Speaker: Dr. Jeong-Hyun Kang (UWG)
Title: Equidistant set in $\mathbb{R}^d$ under $\ell_1$-norm

Abstract:

Kusner (1983) conjectured that the maximum size of a set whose elements are pairwise equidistant under the $\ell_1$-norm in $\mathbb{R}^d$ is $2d$. If true, this would be sharp. The conjecture has been proved for $d = 3$ (in 1998) and $d = 4$ (in 2000). Alon and Pudlak gave an upper bound of $O(d \ln d)$ (in 2003).

We show that the Kusner conjecture is true if given an $\ell_1$-equidistant set of cardinality at least $2d$ in $\mathbb{R}^d$ there exists a subset of size $3d/2$ on a single unit $\ell_1$-sphere. Also, we show that every three points of an $\ell_1$-equidistant set belong to a unit $\ell_1$-sphere, and the centers of these induced unit spheres are contained in a particular hyperplane. This leads to new proofs of Kusner's conjecture for the cases of $d = 3$ and 4.