# INTEGERS CONFERENCE 2009

University of West Georgia
Carrollton, Georgia

## Abstracts

1. **Max Alekseyev**, University of South Carolina

### On the Intersections of Fibonacci, Pell, and Lucas Numbers

We show how to compute the intersection of two Lucas sequences of the forms $\{U_n(P,\pm1)\}_{n=0}^{\infty}$ or $\{V_n(P,\pm1)\}_{n=0}^{\infty}$ with $P \in \mathbb{Z}$, including Fibonacci, Pell, Lucas, and Lucas-Pell numbers. Our approach relies on solving homogeneous quadratic Diophantine equations including Thue equations. We describe how to compute such an intersection and prove that it may be infinite only for two $V$-sequences when the product of their discriminants is a perfect square. In particular, we prove that 0, 1, 2, and 5 are the only numbers that are both Fibonacci and Pell and list similar results for many other pairs of Lucas sequences.

2. **Geoffrey Apel**, Aurora University

### Proving Balanced $T^2$ and $Q^2$ Identities using Modular Forms

Abstract. In our paper "Algorithms for finding and proving balanced $Q^2$ identities", which we presented at Integers Conference 2005, we pointed out that the proof algorithm in that paper had not been able to prove 1,417 of the 97,306 tentative identities which the paper's search algorithm had found. This meant, of course, that this algorithm was inadequate and that we needed a better proof method. In this paper, we develop the following proof algorithm which uses a familiar proof technique in the theory of modular forms: Move the terms of the given $T^2$ or $Q^2$ identity onto the left side of the equation and expand the left side into a power series up to a pre-determined degree $L$. If the first $L+1$ terms of this series are all zero, then the identity is true. When this algorithm was applied to the 1,417 recalcitrant identities, it showed they were all true as hoped.

Joint work with Richard Blecksmith, Northern Illinois University (richard@math.niu.edu) and John Brillhart, The University of Arizona (jdb@math.arizona.edu).

3. **William Banks**, University of Missouri

### Carmichael Numbers in Arithmetic Progressions

Assuming a weak version of a conjecture of Heath-Brown on the least prime in a residue class, we show that for any coprime integers $a$ and $m \geq 1$, there are infinitely many Carmichael numbers in the arithmetic progression $a \bmod m$. This is joint work with Carl Pomerance.

Joint work with Carl Pomerance, Dartmouth College.

4. **Gautami Bhowmik**, Universite de Lille

**On the Alon Dubiner Constant**

One of the topics that we study in zero-sum sequences is the largest size of a zero-sum free sequence. In general, explicit results are not available and we are quite happy to find good bounds instead. The following is one such example. Let $D_k(G)$ be the smallest integer $n$ such that every sequence of $n$ elements of a finite abelian group $G$ contains a zero-sum sequence of length at most $k$. Alon and Dubiner (1992) proved that $D_p(Z_p^r) < c(r)p$, where $p$ is a prime number and $c(r)$ an absolute constant, but this constant is way too big to be computationally useful. We improve it largely by using exponential sums and combinatorial methods.

Joint work with J-C Schlage-Puchta, Universität Freiburg, Germany.

5. **Steve Butler**, University of California Los Angeles

**Tiling Polygons with Lattice Triangles**

Given a simple polygon with rational coordinates having one vertex at the origin and an adjacent vertex on the $x$-axis, we look at the problem of the location of the vertices for a tiling of the polygon using lattice triangles (i.e., triangles which are congruent to a triangle with the coordinates of the vertices being integer). We show that the coordinates of the vertices in any tiling are rational with the possible denominators odd numbers dependent on the cotangents of the angles in the triangles.

Joint work with Fan Chung and Ron Graham, UCSD, and Miklos Laczkovich, Eotvos Lorand University.

6. **Jonathan Chappelon**, Université du Littoral Côte d'Opale

**A Universal Sequence of Integers Generating Balanced Steinhaus Figures**

A Steinhaus triangle in $\mathbb{Z}/n\mathbb{Z}$ is a finite triangle of numbers modulo $n$ that appears in the cellular automaton generating the standard Pascal triangle modulo $n$. In this talk, we discuss an open problem, due to Molluzzo in 1976, about the existence of *balanced* Steinhaus triangles, that are Steinhaus triangles containing all the elements of $\mathbb{Z}/n\mathbb{Z}$ with the same multiplicity. Although its formulation is elementary, this problem turns out to be very difficult. We show that there exists an integer sequence, that I call universal, which generates infinitely many balanced Steinhaus triangles in every $\mathbb{Z}/n\mathbb{Z}$ with $n$ odd. More precisely, we explicitly obtain the existence of balanced Steinhaus triangles for at least $2/3$ of the admissible sizes, i.e. for the Steinhaus triangles in $\mathbb{Z}/n\mathbb{Z}$ of cardinality divisible by $n$, in case $n$ is an odd prime number. Finally, we also consider other figures, like lozenges or parallelograms, in this cellular automaton and the analogous problem of determining the balanced ones.

7. **Yuan-You Fu-Rui Cheng** University of North Carolina at Chapel Hill

**Analytic Implications from the Remainder of the Prime Number Theorem**

It is well known that the distribution of the prime numbers plays a central role in number theory. It has been known, since Riemann's memoir in 1860, that the distribution of prime numbers can be described by the zero-free region of the Riemann zeta function $\zeta(s)$. This function has infinitely many zeros and a unique pole at $s = 1$. Those zeros at $s = -2, -4, -6, \ldots$ are known as trivial zeros. The nontrivial zeros of $\zeta(s)$ are all located in the so-called critical strip $0 < \Re(s) < 1$. Define $\Lambda(n) = \log p$ whenever $n = p^m$ for a prime number $p$ and a positive integer $m$, and zero otherwise. Let $x \geq 2$. The $\psi$-form of the prime number theorem is $\psi(x) = \sum_{n \leq x} \Lambda(n) = x + O\big(x^{H(x)} \log^2 x\big)$, where the sum runs through the set of positive integers and $H(x)$ is a certain function of $x$ with $\frac{1}{2} \leq H(x) < 1$. Turán proved in 1950 this $\psi$-form implies that there are no zeros of $\zeta(s)$ for $\Re(s) > h(t)$, where $t = \Im(s)$, and $h(t)$ is a function connected to $H(x)$ in a certain way with $1/2 \leq h(t) < 1$. We prove a result equivalent to Turan's with $H(x)$ and $h(t)$ being altered to suit our application. Utilizing this equivalent result, we show that the above $\psi$-form implies that $-\frac{\zeta'(s)}{\zeta(s)} - \zeta(s)$ is defined by $\sum_{n=1}^{\infty} \frac{\Lambda(n)-1}{n^s}$ for $\Re(s) > h(t)$. The proof involves Turán's power sum method and the classical integral formula for the Heaviside function $H(x)$, where $H(x)$ is 0 when $x < 1$, $\frac{1}{2}$ when $x = 1$, and 1 when $x > 1$.

8. **Jean-Marc Deshouillers**, Institut de Math'ematiques de Bordeaux

**On the Distribution modulo 1 of Mean Values of Arithmetic Functions**

Arithmetic functions usually have an erratic behaviour (cf. the seventy year old Erdős - Wintner Theorem). One may however expect that averaging those values may lead to a smoother behaviour: we shall discuss this question, focusing on the answer given by Henryk Iwaniec and myself to a question raised by Florian Luca.

9. **Carrie E. Finch**, Washington & Lee University

**Special Sierpiński Numbers**

A Sierpiński number is an odd positive integer $k$ with the property that $k \cdot 2^n + 1$ is composite for all natural numbers $n$. In this talk, we present a survey of results concerning special Sierpiński numbers, such as Sierpiński numbers in the Fibonacci sequence (due to F. Luca and J. Mejía), consecutive Sierpiński numbers (due to Y.-G. Chen), and Sierpiński numbers that are also Riesel numbers (also due to Y.-G. Chen). We then present recent work concerning consecutive integers that are Sierpiński-like (joint work with M. Kozek and G. Dresden), and Sierpiński numbers of particular polynomial forms (joint work with L. Jones and J. Harrington).

Joint work with Gregory P. Dresden, Washington & Lee University; Josh Harrington, University of South Carolina; Lenny K. Jones, Shippensburg University; and Mark R. Kozek, Whittier College.

10. **Kevin Ford**, University of Illinois at Urbana-Champaign

**Prime Chains and Applications**

A sequence of primes $p_1, \ldots, p_k$ is a prime chain if $p_{j+1} \equiv 1 \pmod{p_j}$ for each $j$. For example: 3, 7, 29, 59. We describe new estimates for counts of prime chains satisfying various properties, e.g. the number of chains with $p_k \le x$ and the number of chains with $p_1 = p$ and $p_k \le x$. We discuss some applications of these estimates, in particular the settling of a 50-year old conjecture of Erdős that $\phi(a) = \sigma(b)$ has infinitely many solutions ($\phi$ is Euler's function, $\sigma$ is the sum of divisors function). We also focus on the distribution of $H(p)$, the length of the longest chain ending at a given prime $p$. $H(p)$ has another interpretation as the height of the Pratt tree for a prime $p$, defined recursively as the tree with root node $p$, and below $p$ are links to the prime factors $q$ of $p-1$, below each $q$ are links to the prime factors of $q-1$, and so on. In 1975, Pratt used this tree in conjunction with Lucas' 1876 primality proving method to show that every prime has a short certificate (proof) of primality. We give new, nontrivial bounds for $H(p)$, valid for almost all $p$. We describe a random model of the tree, based on branching random walks, which leads to some surprising conjectures about the distribution of $H(p)$.

Joint work with Sergei Konyagin, Moscow Lomonosov State University, Russia and Florian Luca, Universidad National Autonoma de Mexico.

11. **Aviezri S. Fraenkel**, Weizmann Institute of Science

**Tromping Games**

The game of Domineering is a combinatorial game that has been solved for several boards, including the standard $8 \times 8$ board. We create new partizan – and some impartial – combinatorial games by using trominos instead of and along with dominoes. We analyze these Tromping games for some small boards providing a dictionary of values. Further, we prove properties that permit expressing some connected boards as sums of smaller subboards. We also show who can win in Tromping for some boards of the form $m \times n$, for $m = 2, 3, 4, 5$ and infinitely many $n$.

Joint work with Saúl A. Blanco, Cornell University.

12. **Shanzhen Gao**, Florida Atlantic University

**Some New Sequences Arising From Self-Avoiding Walks**

A self-avoiding walk (SAW) is a sequence of moves on a lattice which does not visit the same point more than once. It was given as one of the two classical combinatorial problems in the Encylopaedia Britannica. A SAW is interesting for simulations because its properties cannot be calculated analytically. Calculating the number of self-avoiding walks in any given lattice is a common computational problem. We will present some interesting problems involving self-avoiding walks and show how to solve a few of them, as well as some new sequences arising from them.

13. **Anant Godbole**, East Tennessee State University

**Random Additive Bases**

A set $A$ of integers is said to be a 2-*additive basis* for an interval $\varphi(n)$ centered at $n$ if each $m \in \varphi(n)$ can be expressed as $m_1 + m_2$ for $m_1, m_2 \in A$. We investigate when a *random* set $A$ (i) is almost certain to be or (ii) almost never is a 2-additive basis for various choices of $\varphi$. Tools employed include Stein's method of Poisson approximation. Connections are drawn to the sequences studied by Erdős, Tetali, and others.

Joint work with Vince Lyzinski, The Johns Hopkins University.

14. **Ron Graham**, University of California San Diego

**Juggling Drops, Descents and Eulerian Numbers**

In this talk, I will describe some recent results concerning the joint distribution of the number of descents and the maximum drop of a permutation. I will also show how these results interconnect with Eulerian numbers and juggling patterns.

15. **Neil Hindman**, Howard University

**Density Recurrence and Difference Sets**

A subset $B$ of $\mathbb{N}$ is *density recurrent* if and only if, whenever $A$ is a subset of $\mathbb{N}$ which has positive Banach density, there is some $x \in B$ such that $A \cap (-x + A)$ also has positive Banach density. We show that the set $\mathcal{R}$ of ultrafilters on $\mathbb{N}$, every member of which is density recurrent, is a subsemigroup of the Stone-Čech compactification $\beta\mathbb{N}$ of $\mathbb{N}$ containing the idempotents of $\beta\mathbb{N}$ as well as $\{-p + p : p \in \mathbb{N}^*\}$, where $-p = \{-A : A \in p\}$. We show further that if $A$ is a subset of $\mathbb{N}$ which has positive Banach density, then $\mathcal{R} \subseteq \overline{A - A}$, where the difference set $A - A = \{x - y : x, y \in A \text{ and } x > y\}$.

Joint work with Vitaly Bergelson, Ohio State University.

16. **Brian Hopkins**, Saint Peter's College

**Partition Dynamics of the Sand Pile Model**

The Sand Pile Model is perhaps the smallest operation on partitions: in terms of the Ferrers diagram, a single dot moves from one part to the next if the result is still non-increasing. E.g., (4,1) maps to (3,2). Introduced in the computer science literature in 1993, the SPM has been studied primarily for the partitions reached by starting with the single part partition $(n)$. Here we will consider the operation on all partitions of $n$, paying attention to fixed points, number of components, and Garden of Eden states. I will survey known results, present some new ones, and outline open questions.

17. **Doug Iannucci**, University of the Virgin Islands

**Primitive Weird Numbers**

We develop a simple algorithm to find all weird numbers of the form $2^k pq$ for fixed $k$ (such numbers must be primitive weird), and we find the largest known primitive weird numbers (also of this form) by using a Lucas-type primality test.

18. **Eugen J. Ionascu**, Columbus State University

**Platonic Solids in $\mathbb{Z}^3$**

Extending previous results on a characterization of all equilateral triangle in space having vertices with integer coordinates ("in $\mathbb{Z}^3$"), we look at the problem of characterizing all regular polyhedra (Platonic Solids) with the same property. To summarize, we show first that there is no regular icosahedron/ dodecahedron in $\mathbb{Z}^3$. On the other hand, there is a finite (6 or 12) class of regular tetrahedra in $\mathbb{Z}^3$, associated naturally to each nontrivial solution $(a, b, c, d)$ of the Diophantine equation $a^2 + b^2 + c^2 = 3d^2$ and for every nontrivial integer solution $(m, n, k)$ of the equation $m^2 - mn + n^2 = k^2$. Every regular tetrahedron in $\mathbb{Z}^3$ belongs, up to an integer translation and/or rotation, to one of these classes. We then show that each such tetrahedron can be completed to a cube with integer coordinates. The study of regular octahedra is reduced to the cube case via the duality between the two. This work allows one to basically give a description the orthogonal group $O(3, \mathbb{Q})$ in terms of the seven integer parameters satisfying the two relations mentioned above.

19. **Veselin Jungic**, Simon Fraser University

**On a Rado Type Problem for Homogeneous Second Order Linear Recurrences**

In 1997 Harborth and Maasberg considered the recurrence $x_i + x_{i+1} = ax_{i+2}$ and obtained a puzzling sequence of results that have inspired a large portion of the work we will present:

i. If $a = 1$ then any finite coloring of positive integers yields a 4-term monochromatic sequence that satisfies the recurrence.

ii. If $a = 2$ then any finite coloring of positive integers yields arbitrarily long monochromatic sequences that satisfy the recurrence.

iii. If $a = 4$ then any 2-coloring of $[1, 71]$ will produce a monochromatic 4-term sequence that satisfies the recurrence.

iv. For any odd prime $a$ there is a 2-coloring of the set positive integers with no monochromatic 4-term sequence that satisfies the recurrence.

We were intrigued with the question what we can learn about monochromatic sequences that satisfy the recurrence $x_i + x_{i+1} = 2^k x_{i+2}$, $k \geq 3$ or the recurrence $x_i + x_{i+1} = 2kx_{i+2}$, $k \geq 3$. In an attempt to examine these and other related problems, we introduce, for $r \in \mathbb{N}$ and non-zero integers $a, b$, and $c$, a new Ramsey type function $S(r; a, b, c)$ as the maximum $s \geq 0$ such that for any $r$-coloring of $\mathbb{N}$ there is a monochromatic sequence $x_1, x_2, \ldots, x_s$ satisfying the recurrence $ax_i + bx_{i+1} + cx_{i+2} = 0$, $1 \leq i \leq s-2$. We write $S(r; a, b, c) = \infty$ if the corresponding set is not bounded. In this talk we present some results about $S(2; a, b, c)$.

Joint work with Hayri Ardal and Zdenek Dvorak, Simon Fraser University; and Tomas Kaiser, University of West Bohemia in Pilsen.

20. **William J. Keith**, Drexel University

**Recursively Self-Conjugate Partitions**

Recursively self-conjugate partitions are self-conjugate partitions for which the portions above and below the Durfee square are also recursively self-conjugate, including the empty partition. This talk explores some of the properties of the set, such as their lacuna and connection to non-squashing partitions, and considers some questions about them that can be explored further.

21. **Mizan R. Khan**, Eastern Connecticut State University

**A Variation on the Number of Sums and Differences in a Finite Field**

Let $\mathbf{P}(\mathbb{F}_q)$ be the power set of a finite field, $\mathbb{F}_q$, of cardinality$q$, with $q$ odd. We define the arithmetical function $D : \mathbf{P}(\mathbb{F}_q) \to \mathbb{Z}$ via

$$D(S) = \#I\left(x + x^{-1}, S\right) - \#I\left(x - x^{-1}, S\right),$$

where for $S \subseteq \mathbb{F}_q$,

$$I\left(x + x^{-1}, S\right) = \left\{\left(x + x^{-1}\right) \,:\, x \in S\right\}$$

and

$$I\left(x - x^{-1}, S\right) = \left\{\left(x - x^{-1}\right) \,:\, x \in S\right\}.$$

Furthermore, let

$$t_q = \begin{cases} k-1, & \text{if } q = 4k+1 \\ k, & \text{if } q = 4k+3, \end{cases}$$

and let $F(k, l)$ denote the multinomial coefficient of $x^l$ in $(x^{-1} + 6 + x)^k$. Our main result is that

$$\#D^{-1}(\{l\}) = \begin{cases} 2^{t_p}(3F(t_q, l-1) + 10F(t_q, l) + 3F(t_q, l+1)), & q \equiv 1 \pmod 4 \\ 2^{t_q}(F(t_q, l-1) + 3F(t_q, l)), & q \equiv 3 \pmod 4. \end{cases}$$

22. **Abdollah Khodkar**, University of West Georgia

**Super Edge-Graceful Labelings of Complete Bipartite Graphs**

Let $[n]^*$ denote the set of integers $\{-\frac{n-1}{2}, \ldots, \frac{n-1}{2}\}$ if $n$ is odd, and $\{-\frac{n}{2}, \ldots, \frac{n}{2}\} \setminus \{0\}$ if $n$ is even. A super edge-graceful labeling $f$ of a graph $G$ of order $p$ and size $q$ is a bijection $f : E(G) \to [q]^*$, such that the induced vertex labeling $f^*$ given by $f^*(u) = \sum_{uv\epsilon\ E(G)} f(uv)$ is a bijection $f^* : V(G) \to [p]^*$. A graph is super edge-graceful if it has a super edge-graceful labeling. We show by construction that all complete bipartite graphs are super edge-graceful except for $K_{2,2}$, $K_{2,3}$, and $K_{1,n}$ if $n$ is odd.

Joint work with Sam Nolen, Vanderbilt University and James T. Perconti, St. Lawrence University.

23. **Dominic Klyve**, Carthage College

**On the Sum of the Reciprocals of Amicable Numbers**

Two numbers $m$ and $n$ are considered amicable if the sum of their proper divisors, $s(n)$ and $s(m)$, satisfy $s(n) = m$ and $s(m) = n$. In 1981, Pomerance showed that the sum of the reciprocals of all such numbers, $P$, is a constant. Since then, however, no one has given any explicit bounds on $P$. In this work, we obtain both a lower and an upper bound on the value of $P$.

Joint work with Jonathan Bayless, Husson University.

24. **Mitsuo Kobayashi**, Dartmouth College

**On the Density of Abundant Numbers**

Following terminology from antiquity, a natural number is said to be abundant if it is smaller than the sum of its proper divisors. Since Davenport, we know the abundant numbers have a positive asymptotic density, and from Behrend we know that this density is between 0.24 and 0.32. Henri Cohen asked if it could be determined whether it is less than, equal to, or more than 1/4. This was settled by Deléglise when he computed that it is $0.247\ldots$. We will discuss recent improvements to the algorithm of Deléglise which allows us to discover the next decimal digit.

25. **Takao Komatsu**, Hirosaki University

**Arithmetical Properties of Reciprocal Sums of Fibonacci-type Numbers**

Reciprocal sums of Fibonacci numbers of the form $\sum_{n=1}^{\infty} 1/F_n^s$ have been extensively studied by various authors, where $F_n$ is the $n$-th Fibonacci number and $s$ is a fixed positive integer. In this talk we shall consider some arithmetical properties of the sum $\sum_{n=k}^{\infty} 1/F_n^s$ or $\sum_{n=1}^{k} 1/F_n^s$

for a fixed positive integer $k$, and the sums where $F_n$ is replaced by any of Fibonacci-type number.

26. **Mark Kozek**, Whittier College

**An Asymptotic Formula for Goldbach's Conjecture with Monic Polynomials**

Let $f(x)$ be a monic polynomial in $\mathbb{Z}[x]$ of degree $d > 1$. We give a proof that the number $\mathfrak{R}(y)$ of representations of $f(x)$ as a sum of two irreducible monic polynomials $g(x)$ and $h(x)$ in $\mathbb{Z}[x]$, with the coefficients of $g(x)$ and $h(x)$ bounded in absolute value by $y$, is asymptotic to $(2y)^{d-1}$.

27. **Urban Larsson**, Chalmers & University of Gothenburg

**Impartial Games and Random Graphs**

Geography is a commonly played children's game. We explore a variation of this game known as Undirected Vertex Geography (UVG), where, given a finite or infinite graph, two players alternate to slide a coin along the edges. Each vertex may only be visited once. A player who cannot move loses. If the graph is finite, one of the players must have a winning strategy, while if the graph is infinite a third possibility arises: The game can be a *draw*, meaning that no player can force a win. If the graph is generated by a Galton-Watson (GW) process with fixed offspring distribution, a natural question is whether the game (where the root may serve as a starting position) is a draw with positive probability. The degree distribution sequence of the Erdös-Rényi (ER) model is Poissonian. The Karp-Sipsers result for a maximum matching on an ER-graph uses a greedy "leaf-removal" algorithm, which analysis leads to a threshold at (Po-)$\lambda = e$. We establish that the probability for draw of UVG on a Poissonian GW-tree is 0 if and only if $\lambda \leq e$. We generalize UVG by introducing a blocking manoeuvre: Fix a $k \in \mathbf{N}$. Before each next player move, the previous player may block off (at most) $k - 1$ of the next player's options. When the next player has moved any blocked options are forgotten and has no further impact on the game. We denote this game $k$-UVG. Graph theory provides a polynomial time algorithm to find a maximal *partial k-factor* (*matching* if $k = 1$) on a finite graph. We show how this algorithm gives a winning strategy for our games. As another example, we show that, with positive probability, there is a draw of 2-UVG on a GW-tree whenever $\lambda > \phi^2 e^{\phi}$, where $\phi$ denotes the Golden ratio.

Joint work with Johan Wästlund, Chalmers & University of Gothenburg.

28. **Thai Hoang Le**, UCLA

**Intersective Polynomials and the Primes**

Intersective polynomials are polynomials in $\mathbb{Z}[x]$ having roots every modulo. For example, $P_1(n) = n^2$ and $P_2(n) = n^2 - 1$ are intersective polynomials. We show, using results of

Green-Tao and Lucier, that for any intersective polynomial $h$, inside any subset of positive relative density of the primes, we can find distinct primes $p_1, p_2$ such that $p_1 - p_2 = h(n)$ for some integer $n$. Such a conclusion also holds in the Chen primes (where by a Chen prime we mean a prime number $p$ such that $p + 2$ is the product of at most 2 primes).

29. **Jaewoo Lee**, City University of New York(BMCC)

### Geometry of Sumsets

Given a finite set of lattice points, we compare its sumsets and lattice points in its dilated convex hulls. Both of these are known to grow as polynomials. Generally, the former are subsets of the latter. In this talk, we will see that sumsets occupy all the central lattice points in convex hulls, giving us a kind of approximation to lattice points in polytopes.

30. **Joon Yop Lee**, POSTECH, Korea

### Polytope Numbers and their Properties

Polytope numbers are generalizations of polygonal numbers, which are 2-dimensional polytope numbers. In case of polygonal numbers, they can be calculated directly, but in higher dimensions, it is not so easy. Therefore we need think about properties of polytopes. In this talk, after defining polytope numbers, some properties of these will be given. To do this, we will consider properties of polytopes in geometric and combinatorial point of view. And these include polytopal complex, triangulation, decomposition of polytopes, etc.

Joint work with Hyun Kwang Kim, POSTECH, Korea.

31. **Hendrik Lenstra**, Universiteit Leiden

### Finding the Ring of Integers in a Number Field

A classical algorithmic problem in algebraic number theory is to find the ring of integers of a given algebraic number field. The lecture is devoted to a new technique for solving this problem. It does not always work, but if it does, then it writes down the answer in one stroke.

32. **Vsevolod F. Lev**, The University of Haifa

### Translation-stable Sets in $\mathbb{Z}/p\mathbb{Z}$

We prove that there is an absolute constant $c > 0$ with the following property: if $\mathbb{Z}/p\mathbb{Z}$ denotes the group of prime order $p$, and a subset $A \subseteq \mathbb{Z}/p\mathbb{Z}$ satisfies $1 < |A| < p/2$, then for any positive integer $m < \min\{c|A|/\ln|A|, \sqrt{p/8}\}$ there are at most $2m$ non-zero elements

$b \in \mathbb{Z}/p\mathbb{Z}$ with $|(A+b)\setminus A| \le m$. This extends onto prime-order groups the result, established earlier by S. Konyagin and ourselves for the group of integers. We notice that if $A \subset \mathbb{Z}/p\mathbb{Z}$ is an arithmetic progression and $m < |A| < p/2$, then there are exactly $2m$ non-zero elements $b \in \mathbb{Z}/p\mathbb{Z}$ with $|(A+b)\setminus A| \le m$. Furthermore, the bound $c|A|/\ln|A|$ is best possible up to the value of the constant $c$. On the other hand, it is likely that the assumption $m < \sqrt{p/8}$ can be dropped or substantially relaxed.

33. **Shuguang Li**, University of Hawaii at Hilo

### Why should Artin's Conjecture hold?

For a positive integer $n$, let us call the residue classes in group $(\mathbb{Z}/n\mathbb{Z})^*$ with the maximal possible order as primitive roots mod $n$ . Research on the primitive roots for composite moduli has resulted a number of publications and led to discovery of many properties of Dirichlet characters. This presentation will focus on how the properties of Dirichlet characters can be used to investigate Artin's conjecture, which claims that, for a fixed an integer $a$, the number $P_a(x)$ of prime moduli up to x that have $a$ as a primitive root is approximately equal to $A(a)\pi(x)$ where $A(a)$ is some constant depending on $a$, and $\pi(x)$ is the number of primes up to $x$.

34. **Florian Luca**, UNAM

### Multiply Perfect Fibonacci Numbers

Let $\sigma(n)$ be the sum of the divisors of the positive integer $n$. A number $n$ is called *multiply perfect* if $n \mid \sigma(n)$. In my talk, I will show that there are no Fibonacci numbers (larger than 1) which are multiply perfect.

Joint work with K. Broughan (Waikato), M. Gonzalez (U. Simon Bolivar), R. Lewis (R.I.T.), V. J. Mejía Huguet (U.A.M. Azcapozalco) and A. Togbé (Purdue).

35. **Neil Lyall**, University of Georgia

### Optimal Polynomial Return Times

Let $P \in \mathbb{Z}[t]$ with $P(0) = 0$. It is a striking and elegant fact (proved independently by Furstenberg and Sárközy) that any subset of the integers of positive upper density necessarily contains a pair of distinct elements whose common difference is given by $P(t)$ for some $t \in \mathbb{Z}$. Using Fourier analysis we establish quantitative bounds for the following strengthening of this result: *Let $\varepsilon > 0$, then there exists $N_\varepsilon$ such that if $N \ge N_\varepsilon$ and $A \subseteq \{1, \dots, N\}$, then there exists $t \neq 0$ such that $A$ contains at least $|A|^2/N - \varepsilon$ pairs of elements whose common differences are all equal to $P(t)$.* If time permits we will also discuss the problem of finding simultaneous $\varepsilon$-optimal return times for a given collection of polynomials $P_1, \dots, P_\ell \in \mathbb{Z}[t]$.

Joint work with Ákos Magyar, University of British Columbia, Canada.

36. **Karl Mahlburg**, Princeton University

### Asymptotics for Crank and Rank Moments

Moments of the partition rank and crank statistics have been studied for their connections to combinatorial objects such as Durfee symbols, and serve as important examples of hypergeometric q-series that are related to harmonic Maass forms (as are Ramanujan's mock theta functions). In this talk I will discuss the proof of a refined version of a conjecture of Garvan that states that all of the moments of the crank function are asymptotically always larger than the moments of the rank function, even though they share the same main asymptotic terms. The proof uses the Hardy-Ramanujan circle method to provide precise asymptotic estimates for rank and crank moments and their differences.

Joint work with Kathrin Bringmann, University of Cologne and Rob Rhoades, Ecole Polytechnique Federale de Lausanne.

37. **Akihiro Matsuura**, Tokyo Denki University

### Analysis of Recurrence Relations Generalized from the 4-Peg Tower of Hanoi

We generalize the recurrence relations for the 3- and 4-peg Tower of Hanoi problems as follows. $T_1(n, p_1, q_1) = p_1 T_1(n-1, p_1, q_1) + q_1$ $(n \geq 1)$, $T_1(0, p_1, q_1) = 0$, $T_2(n, p_1, p_2, q_1, q_2) = \min_{1 \leq t \leq n}\{p_2 T_2(n-t, p_1, p_2, q_1, q_2) + q_2 T_1(t, p_1, q_1)\}$ $(n \geq 1)$, $T_2(0, p_1, p_2, q_1, q_2) = 0$, where $p_1$, $p_2$, $q_1$, and $q_2$ are real numbers. Then for real numbers $p_1$ and $p_2$ such that $p_1 > 1$ and $p_2 > 1$, it is shown that the sequence of differences of $T_2(n, p_1, p_2, q_1, q_2)$'s, i.e., $T_2(n, p_1, p_2, q_1, q_2) - T_2(n-1, p_1, p_2, q_1, q_2)$, consists of numbers of the form $(q_1 q_2)\, p_1^i p_2^j$ $(i, j \geq 0)$ lined in the increasing order.

38. **Neil McKay**, Dalhousie University

### Dicotic Games, a New Name and New Results

A *dicotic game* is a combinatorial game in which from every position of that game either neither player has a move or both players have at least one move. Under the normal play convention the value of every one of these games is infinitesimal, which is why this class of games was called *all-small* by Conway. However, under misre play, dicots are not necessarily all-small (for a variety of reasons) and so the new term dicotic has been suggested to represent the property we really wish to discuss. New results in the area are presented with examples.

39. **Holly Meeks**, University of West Georgia

### $(a, b)$-Triples

An $(a, b)$-triple is a 3-term sequence of the form $x, ax + d, bx + d$, where $1 \leq a \leq b, d > 0$. Define $T(a, b; r)$ as the least positive integer $n$ such that every $r$-coloring of $[1, n]$ emits a monochromatic $(a, b)$-triple. We give improvements to previously known bounds on $T(a, b; r)$ in certain cases.

Joint work with Patrick Allen, University of Maryland, Baltimore County.

40. **Brendan Nagle**, University of South Florida

### Hypergraph Regularity and Quasirandomness

Several approaches to hypergraph regularity have been developed, and in this talk, we examine a few for 3-uniform hypergraphs. In particular, we consider the concept of $\delta$-*regularity* of Frankl and Rödl, $\gamma$-quasirandomness of Gowers and $\mu$-minimality of Haxell, Rödl and the speaker. Regularity and corresponding counting lemmas for 3-uniform hypergraphs have been established on each of these concepts, where the last concept gave rise to an algorithmic regularity lemma. We present recent work with Poerschke, Rödl and Schacht, which shows these concepts are all equivalent (in the 3-uniform setting), and hence, algorithmic versions of all these lemmas follow.

41. **Rishi Nath**, CUNY

### $p$-core Partitions, Welter's Game and the Symmetric Group: Recent Results

The $p$-blocks of irreducible characters of $S(n)$ are labeled by the $p$-core partitions of $n$. We discuss the interplay between the theory of $p$-cores and the representation theory of $S(n)$ and survey recent results and open questions.

42. **Melvyn B Nathanson**, CUNY (Lehman College and the Graduate Center)

### Phase Transitions in Groups with a Prescribed Infinite Set of Generators

Let $G$ be a group and let $A$ be an infinite set of generators for $G$. The length of an element $x \in G$ with respect to the generating set $A$ is the length of the shortest representation of $x$ as a finite product of elements in $A$ or $A^{-1}$. For every nonnegative integer $r$, the sphere $S_A(r)$ is the set of all elements $x$ in $G$ of length exactly $r$. It is proved that the sphere $S_A(r)$ is infinite for all $r$, or there exists a unique integer $r$ such that $S_A(r')$ is empty for all $r' > r$, $S_A(r')$ is infinite for all $r' < r$, and $S_A(r)$ is nonempty. The integer $r$ is called the phase transition of the pair $(G, A)$ and the set $S_A(r)$ is called the transition set. A complete description of phase transitions and transition sets can be given for the integers and for certain other abelian groups.

43. **Saradha Natarajan**, Tata Institute of Fundamental Research

**Generalizations of a Problem of Pillai**

In 1940, Pillai showed that any set of consecutive integers having 16 or less number of integers satisfies the property that there exists an integer in the set which is coprime to all other integers in the set. We generalize this problem and investigate the existence of an integer $M$ such that for $m \geq M$ certain gcd property for some set of $m$ consecutive integers holds. In some particular cases actual values of $M$ is computed by developing some efficient algorithms.

Joint work with Lajos Hajdu, University of Debrecen, Hungary.

44. **Hoi H. Nguyen**, Rutgers University

**A Strong Inverse Littlewood-Offord Theorem and Applications**

Let $V = \{v_1, \dots, v_n\}$ be a subset of size $n$ of a torsion-free abelian group $G$. Let $\eta_i$ be iid Bernoulli random variables. The *concentration probability* $P_1(V)$ of $V$ is defined as $P_1(V) := \sup_{v \in G} P(\sum_i \eta_i v_i = v)$. About forty years ago, A. Sárkőzy and E. Szemerédi proved that $P_1(V) = O(n^{-3/2})$ for any set $V$ of $n$ distinct real numbers. This result is sharp up to a multiplicative factor. In the early 80's, by using tools from Algebraic Geometry, R. Stanley pointed out that $P_1(V)$ is optimal if and only if $V$ is a symmetric arithmetic progression. In this talk, we shall develop an alternative approach which revisits the result of Stanley in an asymptotic way, and also verifies the stability of the optimal examples. Related applications will be discussed as well.

Joint work with Van Vu, Rutgers University.

45. **Heinrich Niederhausen**, Florida Atlantic University

**Pattern in Paths**

Counting the number of Dyck paths containing $k$ occurrences of a given pattern is fairly well understood - we know what can be counted easily, with more effort, or only with great difficulties. So we turn our attention to different types of paths (Motzkin, Schröder, ...) and combination of patterns. The latter will present new challenges, coming from the interactions of the patterns we are considering.

Joint work with Shaun Sullivan, Florida Atlantic University.

46. **Richard J. Nowakowski**, Dalhousie University

### Loopy Subtraction Games

Take a normal subtraction game (as seen on Children's tv programs) but allow one, and only one, player to pass when faced with a non-empty heap. The resulting games have an arithmetic-periodic structure. If both players are allowed to take 1, then the period is based upon a generalization of Atomic Weight.

Joint work with Neil McKay and Angela A. Siegel.

47. **Kevin O'Bryant**, CUNY - Staten Island

### Generalized Sumsets and Differencesets

For a vector $\vec{u}$ of integers, the $\vec{u}$-formset of a set $A$ is the set $\{\vec{u}\cdot\vec{a} \colon a \in A\}$. This generalizes the sumset and difference set. I will summarize recent progress on generalizing sumset theorems to formsets, and enumerate several outstanding conjectures.

48. **Antonio M. Oller-Marcén**, Universidad de Zaragoza

### A New Look at the Trailing Zeroes of $n!$

An usual exercise in Elementary Number Theory involves computing the number of trailing zeroes of the factorial of a given number. In fact, it is easy to compute, for any positive integer, the number of trailing zeroes of the expansion of its factorial in any given base. Let us denote by $Z_b(n)$ the number of trailing zeroes of the base $b$ expansion of $n!$. Clearly the function $Z_b : \mathbb{N} \longrightarrow \mathbb{N}$ is non-decreasing and it is not surjective. The main goals of this talk are:

(a) To characterize the positive integers where $Z_b$ "jumps" and to compute the amplitude of such "jump".

(b) To give some families of integers not belonging to the range of $Z_b$; i.e., such that they are not the number of trailing zeroes of the base $b$ expansion of any $n$.

49. **Paul Ottaway**, Dalhousie University

### The Short Disjunctive Sum of Games

Combinatorial games are typically played using a disjunctive sum of distinct components. On each player's turn, they choose one of the available components and make a legal move in that component. Traditionally, a player cannot choose a component where a legal move does not exist. We examine the case where this is allowed and call it the short disjunctive sum of games. In particular, we show that both interpretations are equivalent under normal play rules but differ under misère play rules. Finally, we show how this interpretation can be extended to the analysis of both normal and misère play games which have non-standard ending conditions.

50. **James T. Perconti**, St. Lawrence University

**On Ramsey Numbers for Sets Free of Prescribed Differences**

For a positive integer $d$, a set $S$ of positive integers is *difference $d$-free* if $|x-y| \neq d$ for all $x, y \in S$. We consider the following Ramsey-theoretical question: Given $d, k, r \in \mathbb{Z}^+$, what is the smallest integer $n$ such that every $r$-coloring of $[1, n]$ contains a monochromatic $k$-element difference $d$-free set? We provide a formula for this $n$. We then consider the more general problem, where the monochromatic $k$-element set must avoid a given *set* of differences rather than just one difference.

Joint work with Bruce M. Landman, University of West Georgia.

51. **Andreas Philipp**, University of Graz

**Orders in Algebraic Number Fields with Half-factorial Localizations**

Non-principal orders in algebraic number fields are not integrally closed, hence they are never factorial, and their arithmetic depends not only on their Picard groups but also on the localizations at singular primes and on a yet not understood interplay between those two data. In general, not much is known about the arithmetic of non-principal orders, even if all localizations are half-factorial. In this situation, we translate the problem of determining the arithmetic of such a non-principal order to combinatorial questions about some block monoids. These problems are solved with methods from combinatorial number theory. Then - after translating everything back - we get precise results for the elasticity $\rho(\mathcal{O})$, the set of distances $\triangle(\mathcal{O})$, and the catenary degree $\mathsf{c}(\mathcal{O})$ of the non-principal order $\mathcal{O}$.

52. **Paul Pollack**, University of Illinois at Urbana-Champaign/Institute for Advanced Study

**Perfect Numbers and their Friends**

The ancient Greeks were fascinated by *perfect numbers*, numbers like 6 which are the sum of their own proper divisors. They were also enchanted by *amicable pairs*, such as 220 and 284, where each number is the sum of the other's proper divisors. Any investigation of these matters quickly leads one to some of the oldest unsolved problems in mathematics. In this talk we discuss some of the problems that have been considered and give a sort of progress report for the 21st century. Some of what we discuss is joint work with Mits Kobayashi and Carl Pomerance.

53. **Aaron Robertson**, Colgate University

**On the Minimum Number of Monochromatic Solutions of $x + y < z$**

For integers $n \geq 1$ and $k \geq 0$, let $M_k(n)$ represent the minimum number of monochromatic solutions to $x + y < z$ over all 2-colorings of $\{k+1, k+2, \ldots, k+n\}$. We show that for any $k \geq 0$, $M_k(n) = C_1 n^3(1 + o_k(1))$, where $C_1 = \frac{3}{196} - \frac{1}{147}\sqrt{2} \approx .005686$. A structural result is also proven, which can be used to determine the exact value of $M_k(n)$ for given $k$ and $n$.

Joint work with Wojciech Kosek, Colorado College; Dusty Sabo, Southern Oregon University; and Daniel Schaal, Dakota State University.

54. **Michael Rowell**, Pacific University

**Conjugate Durfee Squares and the Finite Heine Transformation**

We introduce the idea of a conjugate Durfee square and use it to answer a combinatorial question regarding a finite form of the Heine transformation posed by G. E. Andrews in a recent paper.

Joint work with Ae Ja Yee, The Pennsylvania State University.

55. **András Sárkőzy**, Eőtvős University, Budapest

**A Finite Erdős-Fuchs Theorem**

A finite analogue of the Erdős-Fuchs theorem is presented: it is proved that if $C_m$ is the additively written cyclic group of order $m$, and $A$ is a "not very small" and "not very large" subset of $C_m$, then denoting the number of solutions of $a + b = n$ with $a$, $b$ belonging to $A$ by $f(n)$, the function $f(n)$ can not be almost constant. The proof combines an elementary-combinatorial lemma with the use of characters. It is also shown that the lower bound given for the deviations is sharp. In the second half of the talk several unsolved problems related to the Erdös-Fuchs theorem will be presented.

56. **Wolfgang A. Schmid**, University of Graz, Austria

**Generalizations of the Davenport constant**

The Davenport constant, $\mathsf{D}(G)$, of a finite abelian group $(G, +)$ is the smallest integer $\ell$ such that each sequence over $G$ has a zero-sum subsequence, i.e., the sum of its terms is the neutral element of $G$. Motivated by a question in analytic non-unique factorization theory, F. Halter-Koch introduced the following generalization of the Davenport constant. For $k$ a positive integer, let $\mathsf{D}_k(G)$ denote the smallest integer $\ell$ such that each sequence over $G$ has $k$ *disjoint* zero-sum subsequences. As shown by C. Delorme, O. Ordaz, and D. Quiroz, these invariants, additionally, are important when applying the inductive method to determine or to bound the classical Davenport constant. Some recent results on these invariants are presented. For instance, for each $G$, the sequence $(\mathsf{D}_k(G))_{k\in\mathbb{N}}$ is eventually an arithmetic progression with difference equal to the exponent of $G$. Moreover, in some special cases the precise values of these invariants are given.

Joint work with Michael Freeze, The University of North Carolina at Wilmington.

57. **James Sellers**, Penn State University

**Elementary Proofs of Parity Results for 5-regular Partitions**

In a paper which appeared in INTEGERS in late 2008, Calkin, Drake, James, Law, Lee, Penniston and Radder use the theory of modular forms to examine 5-regular partitions modulo 2 and 13-regular partitions modulo 2 and 3. They obtain and conjecture various results. In this note, we use nothing more than Jacobi's triple product identity to obtain results for 5-regular partitions which are stronger than those obtained by Calkin and his collaborators. In particular, we find infinitely many Ramanujan-type congruences for $b_5(n)$, the number of 5-regular partitions of $n$, in a straightforward manner relying on an easily-proven relationship between $b_5(4n+1)$ and the number of representations of an integer by the quadratic form $2x^2+5y^2$. This is joint work with Michael Hirschhorn of the University of New South Wales.

Joint work with Michael D. Hirschhorn, University of New South Wales.

58. **Mark Shattuck**, University of Tennessee

**Compression Theorems for Periodic Tilings**

We consider a weighted square-and-domino tiling model obtained by assigning real number weights to the cells and the boundaries of an n-board. An important special case apparently arises when these weights form periodic sequences. When the weights of an nm-tiling form sequences having period m, it is shown that such a tiling may be regarded as a meta-tiling of length n whose weights have period 1 except for the first cell (i.e., are constant). We term such a contraction of the period in going from the longer to the shorter tiling as "period compression." It turns out that period compression allows one to provide combinatorial interpretations for certain identities involving continued fractions as well as for several identities involving Fibonacci and Lucas numbers (and their generalizations).

Joint work with Arthur Benjamin, Harvey Mudd and Alex Eustis, San Diego.

59. **Angela Siegel**, Dalhousie University

**Cutting Down the Numbers**

Cutting down the numbers: Roll the Lawn & Cricket Pitch We consider two option-closed combinatorial games, introduced by Nowakowski and Ottaway [1], called Roll the Lawn and Cricket Pitch. Both games use a row of nonnegative integers (or bumps) and a roller that is placed between any two bumps or at either end. Left (Right) moves the roller to the left (right), flattening each bump it passes over by 1 unless already at 0. At least one bump must decrease in size at each move. The roller is allowed to pass over bumps of height zero in Roll the Lawn, while it may not do so in Cricket Pitch. Nowakowski and Ottaway gave values for all positions of Roll the Lawn and determined outcome classes for Cricket Pitch positions, leaving game values as an open problem. We will utilize ordinal sums and introduce some effective reductions to determine the actual values of the game of Cricket Pitch. [1] R.J. Nowakowski, P. Ottaway. Option Closed Games, submitted 2008.

60. **Pantelimon Stanica**, Naval Postgraduate School

### Independence Number and Spectra of Generalized Petersen Graphs

The *generalized Petersen graph* (GPG) $P(n,k)$ has vertices, respectively, edges given by

$$\begin{aligned} V(P(n,k)) &= \{a_i, b_i, 0 \le i \le n-1\}, \\ E(P(n,k)) &= \{a_i a_{i+1}, a_i b_i, b_i b_{i+k} \,|\, 0 \le i \le n-1\}, \end{aligned}$$

where the subscripts are expressed as integers modulo $n$ ($n \ge 5$), and $k$ is the "skip". We investigate the independence number (maximum cardinality subset of $V(P(n,k))$, such that no two vertices are adjacent) and characterize completely the spectrum of $P(n,k)$.

Joint work with Ralucca Gera, Naval Postgraduate School.

61. **Kate Thompson**, University of Georgia

### Additive Structure in Sparse Difference Sets

Many familiar theorems in various areas of mathematics have the following common feature: *The set of differences from a sufficiently large set contains non-trivial structure.* Specifically, we investigate the appearance of dilations of given sets. Using a slight simplification of the arguments of Croot, Ruzsa and Schoen we are able to establish the following result regarding a natural multidimensional generalization of the concept of an arithmetic progression: If $A \subset [1,N]^d$ with $|A|/N^d \ge CN^{-1/\ell}$, then there exists $r \ne 0$ such that $\{rv_1, ..., rv_\ell\} \subseteq A - A$. Time permitting we may also discuss some polynomial variants of these results.

Joint work with Neil Lyall, Mariah Hamel, and Nathan Walters, University of Georgia.

62. **Lola Thompson**, Dartmouth College

### Heights of Divisors of $x^n - 1$

The height of a polynomial with integer coefficients is the largest coefficient in absolute value. Many papers have been written on the subject of bounding heights of cyclotomic polynomials. One result, due to H. Maier, gives a best possible upper bound of $n^{\psi(n)}$ for almost all $n$, where $\psi(n)$ is any function that approaches infinity as $n \to \infty$. We will discuss the related problem of bounding the maximal height over all polynomial divisors of $x^n - 1$ and give an analogue of Maier's result in this scenario.

63. **Enrique Trevino**, Dartmouth College

### Explicit Bounds for the Burgess Bound for Character Sums

Burgess wrote a series of papers giving inexplicit bounds for short character sums where the Polya-Vinogradov inequality is trivial. The Burgess bound is useful to compute $L(1, \chi)$ and give bounds for the least quadratic nonresidue. Iwaniec and Kowalski provide explicit bounds. Booker gives better bounds for quadratic characters in restricted ranges. I widen the range in Booker's results without cost to his constants and I also get explicit bounds for any non-quadratic character

64. **Kurt Vinhage**, Florida State University

### Van der Waerden Numbers on Finite Gap Sets of Order 3

*Preliminary Report* For a set $D$ of positive integers, let $w_D(k; r)$ denote the least positive integer $n$ (if it exists) such that every $r$-coloring of $\{1, 2, 3 \ldots n\}$ contains a monochromatic $k$-term arithmetic progression of the form $\{x, x + d, x + 2d, \ldots x + (k - 1)d\}$, where $d \in D$. If no such $n$ exists, we say that $w_D(k; r) = \infty$. The values for the case of $|D| = 2$ are well understood. We investigate these values when $|D| = 3$, $w_D(3; 2) = \infty$, and consider cases for which $w_D(2; 3) = \infty$. We show that $w_D(2; 3) < \infty$ whenever $D$ is of the form $\{1, 2, 3k\}$, or $\{d_1, d_2, d_1 + d_2\}$ and $3 \mid d_1, d_2$ or $d_1 + d_2$. An explicit formula is provided for the first case, and an upper and lower bound for the other.

Joint work with Stephen Hardy, McDaniel University and Emily McLean, Georgia Institute of Technology.

65. **Paul Young**, College of Charleston

### Bernoulli Numbers and Generalized Factorial Sums

We prove a pair of identities expressing Bernoulli numbers and Bernoulli numbers of the second kind as sums of generalized falling factorials. These are derived from an expression for the Mahler coefficients of degenerate Bernoulli numbers. As corollaries several unusual identities and congruences are derived, involving the Bernoulli numbers, Bernoulli numbers of the second kind, degenerate Bernoulli numbers, and Norlund numbers.

66. **Jianqiang Zhao**, Eckerd College

### Special Values of Multiple Zeta Functions Associated with Lie Algebras

In this talk we will study the relations between the special values of multiple zeta functions associated to Lie algebras defined by Matsumoto and the special values of multiple polylogarithms at roots of unity (MPVs). The main tools we use are some combinatorial identities, the double shuffle relations among MPVs and the regularized renormalization of multiple zeta values defined by Ihara, Kaneko and Zagier.

Joint work with Xia Zhou.