Math 5985, Special Topics in Mathematics, Problem Solving 1 - Counting, Spring 2017

Course Description: This is the first problem solving course in the area of counting and combinatorics. It is to expose students to middle and high school mathematics contest problems and to help them discover efficient problem solving strategies in counting. **After they learn the basic results** and **tools** in a particular topic in Fundamentals of counting, Counting and probability, Pigeonhole principle, Sequences and series, Proofs by Induction, and Number theory, they are **invited to solve typical problems**, where hints will be provided by the instructor if needed. Students will **gradually be introduced** to various classical problems.

Plans: Choose a set of problems with appropriate difficulty and with common themes in counting and combinatorics from Mathcounts, AMC 8, AMC 10/12, AIME, and/or IMO problems. Students will try the problems individually, then discuss them through a blog(?), where they present and exchange their ideas. After general results are introduced, students apply them to different set of problems within the same topic. By the end, student should be able to combine several concepts to build new tools that help solve more difficult problems.

Also, I plan to have 3 to 4 video presentation assignments where you present your solutions to problems and upload to YouTube or somewhere. (You will have an option to present this to the instructor in person on campus.)

Some Sample Problems:

1. When three fair dice are rolled, what is the probability that the product of the three numbers is a prime number?

2. How many four digit positive integers have their digits in strictly ascending order?

3. What is the least positive integer n such that the value of $\frac{2017!}{n!}$ does not have a unit digit of zero?

4. How many positive integers less than or equal to 100 have the same number of odd factors as even factors?

5. In the expansion of $(1+x+x^2+\cdots+x^{27})\,(1+x+x^2+\cdots+x^{14})^2$, what is the coefficient of x^{28}?

6. A permutation $(a_1,\, a_2,\, a_3,\, a_4,\, a_5)$ of $S_5 = \{1,\, 2,\, 3,\, 4,\, 5\}$ is *heavy-tailed* if $a_1 + a_2 < a_4 + a_5$. What is the number of heavy-tailed permutations?