# HYPOTHESIS TESTING

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# CONTENT OUTLINE

- Logic of Hypothesis Testing
- Error & Alpha
- Hypothesis Tests
- Effect Size
- Statistical Power

*how we conceptualize hypotheses*

# LOGIC OF HYPOTHESIS TESTING

# HYPOTHESIS TESTING LOGIC

- **Hypothesis Test**

  *statistical method that uses sample data to evaluate a hypothesis about a population*

- The Logic
  - State a hypothesis about a population, usually concerning a population parameter
  - Predict characteristics of a sample
  - Obtain a random sample from the population
  - Compare obtained data to prediction to see if they are consistent

# STEPS IN HYPOTHESIS TESTING

- Step 1: State the Hypotheses
  - **Null Hypothesis ($H_0$)**

    ***in the general population there is no change, no difference, or no relationship; the independent variable will have no effect on the dependent variable***

    - Example
      - All dogs have four legs.
      - There is no difference in the number of legs dogs have.
  - **Alternative Hypothesis ($H_1$)**

    ***in the general population there is a change, a difference, or a relationship; the independent variable will have an effect on the dependent variable***

    - Example
      - 20% of dogs have only three legs.

# STEP 1: STATE THE HYPOTHESES (EXAMPLE)

- Example

**How to Ace a**

**Statistics Exam**

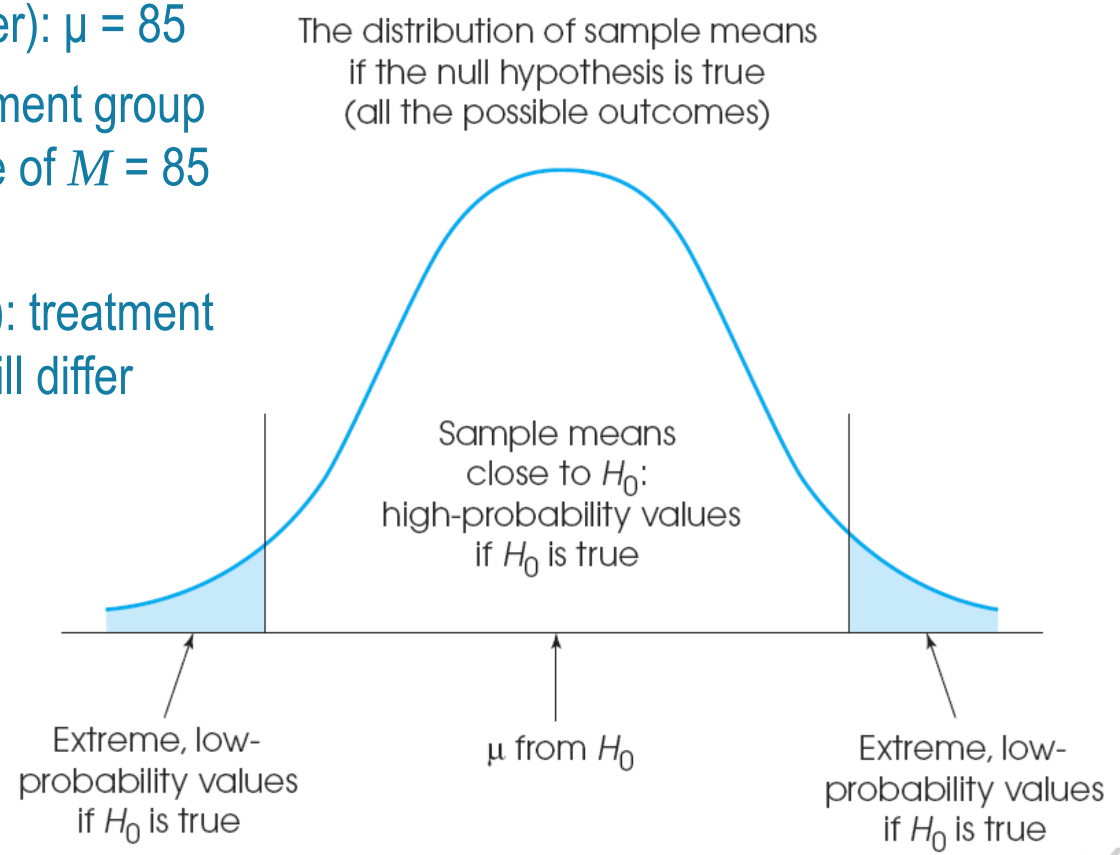
little known facts about
the positive impact of
alcohol on memory
during “cram” sessions

# STEP 1: STATE THE HYPOTHESES (EXAMPLE)

- Dependent Variable
  - Amount of alcohol consumed the night before a statistics exam
- Independent/Treatment Variable
  - Intervention: Pamphlet (treatment group) or No Pamphlet (control group)
- Null Hypothesis ($H_0$)
  - No difference in alcohol consumption between the two groups the night before a statistics exam.
- Alternative Hypothesis ($H_1$)
  - The treatment group will consume more alcohol than the control group.

# STEP 2: SET CRITERIA FOR DECISION

- Example
  - Exam 1 (Previous Semester): $\mu = 85$
  - Null Hypothesis ($H_0$): treatment group will have mean exam score of $M = 85$ ($\sigma = 8$)
  - Alternative Hypothesis ($H_1$): treatment group mean exam score will differ from $M = 85$



# STEP 2: SET CRITERIA FOR DECISION

- **Alpha Level/Level of Significance**

  *probability value used to define the (unlikely) sample outcomes if the null hypothesis is true; e.g., $\alpha = .05$, $\alpha = .01$, $\alpha = .001$*

- **Critical Region**

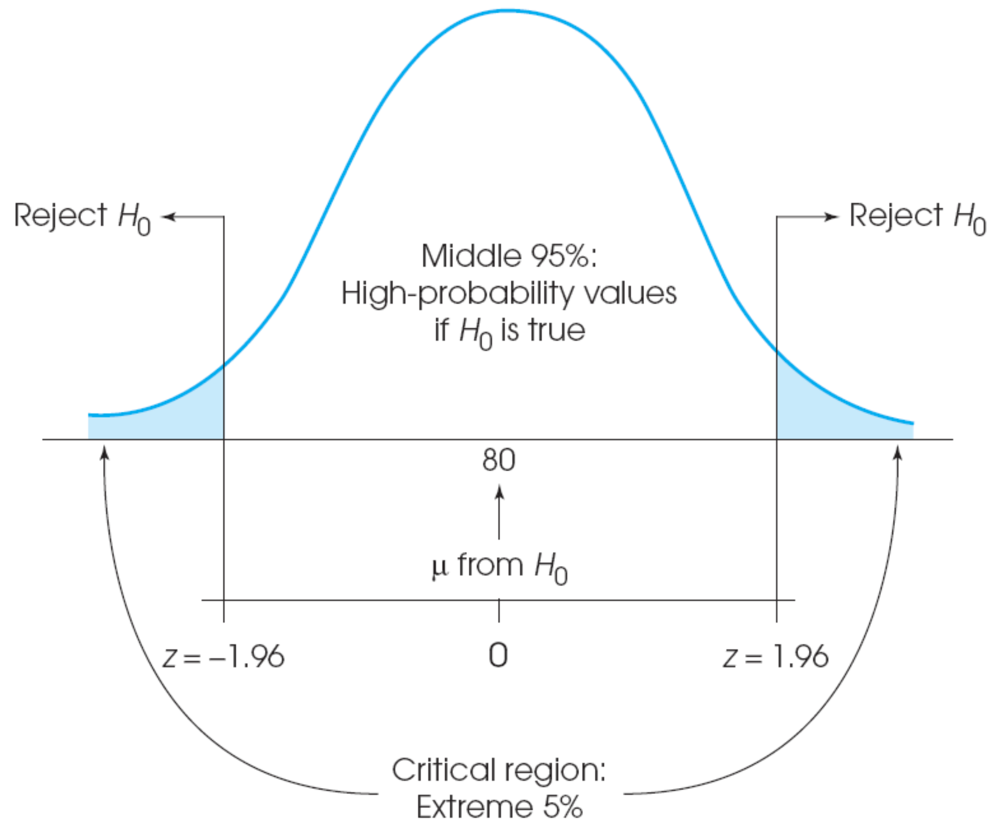
  *extreme sample values that are very unlikely to be obtained if the null hypothesis is true*

  - Boundaries determined by alpha level
  - If sample data falls within this region (the shaded tails), reject the null hypothesis

# STEP 2: SET CRITERIA FOR DECISION

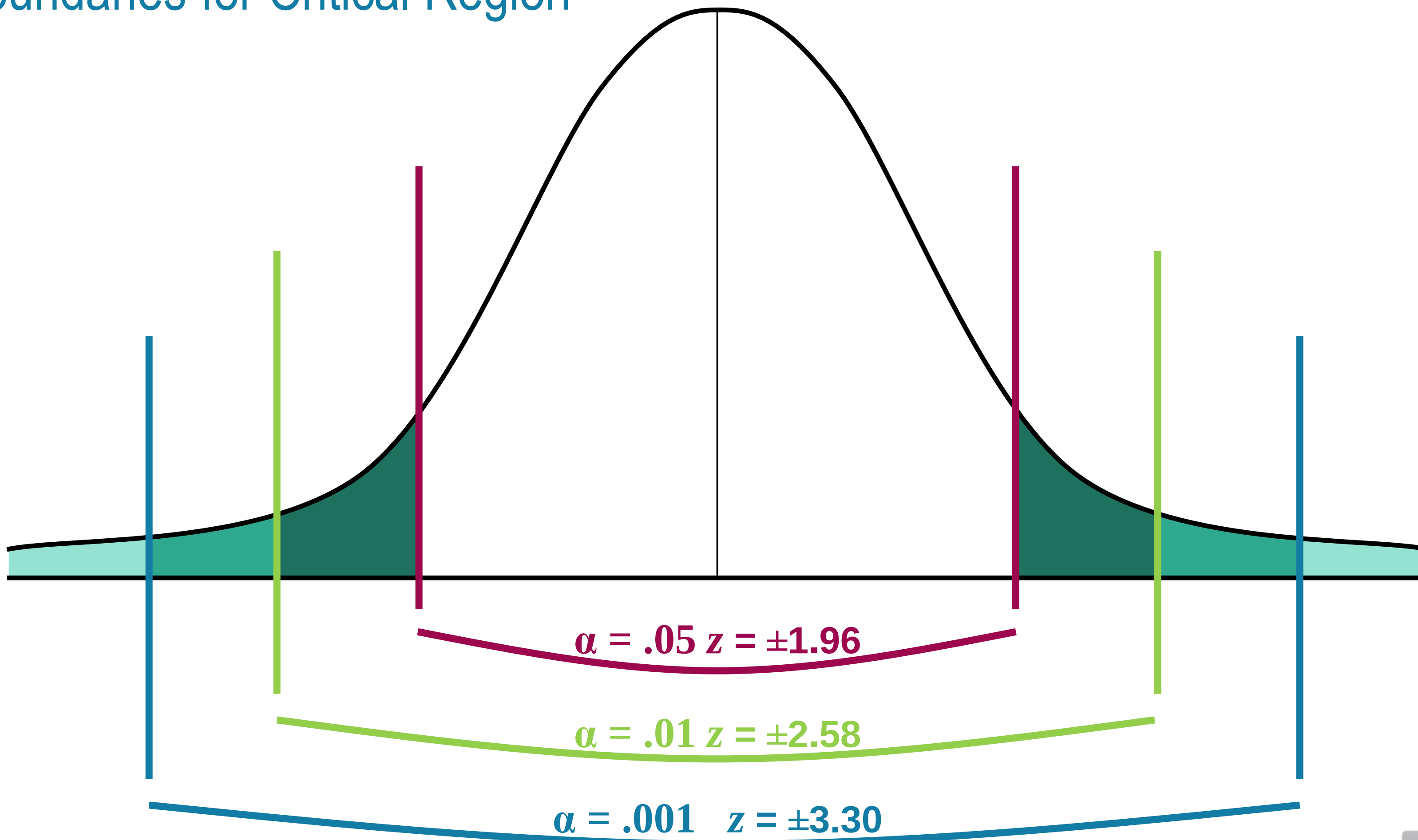
## ➢ Critical Region Boundaries

- ❖ Assume normal distribution
- ❖ Alpha Level + Unit Normal Table
- ❖ Example: if $\alpha = .05$, boundaries extreme 5%
  - ○ 2.5% in each tail (2-tailed)



# STEP 2: SET CRITERIA FOR DECISION

- Boundaries for Critical Region



# STEP 3: COLLECT, COMPUTE

- Collect data
- Compute sample mean
- Transform sample mean $M$ to $z$-score

$$z = \frac{M - \mu}{\sigma_M}$$

- Example #2

$$z = \frac{95 - 85}{1.13} = \frac{10}{1.13} = 8.85$$

# STEP 4: MAKE A DECISION

- Compare $z$-score with boundary of critical region for selected level of significance

- If…
  - $z$-score falls in the tails, our mean is significantly different from $H_0$
    - Reject $H_0$
  - $z$-score falls between the tails, our mean is not significantly different from $H_0$
    - Fail to reject $H_0$

# HYPOTHESIS TESTING: AN EXAMPLE (2-TAIL)

- How to Ace a Statistics Exam…
  - Population: $\mu = 85$, $\sigma = 8$
  - Hypotheses
    - $H_0$: Sample mean will not differ from $M = 85$
    - $H_1$: Sample mean will differ from $M = 85$
  - Set Criteria (Significance Level/Alpha Level)
    - $\alpha = .05$

# EXAMPLE (2-TAIL)

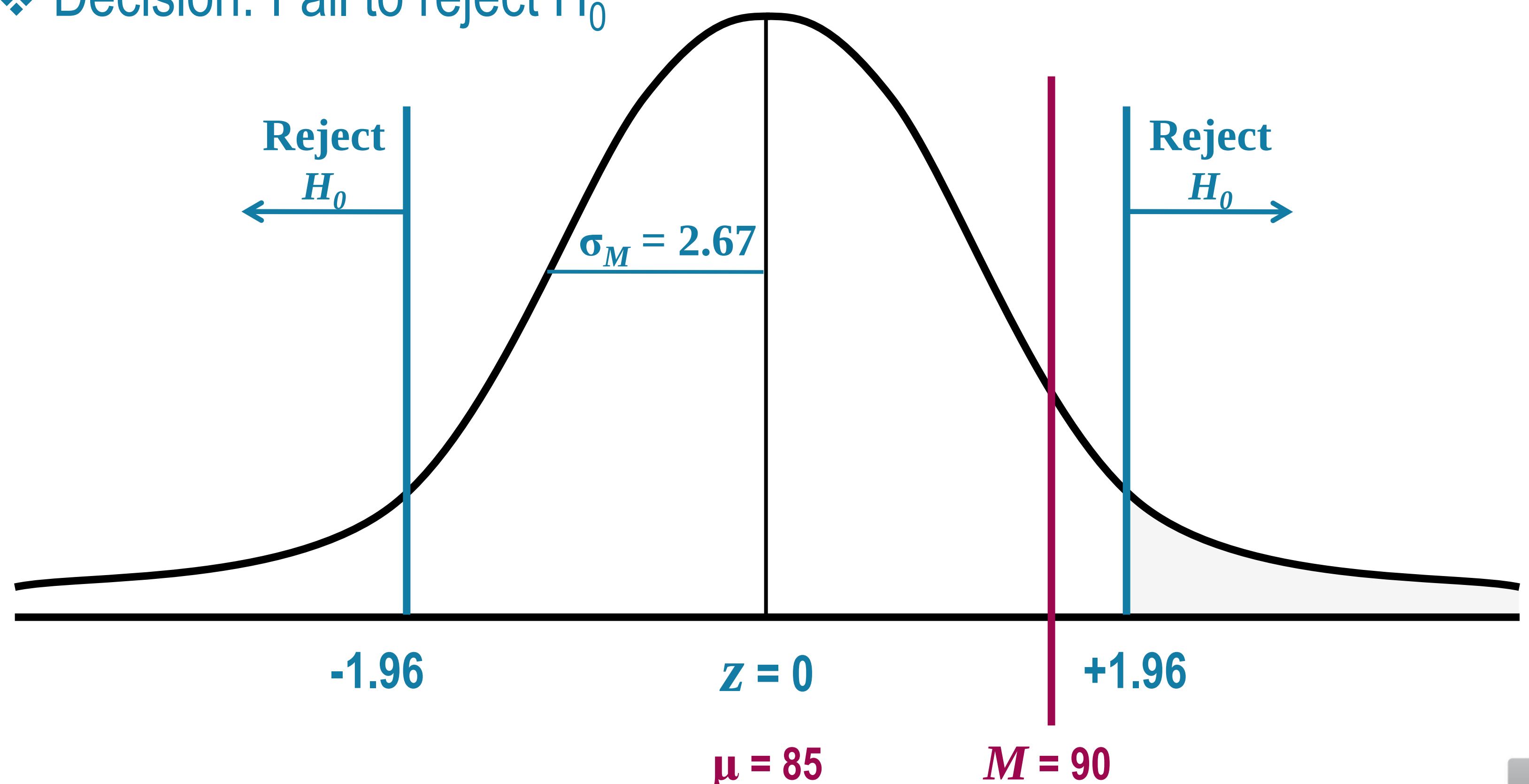
- How to Ace a Statistics Exam…
  - Collect Data & Compute Statistics
    - Intervention to 9 students
    - Mean exam score, $M = 90$

$$\sigma_M = \frac{\sigma}{\sqrt{9}} = \frac{8}{3} = 2.67$$

$$z = \frac{90-85}{2.67} = \frac{5}{2.67} = 1.87$$

# HYPOTHESIS TESTING: EXAMPLE (2-TAIL)

- How to Ace a Statistics Exam…
  - Decision: Fail to reject $H_0$



# REVISITING *Z*-SCORE STATISTICS

- A Test Statistic
  - Single, specific statistic
  - Calculated from the sample data
  - Used to test $H_0$
- Rule of Thumb…
  - Large values of $z$
    - Sample data pry DID NOT occur by chance – result of IV
  - Small values of $z$
    - Sample data pry DID occur by chance – not result of IV

*uncertainty leads to error*

# ERROR & ALPHA

# UNCERTAINTY & ERROR

- Hypothesis Testing = Inferential Process
  - LOTS of room for error

- Types of Error
  - Type I Error
  - Type II Error

# TYPE 1 ERRORS

***error that occurs when the null hypothesis is rejected even though it is really true; the researcher identifies a treatment effect that does not really exist (a false positive)***

- Common Cause & Biggest Problem
  - Sample data are misleading due to sampling error
  - Significant difference reported in literature even though it isn't real
- Type I Errors & Alpha Level
  - Alpha level = probability of committing a Type I Error
  - Lower alphas = less chances of Type I Error

# TYPE II ERRORS

***error that occurs when the null hypothesis is not rejected even it is really false; the researcher does not identify a treatment effect that really exists (a false negative)***

- Common Cause & Biggest Problem
  - Sample mean in not in critical region even though there is a treatment effect
  - Overlook effectiveness of interventions

- Type II Errors & Probability
  - $\beta$ = probability of committing a Type II Error

# TYPE I & TYPE II ERRORS

- Experimenter's Decision

| | Actual Situation | |
|---|---|---|
| | No Effect, $H_0$ True | Effect Exists, $H_0$ False |
| Reject $H_0$ | Type I Error | ☺ |
| Retain $H_0$ | ☺ | Type II Error |

# SELECTING AN ALPHA LEVEL

- Functions of Alpha Level
  - Critical region boundaries
  - Probability of a Type I error

- Primary Concern in Alpha Selection
  - Minimize risk of Type I Error without maximizing risk of Type II Error

- Common Alpha Levels
  - $\alpha = .05$, $\alpha = .01$, $\alpha = .001$

*testing null hypotheses*

# HYPOTHESIS TESTS

# INFLUENTIAL FACTORS

- Magnitude of difference between sample mean and population mean (in $z$-score formula, larger difference ⇨ larger numerator)

$$z = \frac{M - \mu}{\sigma_M} \qquad \sigma_M = \frac{\sigma}{\sqrt{n}}$$

- Variability of scores (influences $\sigma_M$; more variability ⇨ larger $\sigma_M$)

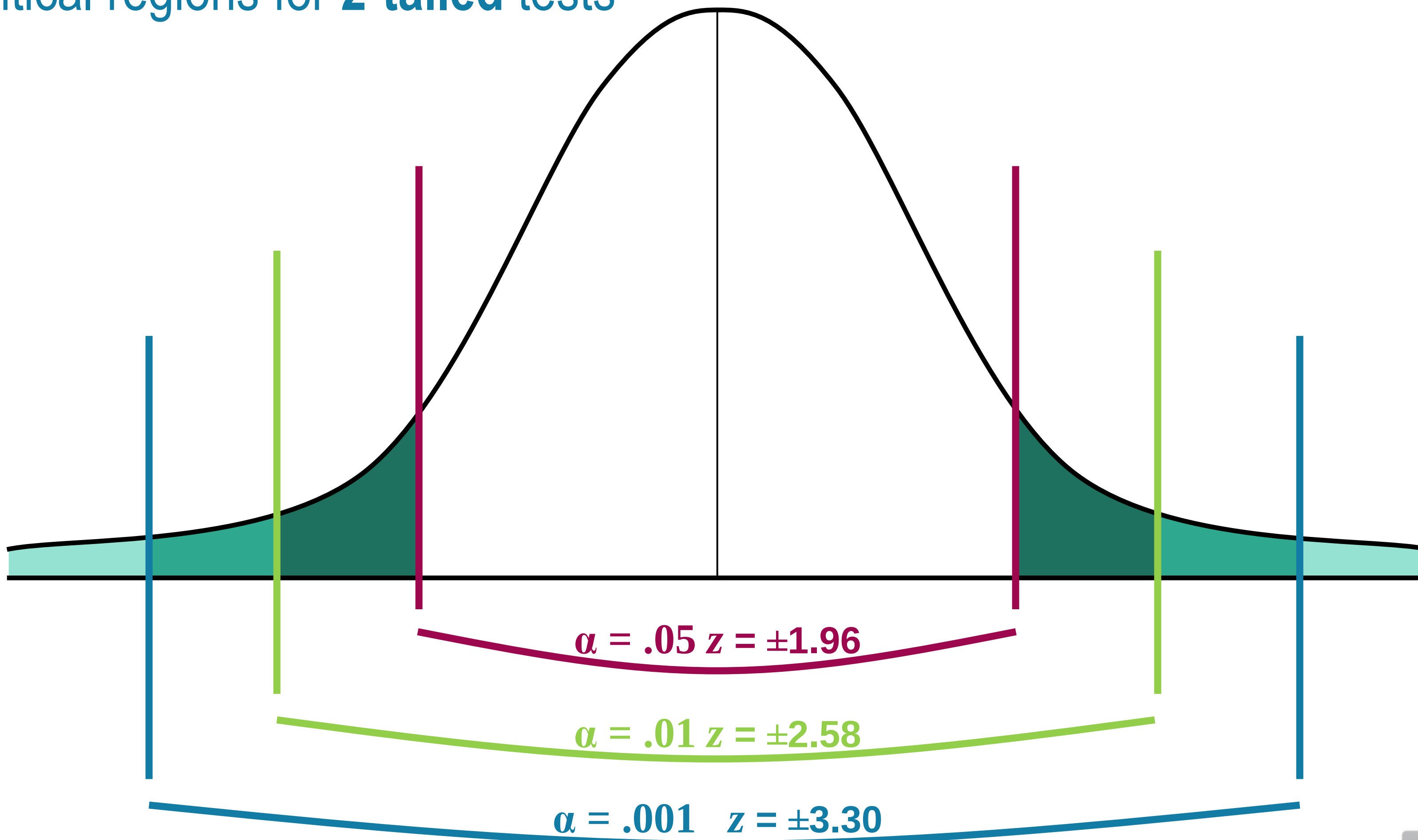
- Sample size (influences $\sigma_M$; larger sample size ⇨ smaller $\sigma_M$)

# HYPOTHESIS TESTS: ASSUMPTIONS

- Random Sampling
- Independent Observations
- Value of σ is Constant
  - Despite treatment
- Normal sampling distribution

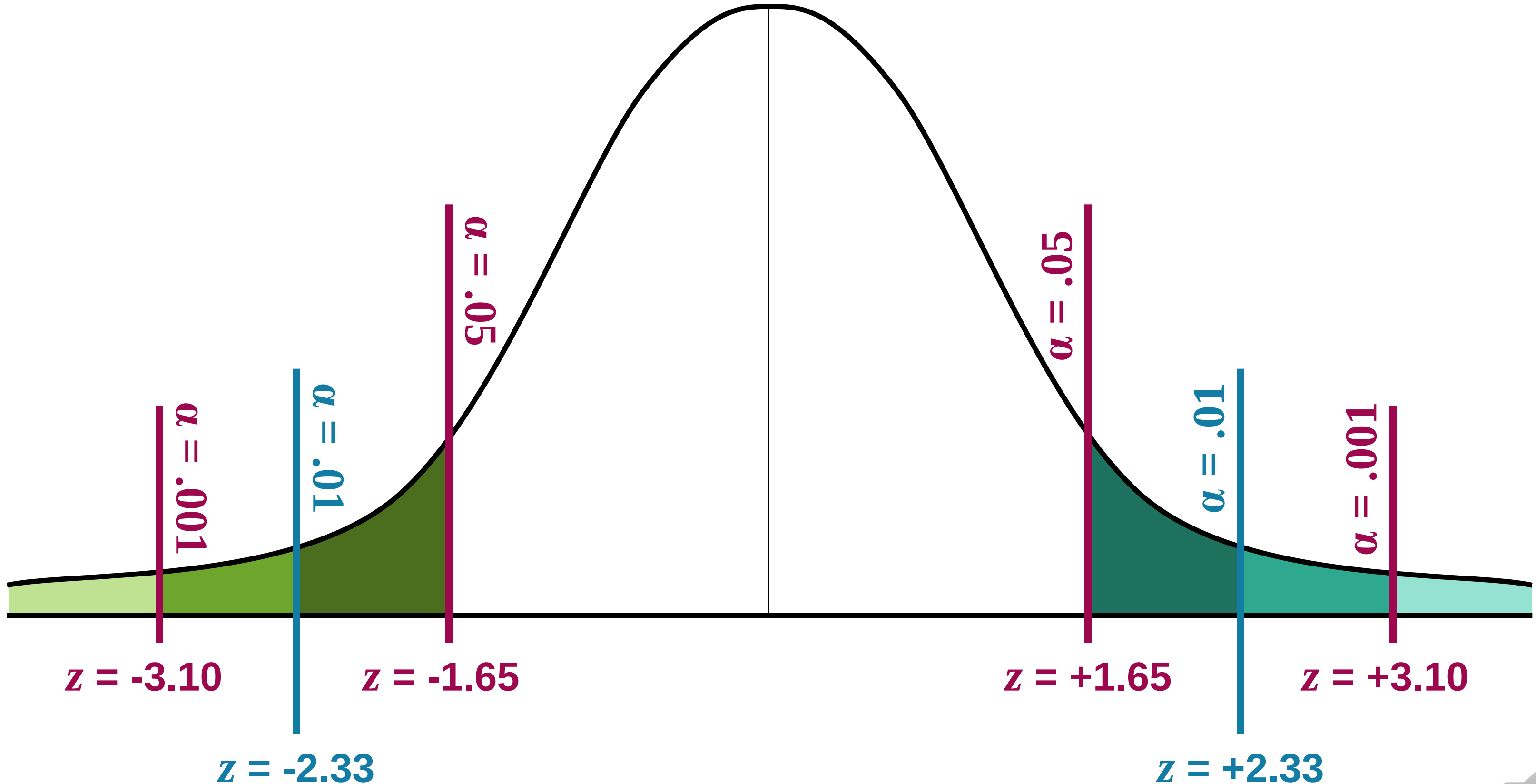
# NON-DIRECTIONAL HYPOTHESIS TESTS

- Critical regions for **2-tailed** tests



# DIRECTIONAL HYPOTHESIS TESTS

- Critical regions for **1-tailed** tests
  - Blue **or** Green tail of distribution – **NOT BOTH**



# ALTERNATIVE HYPOTHESES

- Alternative Hypotheses for **2-tailed** tests
  - Do not specify direction of difference
  - Do not hypothesize whether sample mean should be lower or higher than population mean

- Alternative Hypotheses for **1-tailed** tests
  - Specify a difference
  - Hypothesis specifies whether sample mean should be lower or higher than population mean

# NULL HYPOTHESES

- Null Hypotheses for **2-tailed** tests
  - Specify no difference between sample & population

- Null Hypotheses for **1-tailed** tests
  - Specify the opposite of the alternative hypothesis
  - Example #2
    - $H_0$: $\mu \leq 85$ (There is no increase in test scores.)
    - $H_1$: $\mu > 85$ (There is an increase in test scores.)

# HYPOTHESIS TESTS: AN EXAMPLE (1-TAIL)

- How to Ace a Statistics Exam…
  - Population: $\mu$ = 85, $\sigma$ = 8
  - Hypotheses
    - $H_0$: Sample mean will be less than or equal to $M$ = 85
    - $H_1$: Sample mean be greater than $M$ = 85
  - Set Criteria (Significance Level/Alpha Level)
    - $\alpha$ = .05

# HYPOTHESIS TESTS: AN EXAMPLE (1-TAIL)

- How to Ace a Statistics Exam…
  - Collect Data & Compute Statistics
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$$\sigma_M = \frac{\sigma}{\sqrt{9}} = \frac{8}{3} = 2.67$$

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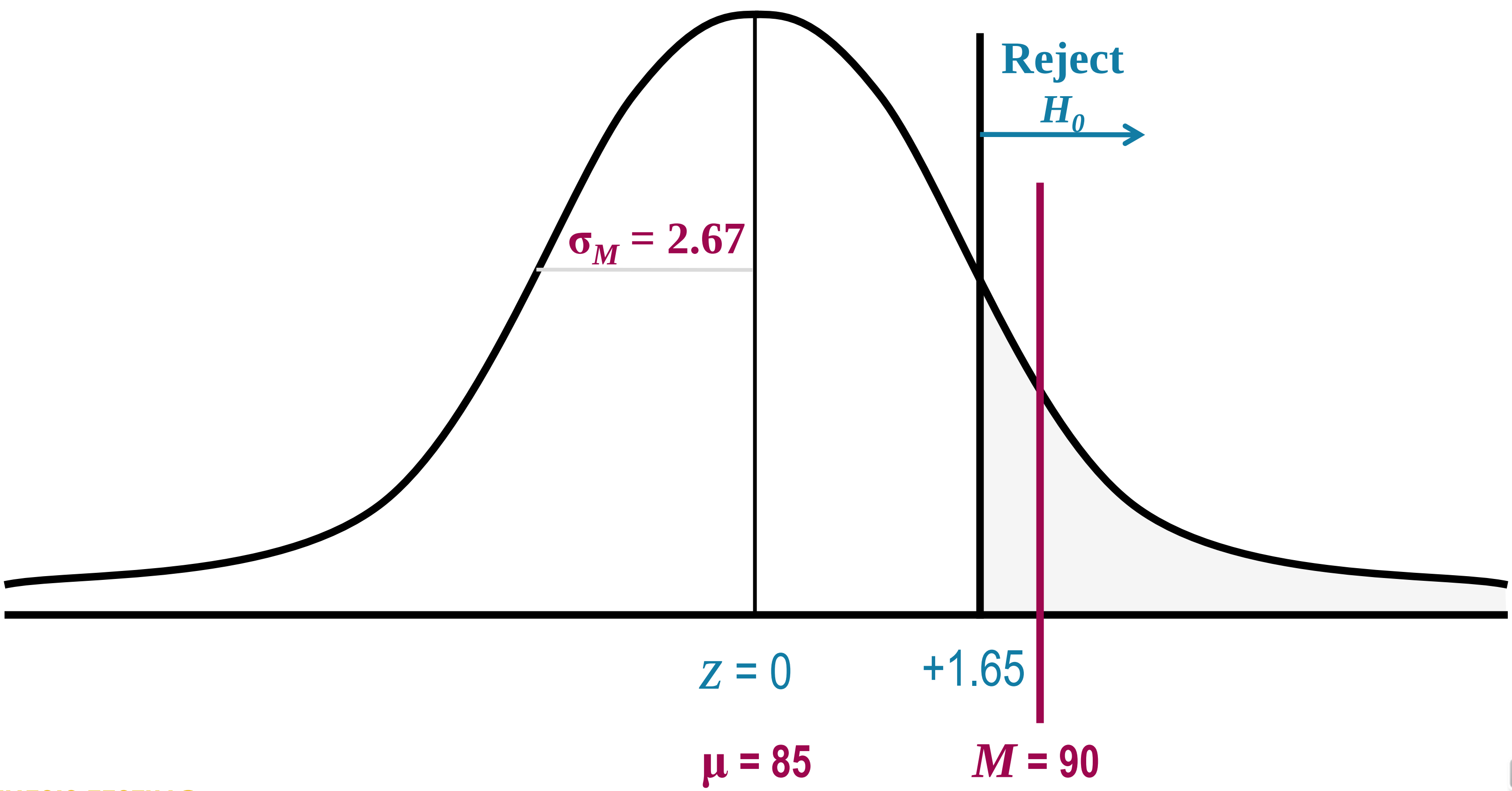
# HYPOTHESIS TESTS: AN EXAMPLE (1-TAIL)

- How to Ace a Statistics Exam…
  - Decision: Reject $H_0$



*estimating the magnitude of an effect*

# EFFECT SIZE

# EFFECT SIZE

- Problem with hypothesis testing
  - Significance ≠ Meaningful/Important/Big Effect
    - Significance is relative comparison: treatment effect compared to standard error

- Effect Size

  ***statistic that describes the magnitude of an effect***

- Measures size of treatment effect in terms of (population) standard deviation

# EFFECT SIZE: COHEN'S *D*

- **Not** influenced by sample size

$$\text{Cohen's } d = \frac{\text{mean difference}}{\text{standard deviation}}$$

- Evaluating Cohen's $d$
  - $d$ = 0.2 – Small Effect (mean difference ≈ 0.2 standard deviation)
  - $d$ = 0.5 – Medium Effect (mean difference ≈ 0.5 standard deviation)
  - $d$ = 0.8 – Large Effect (mean difference ≈ 0.8 standard deviation)

- Calculated the same for 1-tailed and 2-tailed tests *probability of correctly rejecting a false null hypothesis*

# STATISTICAL POWER

# STATISTICAL POWER

***the probability of correctly rejecting a null hypothesis when it is not true; the probability that a hypothesis test will identify a treatment effect when if one really exists***

- A priori
  - Calculate power before collecting data
  - Determine probability of finding treatment effect
- Power is influenced by…
  - Sample size
  - Expected effect size
  - Significance level for hypothesis test ($\alpha$)