PAIRED SAMPLES $t$ & WILCOXON SIGNED RANKS TESTS

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CONTENT OUTLINE

- Within-Subjects Research Designs
- Paired Samples $t$ Test
- Wilcoxon Signed Ranks Test
research design plays a major role in determining correct statistical approach

WITHIN-SUBJECTS DESIGNS
WITHIN-SUBJECTS DESIGN

*research design in which a single sample is measured more than once on the same dependent variable*

- Two sets of data from a single population/sample
  - Example: Comparison of heart rate before & after running a marathon
- A.K.A. Dependent-Measures, Paired Samples, Repeated-Measures
WITHIN-SUBJECTS DESIGN

➢ Hypotheses

  ➢ Null

  o $H_0: X_1 - X_2 = 0 \rightarrow$ There is no change in scores from time 1 to time 2
  o $H_0: X_1 = X_2 \rightarrow$ Scores at time 1 are equal to scores at time 2
  o $H_0: \mu_D = 0 \rightarrow$ The difference in scores from time 1 to time 2 is equal to 0

  ➢ Alternative

  o $H_A: X_1 - X_2 \neq 0 \rightarrow$ There is a change in scores from time 1 to time 2
  o $H_A: X_1 \neq X_2 \rightarrow$ Scores at time 1 are not equal to scores at time 2
  o $H_A: \mu_D \neq 0 \rightarrow$ The difference in scores from time 1 to time 2 is not equal to 0
parametric analysis for within-subjects designs

PAIRED SAMPLES t TEST
PAIRED SAMPLES $t$

- **The Basics**
  - Similar to single-sample $t$ (or z-score to compare sample to population)
  - Based on difference scores instead of raw scores
    - Matched Pairs (individuals from different groups matched by key characteristics)
    - Pre-/Post- Designs
  - Goal: use difference scores to answer questions about population

- **Difference Scores**
  - $D = X_2 - X_1$
  - Measures amount of change for each subject
  - Increase in score = positive value of $D$
  - Decrease in score = negative value of $D$
### Difference Scores (Matched Pairs)

<table>
<thead>
<tr>
<th>Pair</th>
<th>Control Group (X1)</th>
<th>Treatment Group (X2)</th>
<th>Difference (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matched Pair A</td>
<td>215</td>
<td>210</td>
<td>-5</td>
</tr>
<tr>
<td>Matched Pair B</td>
<td>221</td>
<td>242</td>
<td>21</td>
</tr>
<tr>
<td>Matched Pair C</td>
<td>196</td>
<td>219</td>
<td>23</td>
</tr>
<tr>
<td>Matched Pair D</td>
<td>203</td>
<td>228</td>
<td>25</td>
</tr>
</tbody>
</table>

\[
M_D = \frac{\sum D}{n} = \frac{64}{4} = 16
\]
### Difference Scores (Pre-/Post- Design)

<table>
<thead>
<tr>
<th>Person</th>
<th>Before Medication (X1)</th>
<th>After Medication (X2)</th>
<th>Difference (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>215</td>
<td>210</td>
<td>-5</td>
</tr>
<tr>
<td>B</td>
<td>221</td>
<td>242</td>
<td>21</td>
</tr>
<tr>
<td>C</td>
<td>196</td>
<td>219</td>
<td>23</td>
</tr>
<tr>
<td>D</td>
<td>203</td>
<td>228</td>
<td>25</td>
</tr>
</tbody>
</table>

\[
M_D = \frac{\sum D}{n} = \frac{64}{4} = 16
\]
FORMULAS FOR PAIRED SAMPLES $t$

- **Variance**

\[
s^2 = \frac{SS}{df} = \frac{SS}{n - 1}
\]

- **Standard Error Estimate**

\[
s_{MD} = \sqrt{\frac{s^2}{n}} = \frac{s}{\sqrt{n}}
\]

- **$t$ Statistic**

\[
t = \frac{M_D - \mu_D}{s_{MD}}
\]
HYPOTHESIS TESTS: PAIRED $t$

- Step 1: State Hypotheses & Select Alpha Level ($\alpha$)
- Step 2: Identify Critical Regions
  - $t$ distribution table
- Step 3: Compute Statistics
  - Variance
  - Estimate of Standard Error
  - $t$ Statistic
- Step 4: Make decision Regarding $H_0$
EFFECT SIZES: PAIRED $t$

- estimated $d$

$$d = \frac{M_D}{\sqrt{S^2}} = \frac{M_D}{S}$$

- $r^2$

$$r^2 = \frac{t^2}{t^2 + df}$$

http://www.uccs.edu/~lbecker/
Assumptions

- Observations are independent within each treatment
- Population distribution of difference ($D$) scores is normal

Uses/Advantages

- Requires fewer subjects (1 sample instead of 2)
- Evaluation of change over time
- Examine individual differences in response
Research Scenario: Matched Pairs Example

A researcher wants to know if a new ADHD medication reduces the number of behavioral disturbances that take place in the school setting. Students aged 8-10 are recruited from across the state and matched on key characteristics including sex, age, and number of behavioral disturbances during 30 days prior to joining the study. One member of each matched pair is assigned to the treatment condition (they receive the new medication daily), and the other is assigned to the control group. The number of behavioral disturbances of each child is recorded for 1 week. The data file includes the average number of daily behavioral disturbances reported for each child. Is there a difference in the number of behavioral disturbances between children in the treatment and control groups at the $\alpha = 0.05$ level.
Research Scenario: Pre-/Post- Example

A researcher wants to know if a new cold medication impacts reaction time. The reaction times of participants is measured right before and one hour after taking the medication. Using the data provided, determine whether there is a significant change in reaction time at the $\alpha = 0.05$ level.
PAIRED SAMPLES $t$: EXAMPLE

SPSS Output

### Paired Samples Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>N</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair 1 Var1</td>
<td>.500773</td>
<td>50</td>
<td>.2908328</td>
<td>.0411300</td>
</tr>
<tr>
<td>Var2</td>
<td>.986657</td>
<td>50</td>
<td>.3671497</td>
<td>.0519228</td>
</tr>
</tbody>
</table>

### Paired Samples Correlations

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Correlation</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair 1 Var1 &amp; Var2</td>
<td>50</td>
<td>.053</td>
<td>.716</td>
</tr>
</tbody>
</table>

### Paired Samples Test

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
<th>95% Confidence Interval of the Difference</th>
<th>t</th>
<th>df</th>
<th>Sig. (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair 1 Var1 - Var2</td>
<td>-4858838</td>
<td>.4562163</td>
<td>.0645187</td>
<td>-.6155390 - .3562289</td>
<td>-7.531</td>
<td>49</td>
<td>.000</td>
</tr>
</tbody>
</table>
### Excel Output

<table>
<thead>
<tr>
<th></th>
<th>Variable 1</th>
<th>Variable 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.500772757</td>
<td>0.986656537</td>
</tr>
<tr>
<td><strong>Variance</strong></td>
<td>0.0845837</td>
<td>0.13479887</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td><strong>Pearson Correlation</strong></td>
<td>0.052675192</td>
<td></td>
</tr>
<tr>
<td><strong>Hypothesized Mean Difference</strong></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td><strong>df</strong></td>
<td></td>
<td>49</td>
</tr>
<tr>
<td><strong>t Stat</strong></td>
<td>-7.530894641</td>
<td></td>
</tr>
<tr>
<td><strong>P(T&lt;=t) one-tail</strong></td>
<td>5.01483E-10</td>
<td></td>
</tr>
<tr>
<td><strong>t Critical one-tail</strong></td>
<td>1.676550893</td>
<td></td>
</tr>
<tr>
<td><strong>P(T&lt;=t) two-tail</strong></td>
<td>1.00297E-09</td>
<td></td>
</tr>
<tr>
<td><strong>t Critical two-tail</strong></td>
<td>2.009575237</td>
<td></td>
</tr>
</tbody>
</table>
PAIRED SAMPLES $t$: EXAMPLE

R Output

```r
> t.test(mydata$Var1, mydata$Var2, paired=TRUE)

Paired t-test

data:  mydata$Var1 and mydata$Var2
t = -7.5309, df = 49, p-value = 1.003e-09
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
    -0.6155390  -0.3562285
sample estimates:
    mean of the differences
        -0.4858838
```
Interpretation (APA Style Write-Up)

- Matched Pairs Example
  A significant difference in behavioral disturbances was found between students assigned to the new drug trial \((n = 50, \ M = 0.5008, \ SD = 0.29)\) and students not assigned to the new drug trial \((n = 50, \ M = 0.9867, \ SD = 0.37)\), \(t(49) = -7.531, \ p < 0.05, \ d = 1.46\). This indicates that students who were assigned to the new drug trial exhibited significantly fewer behavioral disturbances than their peers in the control group.

- Pre-/Post- Example
  Reaction time was found to have a mean of \(M = 0.5008 \ (n = 50, \ SD = 0.29)\) at time 1, and a mean of \(M = 0.9867 \ (n = 50, \ SD = 0.37)\) at time 2. A paired-samples \(t\) test was calculated for these data and it was determined that a significant increase in response rate was observed, \(t(49) = -7.531, \ p < 0.05, \ d = 1.46\).
WILCOXON SIGNED RANKS TEST

nonparametric analysis for within-subjects designs

WILCOXON SIGNED RANKS TEST

PAIRED SAMPLES T & WILCOXON SIGNED RANKS TESTS
WILCOXON SIGNED RANKS

- **When to Use**
  - Outcome data are not normally distributed
  - Outcome data are ranks instead of interval/ratio

- **Assumptions**
  - Data are paired; members of pairs from same population
  - Pairs selected randomly, independently

- **Process**
  - Same as paired samples $t$, but with ranks
  - Difference scores = 0 omitted from ranking, analysis
  - Ranks assigned by ascending order of absolute values
Research Scenario: Matched Pairs Example

A researcher wants to know if a new ADHD medication reduces the number of behavioral disturbances that take place in the school setting. Students aged 8-10 are recruited from across the state and matched on key characteristics including sex, age, and number of behavioral disturbances during 30 days prior to joining the study. One member of each matched pair is assigned to the treatment condition (they receive the new medication daily), and the other is assigned to the control group. The number of behavioral disturbances of each child is recorded for 1 week. The data file includes the average number of daily behavioral disturbances reported for each child; assume the data are not normally distributed. Is there a difference in the number of behavioral disturbances between children in the treatment and control groups at the $\alpha = 0.05$ level.

**INTERPRETATION OF RESULTS IS SAME AS FOR PAIRED-SAMPLES T TEST.**
Research Scenario: Pre-/Post- Example

A researcher wants to know if a new cold medication impacts reaction time. The reaction times of participants is measured right before and one hour after taking the medication. Assuming the data are not normally distributed, use the data provided to determine whether there is a significant change in reaction time at the $\alpha = 0.05$ level.

*INTERPRETATION OF RESULTS IS SAME AS FOR PAIRED-SAMPLES T TEST.*
WILCOXON SIGNED RANKS

SPSS Output

### Ranks

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean Rank</th>
<th>Sum of Ranks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Var2 - Var1</td>
<td>6a</td>
<td>16.00</td>
<td>96.00</td>
</tr>
<tr>
<td>Negative Ranks</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Positive Ranks</td>
<td>44b</td>
<td>26.80</td>
<td>1179.00</td>
</tr>
<tr>
<td>Ties</td>
<td>0c</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>50</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- a. Var2 < Var1
- b. Var2 > Var1
- c. Var2 = Var1

### Test Statistics

<table>
<thead>
<tr>
<th></th>
<th>Var2 - Var1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z</td>
<td>-5.227b</td>
</tr>
<tr>
<td>Asymp. Sig. (2-tailed)</td>
<td>.000</td>
</tr>
</tbody>
</table>

- a. Wilcoxon Signed Ranks Test
- b. Based on negative ranks.
**WILCOXON SIGNED RANKS**

- Excel Output

<table>
<thead>
<tr>
<th>sum of ranks (smaller)</th>
<th>96</th>
</tr>
</thead>
<tbody>
<tr>
<td>sample size</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>1179</td>
</tr>
</tbody>
</table>

*p < 0.01 (obtained from critical value table)*
WILCOXON SIGNED RANKS

R Output

```r
> wilcox.test(mydata$Var1, mydata$Var2, paired=TRUE)

Wilcoxon signed rank test with continuity correction

data:  mydata$Var1 and mydata$Var2
V = 96, p-value = 1.766e-07
alternative hypothesis: true location shift is not equal to 0
```