PAIRED SAMPLES *t* & WILCOXON SIGNED RANKS TESTS

Frances Chumney, PhD

CONTENT OUTLINE

- Within-Subjects Research Designs
- Paired Samples *t* Test
- Wilcoxon Signed Ranks Test

research design plays a major role in determining correct statistical approach

WITHIN-SUBJECTS DESIGNS

WITHIN-SUBJECTS DESIGN

research design in which a single sample is measured more than once on the same dependent variable

- Two sets of data from a single population/sample
  - Example: Comparison of heart rate before & after running a marathon
- A.K.A. Dependent-Measures, Paired Samples, Repeated-Measures

WITHIN-SUBJECTS DESIGN

- Hypotheses
  - Null
    - H_0: $X_1 - X_2 = 0$ → There is no change in scores from time 1 to time 2
    - H_0: $X_1 = X_2$ → Scores at time 1 are equal to scores at time 2
    - H_0:$\mu_D = 0$ → The difference in scores from time 1 to time 2 is equal to 0
  - Alternative
    - H_A:$X_1 - X_2 \neq 0$ → There is a change in scores from time 1 to time 2
    - H_A:$X_1 \neq X_2$ → Scores at time 1 are not equal to scores at time 2
    - H_A:$\mu_D \neq 0$ → The difference in scores from time 1 to time 2 is not equal to 0

parametric analysis for within-subjects designs

PAIRED SAMPLES *t* TEST

PAIRED SAMPLES *t*

- The Basics
  - Similar to single-sample *t* (or *z*-score to compare sample to population)
  - Based on difference scores instead of raw scores
    - Matched Pairs (individuals from different groups matched by key characteristics)
    - Pre-/Post- Designs
  - Goal: use difference scores to answer questions about population
- Difference Scores
  - $D = X_2 - X_1$
  - Measures amount of change for each subject
  - Increase in score = positive value of *D*
  - Decrease in score = negative value of *D*

PAIRED SAMPLES *t*

- Difference Scores (Matched Pairs)

Pair	Control Group (X1)	Treatment Group (X2)	Difference (D)
Matched Pair A	215	210	-5
Matched Pair B	221	242	21
Matched Pair C	196	219	23
Matched Pair D	203	228	25

$$M_D = \frac{\sum D}{n} = \frac{64}{4} = 16$$

PAIRED SAMPLES *t*

- Difference Scores (Pre-/Post- Design)

Person	Before Medication (X1)	After Medication (X2)	Difference (D)
A	215	210	-5
B	221	242	21
C	196	219	23
D	203	228	25

$$M_D = \frac{\sum D}{n} = \frac{64}{4} = 16$$

FORMULAS FOR PAIRED SAMPLES *t*

- Variance

$$s^2 = \frac{SS}{df} = \frac{SS}{n-1}$$

- Standard Error Estimate

$$s_{M_D} = \sqrt{\frac{s^2}{n}} = \frac{s}{\sqrt{n}}$$

- *t* Statistic

$$t = \frac{M_D - \mu_D}{s_{M_D}}$$

HYPOTHESIS TESTS: PAIRED *t*

- Step 1: State Hypotheses & Select Alpha Level (α)
- Step 2: Identify Critical Regions
  - *t* distribution table
- Step 3: Compute Statistics
  - Variance
  - Estimate of Standard Error
  - *t* Statistic
- Step 4: Make decision Regarding H_0

EFFECT SIZES: PAIRED *t*

- estimated d

$$d = \frac{M_D}{\sqrt{s^2}} = \frac{M_D}{s}$$

  - http://www.uccs.edu/~lbecker/

- r^2

$$r^2 = \frac{t^2}{t^2 + df}$$

PAIRED SAMPLES *t*

- Assumptions
  - Observations are independent within each treatment
  - Population distribution of difference (*D*) scores is normal

- Uses/Advantages
  - Requires fewer subjects (1 sample instead of 2)
  - Evaluation of change over time
  - Examine individual differences in response

PAIRED SAMPLES *t*: EXAMPLE

- Research Scenario: Matched Pairs Example

  A researcher wants to know if a new ADHD medication reduces the number of behavioral disturbances that take place in the school setting. Students aged 8-10 are recruited from across the state and matched on key characteristics including sex, age, and number of behavioral disturbances during 30 days prior to joining the study. One member of each matched pair is assigned to the treatment condition (they receive the new medication daily), and the other is assigned to the control group. The number of behavioral disturbances of each child is recorded for 1 week. The data file includes the average number of daily behavioral disturbances reported for each child. Is there a difference in the number of behavioral disturbances between children in the treatment and control groups at the $\alpha = 0.05$ level.

PAIRED SAMPLES *t*: EXAMPLE

- Research Scenario: Pre-/Post- Example

  A researcher wants to know if a new cold medication impacts reaction time. The reaction times of participants is measured right before and one hour after taking the medication. Using the data provided, determine whether there is a significant change in reaction time at the $\alpha = 0.05$ level.

PAIRED SAMPLES *t*: EXAMPLE

- SPSS Output

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Var1	.500773	50	.2908328	.0411300
	Var2	.986657	50	.3671497	.0519228

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	Var1 & Var2	50	.053	.716

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	Var1 - Var2	-.4858838	.4562163	.0645187	-.6155390	-.3562285	-7.531	49	.000

PAIRED SAMPLES *t*: EXAMPLE

- Excel Output

t-Test: Paired Two Sample for Means		
	Variable 1	*Variable 2*
Mean	0.500772757	0.986656537
Variance	0.0845837	0.13479887
Observations	50	50
Pearson Correlation	0.052675192	
Hypothesized Mean Difference	0	
df	49	
t Stat	-7.530894641	
P(T<=t) one-tail	5.01483E-10	
t Critical one-tail	1.676550893	
P(T<=t) two-tail	1.00297E-09	
t Critical two-tail	2.009575237	

PAIRED SAMPLES *t*: EXAMPLE

- R Output

```
> t.test(mydata$Var1,mydata$Var2, paired=TRUE)

        Paired t-test

data:  mydata$Var1 and mydata$Var2
t = -7.5309, df = 49, p-value = 1.003e-09
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.6155390 -0.3562285
sample estimates:
mean of the differences
             -0.4858838
```

PAIRED SAMPLES *t*: EXAMPLE

- Interpretation (APA Style Write-Up)
  - Matched Pairs Example

    A significant difference in behavioral disturbances was found between students assigned to the new drug trial ($n = 50$, $M = 0.5008$, $SD = 0.29$) and students not assigned to the new drug trial ($n = 50$, $M = 0.9867$, $SD = 0.37$), $t(49) = -7.531$, $p < 0.05$, $d = 1.46$. This indicates that students who were assigned to the new drug trial exhibited significantly fewer behavioral disturbances than their peers in the control group.

  - Pre-/Post- Example

    Reaction time was found to have a mean of $M = 0.5008$ ($n = 50$, $SD = 0.29$) at time 1, and a mean of $M = 0.9867$ ($n = 50$, $SD = 0.37$) at time 2. A paired-samples *t* test was calculated for these data and it was determined that a significant increase in response rate was observed, $t(49) = -7.531$, $p < 0.05$, $d = 1.46$.

nonparametric analysis for within-subjects designs

WILCOXON SIGNED RANKS TEST

WILCOXON SIGNED RANKS

- When to Use
  - Outcome data are not normally distributed
  - Outcome data are ranks instead of interval/ratio
- Assumptions
  - Data are paired; members of pairs from same population
  - Pairs selected randomly, independently
- Process
  - Same as paired samples *t*, but with ranks
  - Difference scores = 0 omitted from ranking, analysis
  - Ranks assigned by ascending order of absolute values

WILCOXON SIGNED RANKS

- Research Scenario: Matched Pairs Example

  A researcher wants to know if a new ADHD medication reduces the number of behavioral disturbances that take place in the school setting. Students aged 8-10 are recruited from across the state and matched on key characteristics including sex, age, and number of behavioral disturbances during 30 days prior to joining the study. One member of each matched pair is assigned to the treatment condition (they receive the new medication daily), and the other is assigned to the control group. The number of behavioral disturbances of each child is recorded for 1 week. The data file includes the average number of daily behavioral disturbances reported for each child; assume the data are not normally distributed. Is there a difference in the number of behavioral disturbances between children in the treatment and control groups at the $\alpha = 0.05$ level.

INTERPRETATION OF RESULTS IS SAME AS FOR PAIRED-SAMPLES T TEST.

WILCOXON SIGNED RANKS

- Research Scenario: Pre-/Post- Example

  A researcher wants to know if a new cold medication impacts reaction time. The reaction times of participants is measured right before and one hour after taking the medication. Assuming the data are not normally distributed, use the data provided to determine whether there is a significant change in reaction time at the $\alpha = 0.05$ level.

INTERPRETATION OF RESULTS IS SAME AS FOR PAIRED-SAMPLES T TEST.

WILCOXON SIGNED RANKS

- SPSS Output

Ranks

		N	Mean Rank	Sum of Ranks
Var2 - Var1	Negative Ranks	6[a]	16.00	96.00
	Positive Ranks	44[b]	26.80	1179.00
	Ties	0[c]		
	Total	50		

a. Var2 < Var1

b. Var2 > Var1

c. Var2 = Var1

Test Statistics[a]

	Var2 - Var1
Z	-5.227[b]
Asymp. Sig. (2-tailed)	.000

a. Wilcoxon Signed Ranks Test

b. Based on negative ranks.

WILCOXON SIGNED RANKS

- Excel Output

sum of ranks (smaller)	96	1179
sample size	50	

$p < 0.01$ (obtained from critical value table)

WILCOXON SIGNED RANKS

- R Output

```
> wilcox.test(mydata$Var1,mydata$Var2, paired=TRUE)

        Wilcoxon signed rank test with continuity correction

data:  mydata$Var1 and mydata$Var2
V = 96, p-value = 1.766e-07
alternative hypothesis: true location shift is not equal to 0
```