



PAIRED SAMPLES t & WILCOXON SIGNED RANKS TESTS

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TRANSCRIPT:

This is Dr. Chumney with a brief overview of the paired samples t test and the Wilcoxon signed ranks test.

CONTENT OUTLINE

- Within-Subjects Research Designs
- Paired Samples t Test
- Wilcoxon Signed Ranks Test

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TRANSCRIPT:

In this brief lecture, I will first provide a quick reminder of what within-subjects research is, and then explain how and when to use paired samples t tests and Wilcoxon tests.

research design plays a major role in determining correct statistical approach

WITHIN-SUBJECTS DESIGNS

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TRANSCRIPT:

Paired samples t and Wilcoxon tests are analytic methods that are appropriate for within-subjects research designs, so let's start with a quick reminder of what a within-subjects design looks like.

WITHIN-SUBJECTS DESIGN

research design in which a single sample is measured more than once on the same dependent variable

- Two sets of data from a single population/sample
 - ❖ Example: Comparison of heart rate before & after running a marathon
- A.K.A. Dependent-Measures, Paired Samples, Repeated-Measures

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TRANSCRIPT:

A within-subjects design is an approach to research in which one sample is measured more than once on the same dependent variable. In other words, we have two sets of responses just like with between-subjects designs, but now those data come from a single sample instead of from two separate samples that we want to compare.

Within-subjects is a type of research design – which is also sometimes called dependent-measures design - is sort of an umbrella term under which design choices such as paired samples, and repeated-measures fall. In a repeated-measures analysis, we have measured each person in the sample twice, as in a pre-/post- design. In a paired samples analysis, data are collected just once on each participant, but the participants come from the same sample and are matched in pairs based on key characteristics. For example, if a paired samples approach were used to examine the effects of a new drug, a sample would be selected and then members of that sample would be assigned to pairs based on sex, age, ethnicity, and other variables that the researcher believes are important for consideration. One member of each pair is then assigned to one condition, while the other member of each pair is assigned to another condition. When the data are analyzed, the data from one group is compared to the data from the other group, but the analysis accounts for the fact that the data are matched or paired.

WITHIN-SUBJECTS DESIGN

➤ Hypotheses

❖ Null

- $H_0: X_1 - X_2 = 0 \rightarrow$ There is no change in scores from time 1 to time 2
- $H_0: X_1 = X_2 \rightarrow$ Scores at time 1 are equal to scores at time 2
- $H_0: \mu_D = 0 \rightarrow$ The difference in scores from time 1 to time 2 is equal to 0

❖ Alternative

- $H_A: X_1 - X_2 \neq 0 \rightarrow$ There is a change in scores from time 1 to time 2
- $H_A: X_1 \neq X_2 \rightarrow$ Scores at time 1 are not equal to scores at time 2
- $H_A: \mu_D \neq 0 \rightarrow$ The difference in scores from time 1 to time 2 is not equal to 0

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TRANSCRIPT:

In terms of hypothesis, the null hypothesis for a within-subjects design is always that there is no change in scores from time 1 to time 2, or that scores at time 1 are equal to scores at time 2, or that the difference in scores from time 1 to time 2 is equal to 0. The alternative hypotheses, then, are the opposites of those statement.

In the case of paired samples data, time 1 refers to the data from the half of each match assigned to group 1, and time 2 refers to the data from the half of each match assigned to group 2.

parametric analysis for within-subjects designs

PAIRED SAMPLES t TEST

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TRANSCRIPT:

When we have data from a repeated measures or paired samples design, and those data consist of a single outcome, the most appropriate statistical methods are the paired samples t test and its nonparametric equivalent, the Wilcoxon test.

PAIRED SAMPLES t

➤ The Basics

- ❖ Similar to single-sample t (or z-score to compare sample to population)
- ❖ Based on difference scores instead of raw scores
 - Matched Pairs (individuals from different groups matched by key characteristics)
 - Pre-/Post- Designs
- ❖ Goal: use difference scores to answer questions about population

➤ Difference Scores

- ❖ $D = X_2 - X_1$
- ❖ Measures amount of change for each subject
- ❖ Increase in score = positive value of D
- ❖ Decrease in score = negative value of D

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TRANSCRIPT:

The paired samples t test is similar to a single-sample t test or z-score test. The difference is that a paired samples t test calculates difference scores between the two sets of observations, and the analysis is based on those difference scores. The goal of paired samples t test – like other inferential statistical approaches – is to use the difference scores of the observed data to make generalizations about the population.

Difference scores are calculated the same way as for other statistics, where one set of scores is subtracted from the other set of scores. Ultimately, it does not matter which variable serves as X_1 or X_2 , but that decision does impact interpretation so it is important to know which variable is being used. It is also important to assign one variable to each conceptual label, and to not switch them back and forth within a single analysis.

PAIRED SAMPLES t

➤ Difference Scores (Matched Pairs)

Pair	Control Group (X1)	Treatment Group (X2)	Difference (D)
Matched Pair A	215	210	-5
Matched Pair B	221	242	21
Matched Pair C	196	219	23
Matched Pair D	203	228	25

$$M_D = \frac{\sum D}{n} = \frac{64}{4} = 16$$

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TRANSCRIPT:

Here are some example data that might be analyzed using a paired samples t test. These data are for matched pairs, where one variable is observations for the control group and the other variable is observations for the treatment group. The data appear on the same row of the data file because the participants have been matched on some relevant characteristics.

PAIRED SAMPLES t

➤ Difference Scores (Pre-/Post- Design)

Person	Before Medication (X1)	After Medication (X2)	Difference (D)
A	215	210	-5
B	221	242	21
C	196	219	23
D	203	228	25

$$M_D = \frac{\sum D}{n} = \frac{64}{4} = 16$$

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TRANSCRIPT:

This is the same set of data as on the previous slide, but now it is organized to reflect the data that might be collected in a repeated measures study. One variable is our pre-test or time 1 data, and the other variable is the post-test or time 2 data. In this example, the multiple observations appear on the same row because they belong to the same participant.

FORMULAS FOR PAIRED SAMPLES t

➤ Variance

$$s^2 = \frac{SS}{df} = \frac{SS}{n - 1}$$

➤ Standard Error Estimate

$$s_{M_D} = \sqrt{\frac{s^2}{n}} = \frac{s}{\sqrt{n}}$$

➤ t Statistic

$$t = \frac{M_D - \mu_D}{s_{M_D}}$$

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TRANSCRIPT:

Here are the key formulas that are used to calculate a paired samples t statistic. Calculate variance, then an estimate of the standard error for the mean difference scores, and then the t statistic.

HYPOTHESIS TESTS: PAIRED t

- Step 1: State Hypotheses & Select Alpha Level (α)
- Step 2: Identify Critical Regions
 - ❖ t distribution table
- Step 3: Compute Statistics
 - ❖ Variance
 - ❖ Estimate of Standard Error
 - ❖ t Statistic
- Step 4: Make decision Regarding H_0

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TRANSCRIPT:

This slide outlines the steps that are followed in a hypothesis test for a paired samples t test. This process is no different than the process for independent t and most other inferential statistics.

EFFECT SIZES: PAIRED t

➤ estimated d

$$d = \frac{M_D}{\sqrt{S^2}} = \frac{M_D}{s}$$

❖ <http://www.uccs.edu/~lbecker/>

➤ r^2

$$r^2 = \frac{t^2}{t^2 + df}$$

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TRANSCRIPT:

As we have discussed before for other statistics, we should calculate an effect size for paired samples t tests. To calculate Cohen's d effect size, use the online effect size calculator, and enter the means and standard deviations for each variable in the analysis.

PAIRED SAMPLES t

➤ Assumptions

- ❖ Observations are independent within each treatment
- ❖ Population distribution of difference (D) scores is normal

➤ Uses/Advantages

- ❖ Requires fewer subjects (1 sample instead of 2)
- ❖ Evaluation of change over time
- ❖ Examine individual differences in response

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TRANSCRIPT:

The advantages of paired samples t over other analytic approaches is that the research design requires fewer subjects and allows an examination of change over time.

Like other methods, though, there are some assumptions that should be met when using the paired samples t test. The most important of these assumptions is that observations should be independent within each time point or treatment group. This means that the value of the dependent variable for each participant should depend only on that participant, and should not be influenced by the data collected for the other participants in the sample. It is also assumed that the difference scores are normally distributed. When this assumption is not met, the nonparametric alternative to the paired samples t test should be used instead.

PAIRED SAMPLES t : EXAMPLE

➤ Research Scenario: Matched Pairs Example

A researcher wants to know if a new ADHD medication reduces the number of behavioral disturbances that take place in the school setting. Students aged 8-10 are recruited from across the state and matched on key characteristics including sex, age, and number of behavioral disturbances during 30 days prior to joining the study. One member of each matched pair is assigned to the treatment condition (they receive the new medication daily), and the other is assigned to the control group. The number of behavioral disturbances of each child is recorded for 1 week. The data file includes the average number of daily behavioral disturbances reported for each child. Is there a difference in the number of behavioral disturbances between children in the treatment and control groups at the $\alpha = 0.05$ level.

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TRANSCRIPT:

Here we have an example research scenario laid out as if we are going to examine data from a matched pairs design. Notice that the sample is matched into pairs, and one member of each pair is assigned to each treatment group.

PAIRED SAMPLES t : EXAMPLE

➤ Research Scenario: Pre-/Post- Example

A researcher wants to know if a new cold medication impacts reaction time. The reaction times of participants is measured right before and one hour after taking the medication. Using the data provided, determine whether there is a significant change in reaction time at the $\alpha = 0.05$ level.

PAIRED SAMPLES T & WILCOXON SIGNED RANKS TESTS



TRANSCRIPT:

Here we have an example research scenario laid out as if we are going to examine data from a repeated measures design with pre-/post- data. Notice that each participant is measured twice.

PAIRED SAMPLES t : EXAMPLE

➤ SPSS Output

Paired Samples Statistics

	Mean	N	Std. Deviation	Std. Error Mean
Pair 1 Var1	.500773	50	.2908328	.0411300
Var2	.986657	50	.3671497	.0519228

Paired Samples Correlations

	N	Correlation	Sig.
Pair 1 Var1 & Var2	50	.053	.716

Paired Samples Test

	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference		t	df	Sig. (2-tailed)
				Lower	Upper			
Pair 1 Var1 - Var2	-.4858838	.4562163	.0645187	-.6155390	-.3562286	-7.531	49	.000

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TRANSCRIPT:

For these output examples, the data used would have been appropriate for either of the two scenarios you just viewed – either the matched pairs or repeated measures examples.

This excerpt from SPSS output includes the descriptive information for each variable, the bivariate correlation between the two variables, the t statistic, degrees of freedom, and p value. When the p value is less than the alpha level selected for the test, we determine that the test is significant and we reject the null hypothesis.

PAIRED SAMPLES t : EXAMPLE

➤ Excel Output

t-Test: Paired Two Sample for Means		
	Variable 1	Variable 2
Mean	0.500772757	0.986656537
Variance	0.0845837	0.13479887
Observations	50	50
Pearson Correlation	0.052675192	
Hypothesized Mean Difference	0	
df	49	
t Stat	-7.530894641	
P(T<=t) one-tail	5.01483E-10	
t Critical one-tail	1.676550893	
P(T<=t) two-tail	1.00297E-09	
t Critical two-tail	2.009575237	

PAIRED SAMPLES T & WILCOXON SIGNED RANKS TESTS



TRANSCRIPT:

Here is an excerpt of the same output when the data have been analyzed in Excel.

PAIRED SAMPLES t : EXAMPLE

➤ R Output

```
> t.test(mydata$Var1,mydata$Var2, paired=TRUE)

Paired t-test

data: mydata$Var1 and mydata$Var2
t = -7.5309, df = 49, p-value = 1.003e-09
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.6155390 -0.3562285
sample estimates:
mean of the differences
      -0.4858838
```

PAIRED SAMPLES T & WILCOXON SIGNED RANKS TESTS



TRANSCRIPT:

Here is an excerpt of the same output when the data have been analyzed in R.

PAIRED SAMPLES t : EXAMPLE

➤ Interpretation (APA Style Write-Up)

❖ Matched Pairs Example

A significant difference in behavioral disturbances was found between students assigned to the new drug trial ($n = 50$, $M = 0.5008$, $SD = 0.29$) and students not assigned to the new drug trial ($n = 50$, $M = 0.9867$, $SD = 0.37$), $t(49) = -7.531$, $p < 0.05$, $d = 1.46$. This indicates that students who were assigned to the new drug trial exhibited significantly fewer behavioral disturbances than their peers in the control group.

❖ Pre-/Post- Example

Reaction time was found to have a mean of $M = 0.5008$ ($n = 50$, $SD = 0.29$) at time 1, and a mean of $M = 0.9867$ ($n = 50$, $SD = 0.37$) at time 2. A paired-samples t test was calculated for these data and it was determined that a significant increase in response rate was observed, $t(49) = -7.531$, $p < 0.05$, $d = 1.46$.

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TRANSCRIPT:

Here is an example interpretation of these results for each of the research scenarios presented. Notice that the sample size, mean, and standard deviation for each variable is included, as well as the t statistic and effect size.

nonparametric analysis for within-subjects designs

WILCOXON SIGNED RANKS TEST

PAIRED SAMPLES T & WILCOXON SIGNED RANKS TESTS 

TRANSCRIPT:

In instances when the data are ordinal or the assumptions of the paired samples t test are not met, it is appropriate to use the nonparametric alternative to the paired samples t test, and that is the Wilcoxon signed ranks test, which we often just refer to as the Wilcoxon test.

WILCOXON SIGNED RANKS

- When to Use
 - ❖ Outcome data are not normally distributed
 - ❖ Outcome data are ranks instead of interval/ratio
- Assumptions
 - ❖ Data are paired; members of pairs from same population
 - ❖ Pairs selected randomly, independently
- Process
 - ❖ Same as paired samples t , but with ranks
 - ❖ Difference scores = 0 omitted from ranking, analysis
 - ❖ Ranks assigned by ascending order of absolute values

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TRANSCRIPT:

As I mentioned, the Wilcoxon test is the nonparametric equivalent to the paired samples t test, and should be used when the dependent variable consists of ranks, ordinal data, or is not normally distributed.

The Wilcoxon test does impose its own assumptions, and they are that the data are paired and that the members of each pair are members of a single population.

The process for calculating the test statistic and p value for the Wilcoxon test are the same as the process used for the paired samples t test. The only difference is that the Wilcoxon test conducts the analysis based on ranks assigned to the data instead of the raw data values themselves. First, the difference scores are calculated, and all difference scores equal to the value specified by the null hypothesis (which is typically 0) are omitted. The remaining difference scores are ranked in ascending order based on their absolute values.

WILCOXON SIGNED RANKS

➤ Research Scenario: Matched Pairs Example

A researcher wants to know if a new ADHD medication reduces the number of behavioral disturbances that take place in the school setting. Students aged 8-10 are recruited from across the state and matched on key characteristics including sex, age, and number of behavioral disturbances during 30 days prior to joining the study. One member of each matched pair is assigned to the treatment condition (they receive the new medication daily), and the other is assigned to the control group. The number of behavioral disturbances of each child is recorded for 1 week. The data file includes the average number of daily behavioral disturbances reported for each child; assume the data are not normally distributed. Is there a difference in the number of behavioral disturbances between children in the treatment and control groups at the $\alpha = 0.05$ level.

INTERPRETATION OF RESULTS IS SAME AS FOR PAIRED-SAMPLES T TEST.

PAIRED SAMPLES T & WILCOXON SIGNED RANKS TESTS



TRANSCRIPT:

Here is an example research scenario for a matched pairs design for which the Wilcoxon test is an appropriate data analytic technique.

WILCOXON SIGNED RANKS

➤ Research Scenario: Pre-/Post- Example

A researcher wants to know if a new cold medication impacts reaction time. The reaction times of participants is measured right before and one hour after taking the medication. Assuming the data are not normally distributed, use the data provided to determine whether there is a significant change in reaction time at the $\alpha = 0.05$ level.

INTERPRETATION OF RESULTS IS SAME AS FOR PAIRED-SAMPLES T TEST.

PAIRED SAMPLES T & WILCOXON SIGNED RANKS TESTS



TRANSCRIPT:

Here is an example research scenario for a repeated measures design for which the Wilcoxon test is an appropriate data analytic technique.

Once the analysis is complete, the interpretation of the Wilcoxon test results is identical to the interpretation of the results of a paired samples t test.

WILCOXON SIGNED RANKS

➤ SPSS Output

Ranks				
		N	Mean Rank	Sum of Ranks
Var2 - Var1	Negative Ranks	6 ^a	16.00	96.00
	Positive Ranks	44 ^b	26.80	1179.00
	Ties	0 ^c		
	Total	50		

- a. Var2 < Var1
- b. Var2 > Var1
- c. Var2 = Var1

Test Statistics^a

	Var2 - Var1
Z	-5.227 ^b
Asymp. Sig. (2-tailed)	.000

- a. Wilcoxon Signed Ranks Test
- b. Based on negative ranks.

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TRANSCRIPT:

This slide includes an excerpt of SPSS output from a Wilcoxon test. The test statistic and p value are calculated based on the sum of the negative ranks. If the p value is smaller than the critical value or the selected alpha value, then we conclude the test is significant and we reject the null hypothesis.

WILCOXON SIGNED RANKS

➤ Excel Output

sum of ranks (smaller)	96	1179
sample size	50	
p < 0.01 (obtained from critical value table)		

PAIRED SAMPLES T & WILCOXON SIGNED RANKS TESTS



TRANSCRIPT:

Here is an example of some Excel output. The only catch with Excel is that once you have the sum of the ranks, you use the smaller of the two sum of ranks values along with sample size and then compare those numbers to a table of critical values similar to the unit normal table to determine the approximate p value.

WILCOXON SIGNED RANKS

➤ R Output

```
> wilcox.test(mydata$Var1,mydata$Var2, paired=TRUE)

Wilcoxon signed rank test with continuity correction

data: mydata$Var1 and mydata$Var2
V = 96 p-value = 1.766e-07
alternative hypothesis: true location shift is not equal to 0
```

PAIRED SAMPLES T & WILCOXON SIGNED RANKS TESTS



TRANSCRIPT:

Finally, here is an excerpt of output for a Wilcoxon test computed in R.