INTRODUCTION TO THE *t* STATISTIC

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z-SCORES REVISITED

- Basic Assumptions
  - M approximates μ
  - σ_M estimates how well M approximates μ
  - Quantifies inferences made about the population
- The Problem with z-scores
  - Requires knowledge of σ

ESTIMATE OF STANDARD ERROR

estimate of the standard error when the population standard deviation is unknown; estimate of standard difference between M and μ

$$s_M = \sqrt{\frac{s^2}{n}}$$

DEGREES OF FREEDOM

number of scores in a sample that are independent and free to vary

- Larger df → better s^2 represents σ^2
- df associated with s^2 describes how well t represents z
  - Larger df → better t statistic approximates z statistic

t STATISTIC

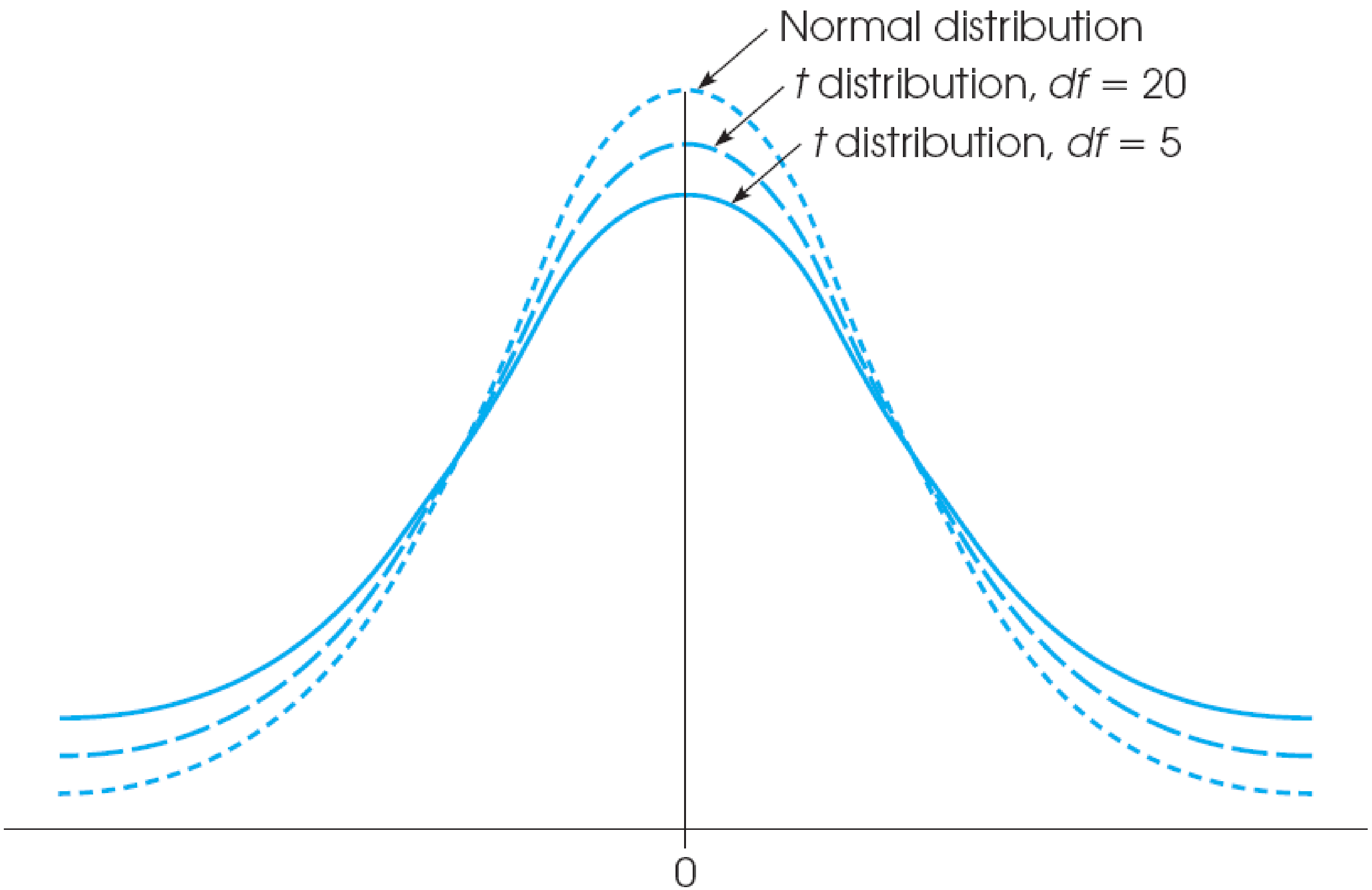
statistic used to test hypotheses about unknown population μ when the value of σ is unknown

- May be positive or negative (absolute value = magnitude)
- No upper- or lower- bound values

$$z = \frac{M - \mu}{\sigma_M} = \frac{M - \mu}{\sqrt{\frac{\sigma^2}{n}}}$$

$$t = \frac{M - \mu}{s_M} = \frac{M - \mu}{\sqrt{\frac{s^2}{n}}}$$

t DISTRIBUTION



t DISTRIBUTION: PROPORTIONS

	Proportion in One Tail					
	0.25	0.10	0.05	0.025	0.01	0.005
	Proportion in Two Tails Combined					
df	0.50	0.20	0.10	0.05	0.02	0.01
1	1.000	3.078	6.314	12.706	31.821	63.657
2	0.816	1.886	2.920	4.303	6.965	9.925
3	0.765	1.638	2.353	3.182	4.541	5.841
4	0.741	1.533	2.132	2.776	3.747	4.604
5	0.727	1.476	2.015	2.571	3.365	4.032
6	0.718	1.440	1.943	2.447	3.143	3.707

HYPOTHESIS TESTING WITH *t*

- Assumptions
  - Values/Observations in sample are independent
  - Sampled population is normal
- Process of Hypothesis Testing
  - Start with population with unknown μ and σ^2
  - Goal: Sample to determine effect (if any) of treatment
  - H_0: the treatment had no effect
  - s^2 and σ_s computed from sample data

$$t = \frac{M - \mu}{s_M}$$

HYPOTHESIS TESTING WITH *t*

- Step 1: State H_0 and H_1
- Step 2: Locate critical region
  - Compute df and refer to t distribution table
- Step 3: Calculate Test Statistic
  - Calculate sample s^2
  - Compute s_M
  - Compute t statistic
- Step 4: Make decision regarding H_0

$$s^2 = \frac{SS}{df} \qquad s_M = \sqrt{\frac{s^2}{n}} \qquad t = \frac{M - \mu}{s_M}$$

EFFECT SIZES

- Cohen's d

$$d = \frac{M - \mu}{s}$$

  - $d = 0.2$ – Small Effect (mean difference ≈ 0.2 standard deviation)
  - $d = 0.5$ – Medium Effect (mean difference ≈ 0.5 standard deviation)
  - $d = 0.8$ – Large Effect (mean difference ≈ 0.8 standard deviation)

- r^2

$$r^2 = \frac{t^2}{t^2 + df}$$

  - $r^2 = 0.01$ – Small Effect
  - $r^2 = 0.09$ – Medium Effect
  - $r^2 = 0.25$ – Large Effect

INFLUENCE OF n AND s ON s_M

- Larger s_M → smaller values of t (closer to zero)
- Any factor that increases s_M will reduce likelihood of rejecting H_0
- Large Variance
  - Difference less likely to be significant
  - Scores widely scattered, so harder to see consistent patterns in data
  - Reduced effect size
- Large Sample
  - Difference more likely to be significant
  - smaller s_M