

**DISCRETE MATHEMATICS SEMINAR**  
CENTER FOR APPLIED MATHEMATICS AND SCIENCE  
DEPARTMENT OF MATHEMATICS  
UNIVERSITY OF WEST GEORGIA

**2:00 - 2:50 PM, FRIDAY, OCTOBER 23, 2015**

**BOYD 306**

**Speaker: Dr. Guantao Chen**

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**Title: On Goldberg's Conjecture**

**Abstract:**

Given a graph  $G$  possibly with multiple edges but no loops, denote by  $\Delta$  the *maximum degree*,  $\mu$  the *multiplicity*,  $\chi'$  the *chromatic index* and  $\chi'_f$  the *fractional chromatic index* of  $G$ , respectively. It is known that  $\Delta \leq \chi'_f \leq \chi' \leq \Delta + \mu$ , where the upper bound is a classic result of Vizing. While deciding the exact value of  $\chi'$  is a classic NP-complete problem, the computing of  $\chi'_f$  is in polynomial time. In fact, it is shown that if  $\chi'_f > \Delta$  then  $\chi'_f = \max \frac{|E(H)|}{\lfloor |V(H)|/2 \rfloor}$ , where the maximality is over all induced subgraphs  $H$  of  $G$ . Goldberg (1973), Andersen (1977), and Seymour (1979) conjectured that  $\chi' = \lceil \chi'_f \rceil$  if  $\chi' \geq \Delta + 2$ . Chen, Gao and Shan showed that if  $\chi' > \Delta + \sqrt[3]{\Delta/2}$  then  $\chi' = \lceil \chi'_f \rceil$ . The previous best known result was for graphs with  $\chi' > \Delta + \sqrt{\Delta/2}$  obtained by Scheide and by Chen, Yu and Zang, independently. It has been shown that Goldberg's Conjecture is equivalent to the following conjecture of Jakobsen: *For any positive integer  $m$  with  $m \geq 3$ , every graph  $G$  with  $\chi' > \frac{m}{m-1}\Delta + \frac{m-3}{m-1}$  satisfies  $\chi' = \lceil \chi'_f \rceil$ .* Jakobsen's conjecture has been verified for  $m$  up to 15 by various researchers in the last four decades. Chen, Gao and Shan showed that it is true for  $m \leq 23$ . Moreover, They showed that Goldberg's Conjecture holds for graphs  $G$  with  $\Delta \leq 23$  or  $|V(G)| \leq 23$ .