SPECIAL AFTERNOON ON DISCRETE MATHEMATICS

CENTER FOR APPLIED MATHEMATICS AND SCIENCE
DEPARTMENT OF MATHEMATICS
UNIVERSITY OF WEST GEORGIA

Friday, November 1st, 2013, Boyd 306

3:00 PM, Gexin Yu, Department of Mathematics, The College of William and Mary, **Permutations as product of parallel transpositions**

Abstract: Let G be a connected graph. Initially, each vertex v of G is occupied by a "pebble" that has a unique destination $\pi(v)$ in G (so that π is a permutation of the vertices of G). It is required that all the pebbles be routed to their respective destinations by performing a sequence of moves of the following type: A disjoint set of edges is selected, and the pebbles at each edges endpoints are interchanged. Define $rt(G, \pi)$ to be the minimum number of steps to route the permutation π and the routing number $rt(G)$ of G to be the maximum of $rt(G, \pi)$ over all permutation π.

We will investigate two conjectures related to the routing numbers of graphs: Strang's conjecture on decomposition of permutation matrix with bandwidth b and Li-Lu-Yang's conjecture on extremal permutations on cycles.

4:00 PM, Dr. Xiaoya Zha, Department of Mathematical Sciences, Middle Tennessee State University, **Isoperimetric Constant of Non-Regular plane Graphs**

Abstract: A plane graph G (finite or infinite) can be a discrete model of the sphere, the flat plane, or the hyperbolic plane. A (d, f)-*graph* is a vertex-d-regular and face-f-regular plane graph. A (d, f)-graph is *spherical* if $1/d + 1/f > 1/2$ (such as the five Platonic solids), *flat* if $1/d + 1/f = 1/2$ (such as the $(3, 6)$, $(4, 4)$ and $(6, 3)$ plane tessellation), and *hyperbolic* if $1/d + 1/f < 1/2$.

The *discrete curvature* of a vertex v of G is $\phi(v) = 1 - deg/2 + \sum 1/|f_i|$ (the summation is taken over all faces incident to v) measures whether the vertex v is positively curved, flat, or negatively curved, and essentially determines whether G is spherical, flat, or hyperbolic. If $1/d + 1/f < 1/2$, then $\phi(v) < 0$, and G is negatively curved at every vertex v, and hence G is hyperbolic and infinite.

The *isoperimetric constant* $\alpha(G)$ of G is the infermum of the ratio of the number of edges of a

cycle over the number of all faces inside the finite region bounded by this cycle. The infermum is taking over all cycles and the finite regions bounded by these cycles. This isoperimetric constant is discrete analogue of the Cheeger's constant of a compact Riemannian manifold. Huguchi/Shiral, and Häggström/Jonasson/Lyons proved that $\alpha(G) = (f-2)\sqrt{1 - \frac{4}{(d-2)(f-2)}}$.

Now assume that G is not vertex-regular and face-regular, instead, G has minimum degree d and minimum face size f. Intuitively G should be more negatively curved at each vertex v if either degree of v is greater than d, or some face incident to v has size greater than f. It is expected that the isoperimetric constant for a (d, f)-graph should be an lower bound for the isoperimetric constant of the non-regular graph G. With Whitlatch we proved this.

We will give a survey on the isoperimetric constant, and outline the proof for the tight lower bound of the isoperimetric constant for non-regular graphs.