SEMINAR
CENTER FOR APPLIED MATHEMATICS AND SCIENCE
DEPARTMENT OF MATHEMATICS
UNIVERSITY OF WEST GEORGIA

11:00 AM, THURSDAY 15, 2016, BOYD 306

Speaker: **Dr. Vu Kim Tuan, Department of Mathematics, UWG**

Title: **An inverse problem for the heat equation on a star graph**

Abstract: Consider the heat equation on a simple star graph with three equal edges

$$\begin{cases} \partial_t u^{(j)}(x,t) = \partial_x^2 u^{(j)}(x,t) - q^{(j)}(x)u^{(j)}(x,t) & \text{for } 0 \le x \le a \quad \text{and} \quad j = 1,2,3, \\ u^{(1)}(0,t) = u^{(2)}(0,t) = u^{(3)}(0,t) = 0 & (a) \\ u^{(1)}(a,t) = u^{(2)}(a,t) = u^{(3)}(a,t) & (b) \\ \partial_x u^{(1)}(a,t) + \partial_x u^{(2)}(a,t) + \partial_x u^{(3)}(a,t) = 0 & (c) \\ u^{(j)}(x,0) = f(x). \end{cases} \tag{1}$$

Equation (1) describes a heat process with internal sinks ($q^{(j)}(x) \ge 0$) proportional to the temperature distributions $u^{(j)}(x,t)$. For simplicity we take equal edges in length, and the boundary condition (a) means that the three outer vertices are maintained at temperature zero. Condition (b) describes the fact that they have the same temperature at the common vertex $x = a$ while condition (c) is Kirchhoff's law, which says that the heat transferred through the node $x = a$ is conserved. The function $f \in L^2(0,\ a)$ is the same prescribed initial condition for the three edges. The inverse problem is to recover the heat proportional coefficients $\left\{q^{(1)},\ q^{(2)}, q^{(3)}\right\}$ from readings of the heat flux at the vertices of the graph and of the temperature at the common vertex, which defines a map

$$f \to \left\{\partial_x u^{(j)}(0,t),\ \partial_x u^{(j)}(a,t),\quad u^{(1)}(a,t) \quad \text{for} \quad j = 1,2,3\right\} \qquad \text{for} \quad 0 < T \le t \le T_1 < \infty. \tag{2}$$

We are also interested in finding the best choice for f, that would help uniquely recover all $\left\{q^{(j)}\right\}_{j=1}^{j=3}$ by using only one measurement (2). At the end we shall indicate how we can generalize (1) to deal with more general boundary conditions.

This is a joint work with Dr. Amin Boumenir.
All are welcome.