & & & & & & 3.5684 \\
\end{array}
\]

YTC = 3.5684 \times 2 = 7.137\%
Find current yield and capital gains yield for a 9%, 10-year bond when the bond sells for $887 and YTM = 10.91%.

Current yield = \( \frac{\$90}{\$887} \)  
= 0.1015 = 10.15%.

YTM = Current yield + Capital gains yield.  
Cap gains yield = YTM – Current yield  
= 10.91% – 10.15%  
= 0.76%. 

Definitions

Current yield = \( \frac{\text{Annual coupon pmt}}{\text{Current price}} \)

Capital gains yield = \( \frac{\text{Change in price}}{\text{Beginning price}} \)

Exp total return = YTM = \( \frac{\text{Exp curr yld}}{\text{Exp cap gains yld}} \)
What’s interest rate (or price) risk? Does a 1-year or 10-year 10% bond have more risk?

**Interest rate risk:** Rising $k_d$ causes bond’s price to fall.

<table>
<thead>
<tr>
<th>$k_d$</th>
<th>1-year</th>
<th>Change</th>
<th>10-year</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>$1,048</td>
<td>+4.8%</td>
<td>$1,386</td>
<td>+38.6%</td>
</tr>
<tr>
<td>10%</td>
<td>1,000</td>
<td>-4.4%</td>
<td>1,000</td>
<td>-25.1%</td>
</tr>
<tr>
<td>15%</td>
<td>956</td>
<td></td>
<td>749</td>
<td></td>
</tr>
</tbody>
</table>

What is reinvestment rate risk?

The risk that CFs will have to be reinvested in the future at lower rates, reducing income.

If you invest in a multi-year bond, you get the same interest rate each year until maturity.

If you invest in a one-year bond, you get the stated interest rate the first year. The next year you have to reinvest at the prevailing interest rate.
Interest rate and Reinvestment rate risk

- Long-term bonds: High interest rate risk, low reinvestment rate risk.
- Short-term bonds: Low interest rate risk, high reinvestment rate risk.
- Nothing is completely riskless!

Do all bonds of the same maturity have the same price and reinvestment rate risk?

No, low coupon bonds have less reinvestment rate risk but more price risk than high coupon bonds.
Types of Bonds

- Mortgage bond – a bond backed by fixed assets.
- Debenture – a long-term bond that is not secured by a mortgage on specific property.
- Subordinated debenture – a bond having a claim on assets only after the senior debt has been paid off in the event of liquidation.

Bond Default Risk

- If the issuer defaults, investors receive less than the promised return.
- Therefore, the expected return on corporate and municipal bonds is less than the promised return.
- The difference is dependent on the risk of default.
### Bond Ratings Provide One Measure of Default Risk

<table>
<thead>
<tr>
<th>Investment Grade</th>
<th>Junk Bonds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moody’s</td>
<td></td>
</tr>
<tr>
<td>Aaa</td>
<td>Ba</td>
</tr>
<tr>
<td>Aa</td>
<td>B</td>
</tr>
<tr>
<td>A</td>
<td>Caa</td>
</tr>
<tr>
<td>Baa</td>
<td>C</td>
</tr>
<tr>
<td>S&amp;P</td>
<td></td>
</tr>
<tr>
<td>AAA</td>
<td>BB</td>
</tr>
<tr>
<td>AA</td>
<td>B</td>
</tr>
<tr>
<td>A</td>
<td>CCC</td>
</tr>
<tr>
<td>BBB</td>
<td>D</td>
</tr>
</tbody>
</table>

### What factors affect default risk and bond ratings?

- Financial performance
  - Debt ratio; TIE ratio; Current ratio

- Provisions in the bond contract
  - Secured vs. unsecured debt; Senior vs. subordinated debt; Guarantee provisions; Sinking fund provisions; Debt maturity

- Other factors
  - Earnings stability; Regulatory environment; Potential product liability; Accounting policies
What is a sinking fund?

- Provision to pay off a loan over its life rather than all at maturity.
- Similar to amortization on a term loan.
- Reduces risk to investor, shortens average maturity.
- But not good for investors if rates decline after issuance.

How are sinking funds executed?

- Call x% of the issue at par, for sinking fund purposes.
  - Likely to be used if $k_d$ is below the coupon rate and the bond sells at a premium.
- Buy bonds in the open market.
  - Likely to be used if $k_d$ is above the coupon rate and the bond sells at a discount.