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Title: Connectivity and non-revisiting paths in polyhedral maps on surfaces

Abstract:

The $W_v$-path conjecture states that any two vertices of a simple polytope can be joined by a path that does not revisit any facet. This is equivalent to the well-known Hirsch Conjecture. Klee conjectured even more, namely that the $W_v$-conjecture is true for all general cell complexes. Klee proved that the $W_v$-conjecture is true for 3-polytope (3-connected plane graphs). Later, the general $W_v$-conjecture was verified for polyhedral maps on the projective plane and torus by Barnette, and on the Klein-bottle by Pulapaka and Vince. Recently, however, Santos proved that the Hirsch conjecture is false.

In this talk, we will present some recent development on the $W_v$-path problem in polyhedral maps on surfaces. We show that the $W_v$-path problem is closely related to both the local connectivity $\kappa_G(x,y)$, and the number of non-homotopy classes of $(x,y)$-paths as well as the number of $(x,y)$-paths in each homotopy class. For every surface $\Sigma$, define a function $f(\Sigma)$ such that if for every graph polyhedrally embedded in $\Sigma$ and for every pair of vertices $x$ and $y$ in $V(G)$, $\kappa_G(x,y) \geq f(\Sigma)$, then there exists a $W_v$-path joining $x$ and $y$. We show that $\max\{3, 1 - \chi(\Sigma)\} \leq f(\Sigma) \leq 9 - 4\chi(\Sigma)$, where $\chi(\Sigma)$ is the Euler characteristic. Further, if $x$ and $y$ are not co-facial, we show that $G$ has at least $\kappa_G(x,y) + 4\chi(\Sigma) - 8$ internally disjoint $W_v$-paths joining $x$ and $y$. The bound is sharp for the sphere.

This is joint work with Michael D. Plummer and Xiaoya Zha.