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Title: On Goldberg’s Conjecture

Abstract:

Given a graph $G$ possibly with multiple edges but no loops, denote by $\Delta$ the maximum degree, $\mu$ the multiplicity, $\chi'$ the chromatic index and $\chi'_{f}$ the fractional chromatic index of $G$, respectively. It is known that $\Delta \leq \chi'_{f} \leq \chi' \leq \Delta + \mu$, where the upper bound is a classic result of Vizing. While deciding the exact value of $\chi'$ is a classic NP-complete problem, the computing of $\chi'_{f}$ is in polynomial time. In fact, it is shown that if $\chi'_{f} > \Delta$ then $\chi'_{f} = \max \frac{|E(H)|}{|V(H)|/2}$, where the maximality is over all induced subgraphs $H$ of $G$. Goldberg (1973), Andersen (1977), and Seymour (1979) conjectured that $\chi' = \lceil \chi'_{f} \rceil$ if $\chi' \geq \Delta + 2$. Chen, Gao and Shan showed that if $\chi' > \Delta + \sqrt[3]{\Delta/2}$ then $\chi' = \lceil \chi'_{f} \rceil$. The previous best known result was for graphs with $\chi' > \Delta + \sqrt[3]{\Delta/2}$ obtained by Scheide and by Chen, Yu and Zang, independently. It has been shown that Goldberg’s Conjecture is equivalent to the following conjecture of Jakobsen: For any positive integer $m$ with $m \geq 3$, every graph $G$ with $\chi' > \frac{m}{m-1} \Delta + \frac{m-3}{m-1}$ satisfies $\chi' = \lceil \chi'_{f} \rceil$. Jakobsen’s conjecture has been verified for $m$ up to 15 by various researchers in the last four decades. Chen, Gao and Shan showed that it is true for $m \leq 23$. Moreover, They showed that Goldberg’s Conjecture holds for graphs $G$ with $\Delta \leq 23$ or $|V(G)| \leq 23$. 
